DISCUSSION

Some comments on the paper "Microplane constitutive model and metal plasticity" (Brocca M and Bažant ZP, 2000, *Appl Mech Rev* 53(10) 265–281)

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First of all, here are some comments on the microplane model MP1.

On the microplane level, the yield condition is given by the function (5.11) [1]

$$f(\sigma)=\sigma_D^2+\sigma_L^2+\sigma_M^2-k^2=0 \tag{1}$$

It should be stressed that, for obtaining a global yield condition from the local one, some global criterion independent of the chosen coordinate system must be used eg,

$$\max_n f(\boldsymbol{\sigma},\boldsymbol{n})=0 \tag{2}$$

or

$$\int_{S_+} f(\boldsymbol{\sigma},\boldsymbol{n})\,ds(\boldsymbol{n})=c>0, \tag{3}$$

where $\boldsymbol{n}$ is a normal to the unit sphere S, and S_+ is the domain on S where

$$f(\sigma)\geqslant k^2. \tag{4}$$

Criterion (2) defines a surface (in the σ-space) of the first onset of global plasticity, criterion (3)—a surface of more extended plasticity. It is clear that, for $k>0$, the $S_+\subset S$ and S_+ may never coincide with the full unit sphere.

Let us consider the yield criterion of type (1), (2) for a more general function f,

$$f(\boldsymbol{\sigma},\boldsymbol{n})=Y(\boldsymbol{\sigma},\boldsymbol{n})-k^2 \quad Y=a_1\sigma_D^2+a_2(\sigma_L^2+\sigma_M^2), \tag{5}$$

where a_1 and a_2 are some non-negative constants.

Equations (2) and (5) define a family of cylinders in the 6D σ-space.

For the convenience of classification, let us introduce a parameter $\kappa=(\sigma_y^{\pm}/\tau_y)^2$, where $\sigma_y^{\pm}$ and τ_y are the yield points in the uniaxial tension (compression) and shear, respectively.

An analysis of Eqs. (5) and (2) reveals that the following cases are possible [2]:

if $a_1\geqslant a_2\geqslant 0$, then $\kappa=9/4$—the Schmidt cylinder;
if $a_2>a_1>3/4\,a_2$, then $9/4<\kappa<3$—cylinders intermediate between the Schmidt and von Mises cylinders;
if $a_1=3/4\,a_2>0$, then $\kappa=3$—the von Mises cylinder;
if $3/4\,a_2>a_1>0$, then $3<\kappa<4$—cylinders intermediate between the von Mises and Tresca cylinders;
if $a_1=0$, $a_2>0$, then $\kappa=4$—the Tresca cylinder.

Examples:

a)

$$a_1=a_2=1;$$
$$|2\sigma_\alpha-\sigma_{\alpha+1}-\sigma_{\alpha+2}|=\kappa;$$
$$\alpha=1,2,3; \text{ —the Schmidt cylinder,} \tag{6}$$

b)

$$a_1=0,\ a_2=1;$$
$$|\sigma_\alpha-\sigma_{\alpha+1}|=2\kappa;\quad \alpha=1,2,3; \text{ —the Tresca cylinder,} \tag{7}$$

c)

$$a_1=\frac{3}{4},\ a_2=1;$$
$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=6\kappa^2,$$
$$\text{—the von Mises cylinder,} \tag{8}$$

where σ_α, $\alpha=1,2,3$, are the principal stresses.

On the other hand, the integration of Y over the unit sphere S gives

$$\frac{1}{4\pi}\int_S Y(\boldsymbol{\sigma},\boldsymbol{n})\,ds=\frac{1}{15}(2a_1+3a_2)J_2(\boldsymbol{\sigma}) \tag{9}$$

(formula (5.2) [1] follows from here when $a_1=a_2=1$).

However, as mentioned above, only the domains S_+ on the unit sphere where the local yield condition (4) is fulfilled, must be taken into account and therefore S_+ may never coincide with S for $k>0$.

Then the resulting function will also depend on the third deviatoric invariant $J_3(\sigma)$, and the global yield criterion (3) gives

$$\frac{1}{4\pi}\int_{S_+} f(\boldsymbol{\sigma},\boldsymbol{n})\,ds(\boldsymbol{n})=F(J_2,J_3). \tag{10}$$

So the local yield condition (5.1) together with the global criterion (2) defines the Schmidt cylinder [3] or, in the case of (5.1) and (3), the yield condition (10) and does not correspond to the J_2-flow theory.

Finally, we should note that the so called "microplane model version MP2" (5.8) was first put forward by Malmeister in 1955 [4] and was further elaborated by him and by his collaborators and followers in numerous Russian and English papers. We have also considered a number of other models based on general integral representations of arbitrary second-rank tensors. For more details and references see our book [2].

REFERENCES

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Author's Reply to "Some comments on the paper 'Microplane constitutive model and metal plasticity' (Brocca M and Bažant ZP, 2000, *Appl Mech Rev* 53(10) 265–281)," by A Lagzdiņš and V Tamužs

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The discussion by Lagzdiņš and Tamužs is deeply appreciated. It introduces interesting points worthy of consideration.

One point made in the discussion is that the microplane model MP1—the first of the three versions of the microplane model examined by numerical simulations— is not equivalent to the J_2 theory of plasticity if yielding does not occur on the microplanes of all possible orientations, and that the third invariant J_3 must also influence the response. This is of course true. But this fact was not really contradicted in the paper. Indeed, even though the dependence on the third invariant was not explicitly mentioned in the paper, it was stated below Eq. (5.5) that "after yielding saturation, the performance of this model (ie, MP1) is very similar to that of the usual models of the J_2 flow theory." This statement implies that before the yielding saturation (ie, before the yielding occurs on all the microplanes), the response is not equivalent to the J_2 theory, which is roughly sketched in Fig. 5.1. This in turn implies that the response must also depend on the invariants J_3 and I_1, of which the latter must be ruled out since the response is insensitive to pressure.

The discussers' statement below their Eq. (9), namely that "only the domain S_+ on the unit sphere...must be taken into account" (in the integration), also does not conflict with the microplane model formulation in the paper. Although the integration is always carried out over the complete unit hemisphere, the contributions to the integral from the planes with no plastic strains vanish, which is equivalent to deleting from the integration the part of the surface other than what the discussers label S_+. Numerical computations also confirm that the asymptotic case of yielding on all the microplanes (ie, yielding over the entire surface of the unit hemisphere) is hardly ever achieved, which is what the discussers intuitively suggest (the gradual approach to the J_2 asymptote, pictured in Fig. 5.1, documents this behavior graphically). Nevertheless, the simple asymptotic case of J_2 theory can be approached quite closely, and so its knowledge is not useless.

The discussers' Eq. (5) represents a possible generalization of the model labeled MP1. Although this generalization seems purely phenomenological and intuitive, it may be worthwhile to investigate its data fitting capability numerically. The comments on the meaning of this possible generalization in the discussers' Eqs. (6)–(8) provide an interesting geometrical perspective.

The authors wish to thank the discussers for sending them a copy of the Russian proceedings article by Malmeister (1955), which has been difficult to obtain in the US. Indeed, Malmeister's yield condition for a slip plane, reproduced in the discussion as Eq. (1), is the same as that adopted for microplane model MP1. However, the usefulness of this particular alternative examined in the numerical simulations in the paper is questionable because this alternative is found to exhibit no vertex effect and to be incapable of fitting the test data in Fig. 6.2. It does not represent a recommended version of the microplane model.

Finally, the authors are glad to know that Malmeister (1955), independently of the earlier work of Batdorf and Budianski (1949) referenced in the paper, proposed determining the plastic strain on what is now called the microplanes from the shear stress resultant on each particular microplane, as adopted for the microplane model version MP2. However, unlike Batdorf and Budianski, Malmeister (in his article) did not investigate this version numerically and did not study with his model the vertex effect which was recognized already in the 1950s as a phenomenon of paramount importance. Also, he did not introduce other salient characteristics of the microplane model, such as an optimal Gaussian numerical integration on the hemispherical surface and the variational relation between the microplane vectors and the continuum tensors, which are essential for more effective generalizations.

The discussers' interest is very welcome. Hopefully, it will stimulate further progress.