Examining tendencies of in-plane rupture to migrate to material interfaces

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\textbf{SUMMARY}

We perform a numerical parameter-space study of 2-D in-plane ruptures in a model consisting of two different half-spaces separated by a low-velocity layer and possible simultaneous slip along multiple faults. Ruptures are nucleated by a bilateral expanding stress drop in a limited source region, and may continue to propagate spontaneously (or not) along one or several faults. Most calculations are done for purely elastic media and faults governed by Coulomb friction, but some simulations employ Prakash–Clifton friction and Kelvin-Voigt viscosity. The faults, two of which are material interfaces, are situated equidistant and parallel to each other. Using different nucleation locations, different initial stress, different velocity contrasts, different frictional fault separations, different widths of a low-velocity zone, and different number of faults, we examine the range of conditions for which ruptures migrate to other faults and continue to propagate in a self-sustaining manner. The model produces diverse migration and propagation phenomena represented by several phase diagrams. However, a general result of the study is that ruptures tend to migrate to the material interfaces and become self-sustained wrinkle-like pulses for wide ranges of conditions. The wrinkle-like pulses propagate along each material interface unilaterally in the direction of motion on the more compliant side of the interface (referred to as the ‘positive’ direction). The existence of a large number of faults produces, like viscosity, distributed deformation that reduces the divergent behaviour of the wrinkle-like pulses. In many cases, ruptures migrate to the interface with the stronger contrast and propagate unilaterally in the positive direction associated with that interface and the overall contrast across the fault zone. In smaller number of cases, ruptures migrate to the interface with the weaker contrast and propagate unilaterally in the opposite positive direction associated with that interface. For various parameter combinations, self-sustained unilateral pulses travel simultaneously, in the two opposite positive directions, along the interfaces on the opposite sides of the low-velocity layer. A low-resolution imaging of these ruptures would lead to an inference on bilateral propagation. The M6, 2004 September, Parkfield California earthquake may provide a natural example of such a case.

\textbf{Key words:} dynamic rupture, fault-zone structure, friction, material interfaces, numerical simulations, rupture migration.

\section{1 INTRODUCTION}

Active faulting over geological time brings into contact materials that were originally separated and are thus likely to have different elastic properties. In some cases, large faults nucleate and grow along a pre-existing suture that separates different lithologies. Recent geological mapping in the structure of several large strike slip fault zones indicates that the principal slip zones of large earthquakes are localized along interfaces that separate rock units with considerably different properties (Dor et al. 2006a,b). The slip zones in lab experiments that include material interfaces (e.g. due to the presence of a gouge layer) also tend to localized along such interfaces. Contrasts of elastic properties across large faults have been imaged by seismic reflection and refraction studies (e.g. Fuis et al. 2001, 2003; Lutter et al. 2004), body and coda wave tomography (e.g. Eberhart-Phillips & Michael 1998; Magistrale & Sanders 1995; Shapiro et al. 2005), modelling of geodetic data (Le Pichon et al. 2005) and analysis of head waves that refract along material interfaces in the fault zone structure (Ben-Zion & Malin 1991; Ben-Zion et al. 1992; McGuire & Ben-Zion 2005). Estimates of the seismic
velocity contrasts across the San Andreas and other large faults range from a few percent to more than 30 per cent, depending on the geographical location and resolution of the employed imaging method. In addition, the faulting process produces in the top few km of the crust localized belts of damaged fault zone rocks that act as trapping structures for seismic waves (e.g. Ben-Zion et al. 2003; Peng et al. 2003; Fohrmann et al. 2004; Lewis et al. 2005) and are also manifested by fault-related anisotropy (e.g. Zhang & Schwartz 1994; Peng & Ben-Zion 2004). In some cases, the opposite sides of a fault are separated by a layer (sliver) of rock with faster velocity than one or both sides of the fault. This holds, for example, for a portion of the Bear valley section of the San Andreas fault (SAF) (McGuire & Ben-Zion 2005).

There are fundamental differences between properties of dynamic ruptures on faults that do or do not separate different elastic solids. On a planar fault between solids with identical elastic properties, there is no coupling between slip and changes of normal traction. On the other hand, mode II (in-plane) rupture along a material interface generates local changes of normal stress that are proportional to the spatial derivative of slip (Weertman 1980; Adams 1995; Ben-Zion 2001; Ranjith & Rice 2001). This produces dynamic dilation at the tip that propagates in the direction of slip on the more compliant side of the fault (referred to below as the ‘positive’ direction) and dynamic compression at the tip propagating in the opposite direction (referred to as the ‘negative’ direction). The magnitudes of these effects increase with the rupture velocity and the degree of velocity contrast across the fault, up to about 30–40 per cent contrast beyond which the generalized Rayleigh wave speed $C_{GR}$ does not exist (e.g. Weertman 1980; Ben-Zion & Andrews 1998; Ben-Zion 2001). In addition, the dynamic changes of normal stress increase with propagation distance along the material interface due to a dynamic instability (Adams 1995, 1998) that produces a continual transfer of energy to shorter wavelengths during rupture propagation. The Adams (1995, 1998) instability reduces dynamically the physical length scales such as the width of the rupture pulse or the crack-tip region with high slip velocity. This leads to a grid-size dependency in simulations with purely elastic materials and instantaneous Coulomb-like relation between the frictional strength and normal stress (e.g. Cochard & Rice 2000; Ranjith & Rice 2001; Ben-Zion & Huang 2002). The Adams instability can be regularized using a Prakash–Clifton friction law (Prakash & Clifton 1993; Prakash 1998) with a gradual response to changes of normal stress (Cochard & Rice 2000; Ben-Zion & Huang 2002), using artificial viscosity that damps short wavelength features (S. Day, personal communication, 2003), incorporation of plastic yielding off the fault (Ben-Zion & Shi 2005), and other mechanisms that suppress the development of small-scale aspects of the response (e.g. Ben-Zion 2001). However, the inclusion of any such ingredient changes the physical problem that is being solved and increases the number of model parameters.

Previous analytical and numerical parameter-space studies indicate (Weertman 1980; Adams 1995; Andrews & Ben-Zion 1997; Ben-Zion & Andrews 1998; Ben-Zion & Huang 2002; Cochard & Rice 2000; Ranjith & Rice 2001; Shi & Ben-Zion 2006) that mode II rupture along a material interface with slip-independent friction can propagate in a self-sustaining manner, for ranges of frictional parameters, material contrasts, and stress/strength heterogeneities, as a unidirectional ‘wrinkle-like’ pulse that propagates in the positive direction with a speed close to $C_{GR}$. Adams (2001), Ranjith & Rice (2001) and Cochard & Rice (2000) showed that slip pulses with a velocity near that of the slower $P$ wave can also propagate along a material interface in the negative direction. However, these pulses are considerably weaker than the primary wrinkle-like pulses in the positive direction and are unlikely to evolve to self-sustaining ruptures. Characteristic features of the wrinkle-like pulse include: (1) strong correlation between variations of normal stress and slip, (2) strongly asymmetric motion across the fault, (3) self-sharpening with propagation distance and, (4) preferred direction of rupture propagation. Cochard & Rice (2000) and Ranjith & Rice (2001) suggested that the regularized Prakash–Clifton friction law can suppress the divergent behaviour associated with feature (3). However, subsequent calculations for large propagation distance (Ben-Zion & Huang 2002) and analytical work (Adda-Bedia & Ben Amar 2003) showed that the divergent behaviour persists even with the Prakash–Clifton friction. Ben-Zion & Huang (2002) found that the parameters of the regularized Prakash–Clifton friction law have to be fine-tuned to produce (with a fixed set of material properties) apparent stability for a given propagation distance, rendering that friction law unsuitable for a systematic parameter-space study.

Numerical simulations of rupture along a material interface governed by slip-weakening friction produced results that depend strongly on the nucleation procedure. The employed procedures go generally to the following two classes (Ben-Zion 2006a,b). Class (I) is associated with relatively small and strong nucleation phases mimicking the initiation of a cascade-type process by a failure of a strong asperity (e.g. Andrews & Ben-Zion 1997). Such cases generate for wide ranges of frictional and material contrast ruptures that evolve with sufficient propagation distance to wrinkle-like pulses similar to those generated with slip-independent friction (Shi & Ben-Zion 2006). Class (II) studies with slip-weakening friction involves relatively large nucleation phases mimicking the final stage of a quasi-static growth of a slip patch to a critical size needed to produce macroscopic dynamic instability in a homogenous medium. The associated critical patch size scales in laboratory experiments with the largest wavelength of the roughness characterizing the sliding surfaces (e.g. Ohnaka 1996). Such cases generate bilateral cracks with a wrinkle-like pulse superposed at the tip propagating in the positive direction (e.g. Harris & Day 1997). The superposed wrinkle-like pulse produces higher slip velocity near the tip propagating in the positive direction than at the other rupture front. Recent simulations with fine resolution of sharp dynamical features generated (Rubin & Ampuero 2006) very prominent asymmetry of slip velocities at the opposite rupture fronts. In those calculations, the strong asymmetry of slip velocities at the opposite crack tips cannot manifest itself into macroscopic rupture asymmetry. However, incorporating in the simulations rate-dependent friction compatible with experiments of rock friction at high slip rates (e.g. Tsutsumi & Shimamoto 1997; Di Toro et al. 2004) is expected to produce larger stress drop in the positive direction, leading to asymmetric rupture with larger energy release in the positive direction (Ben-Zion 2006b). This expectation should be tested in a future work.

On polished man-made interfaces, having roughness only over very short wavelengths, dynamic instabilities are likely to be initiated by nucleation phases of class (II). On large fractal-like surfaces, however, class (I) may be realized first since the required critical patch of class (II) is essentially the size of the entire surface. On realistic natural surfaces with roughness over broad bandwidth, a nucleation phase of class (II) in a small spatial domain may trigger a stronger nucleation phase of class (I). The same may hold for other types of strength heterogeneities. These cases are likely to excite on a material interface the wrinkle-like mode of rupture (Ben-Zion 2006b). Clarifying the ability of different nucleation
phases to excite different modes of rupture, and the relation of the various proposed nucleation mechanisms to natural faulting, are important topics for continuing theoretical and observational studies. Here we simply note that rupture on a material interface tends to evolve, for realistic classes of nucleation mechanisms and constitutive laws, to a wrinkle-like pulse with properties similar to those associated with the simple Coulomb friction.

One shortcoming of all previous works on this topic is that the path of rupture propagation was prescribed rather than being allowed to develop spontaneously. While material interfaces are mechanically efficient failure surfaces due to the dynamic reduction of normal stress in the positive direction, it is not clear for which conditions ruptures that start in the bulk would migrate on their own to material interfaces. In this paper we test with a well-defined model, having a typical fault-zone velocity structure, how unstable slip on multiple possible faults localize as a pure material contrast effect. Resolution of this issue is important to clarifying whether the remarkable dynamic phenomena associated with rupture along material interfaces occur only for a (perhaps small) subset of ruptures with hypocentres at the interface, or whether they also tend to occur in the more general case of hypocentres in a volume surrounding material interfaces. We address this issue by performing a numerical parameter-space study in a model having a compliant layer between two different elastic blocks and a number of possible rupture planes, two of which are material interfaces. Ruptures are nucleated in different positions within the compliant layer or in the surrounding blocks and we examine the conditions (velocity structure, initial stress, assumed rheology) for which ruptures migrate spontaneously to a material interface.

The results of Ben-Zion & Huang (2002) and Shi & Ben-Zion (2006), persistence of features (1)–(4) of the wrinkle-like pulse in simulations with Coulomb, Prakash–Clifton, and slip-weakening friction laws, and the foregoing discussion indicate that the simple Coulomb friction provides a useful constitutive law for exploring general tendencies associated with rupture along a material interface. Given the complexity of the problem and large number of possible parameters, most simulations of this work employ the simple Coulomb friction and purely elastic materials. Some simulations are performed with Kelvin–Voigt viscosity in the bulk or the regularized Prakash–Clifton friction on faults.

The results show that ruptures tend to migrate spontaneously to the material interfaces and continue to propagate there in a self-sustaining manner for broad ranges of conditions. In some cases, ruptures migrate simultaneously to the material interfaces on the opposite sides of the low-velocity layer and propagate along both interfaces in the two (opposite) positive directions, creating together an apparent bilateral rupture. In other cases, ruptures propagate in the positive direction along the interface with weaker material contrast, which is the opposite preferred direction associated with the overall contrast across the fault. Low-resolution imaging of such cases may be interpreted erroneously as violating the prediction of a preferred propagation direction along a material interface.

2 METHODS AND MODEL SET-UP

2.1 Finite-difference method and fault model

We perform 2-D numerical simulations of in-plane rupture in a model (Fig. 1) consisting of two different half-spaces separated by a fault zone layer, which is the medium with lowest seismic velocity, and a number of possible rupture planes. As noted by Ben-Zion (2006a), the 2-D in-plane calculations may be understood to represent ruptures that already saturated the seismogenic zone of a strike-slip fault and continue to propagate as in-plane ruptures. Modelling the initial transient regime, when small earthquakes grow as mixed in-plane and anti-plane ruptures, requires 3-D calculations. Some results associated with this transient mixed-mode regime can be found in Harris & Day (2005).

The simulations employ a version of the code used by Andrews & Ben-Zion (1997), Ben-Zion & Andrews (1998) and Ben-Zion & Huang (2002) that can account for simultaneous rupture on multiple faults. The calculations are based on a staggered velocity-stress finite-difference formulation of the governing elastodynamic equations on a triangular grid. Frictional sliding on a set of predetermined possible rupture planes is calculated using the traction at split-node sliding logic described by Andrews (1973). If the ratio of the x- and y-traction components exceeds the static coefficient of friction on a fault, slip begins and continues until the slip velocity tends to change sign. The frictional strength is proportional to the compressive normal stress and most calculations employ a constant coefficient of friction (i.e. Coulomb friction). The set of possible fault planes are defined along the space dimension (the x coordinate) parallel to the material interfaces, which are two possible rupture plane. The other faults are situated equidistantly from each other across the material interfaces, so that each medium has a number of possible rupture planes.

In principle the number of faults that can be defined in our code is limited by the number of gridpoints along one dimension. However, in most of our calculations the number of frictional interfaces is fixed to nine faults: four faults outside the fault zone layer (two on each side), three faults inside the fault zone layer, and two faults on the material interfaces. We use the following different fault-separations $y_{fs} = 7, 14, 21$ m. The width of the fault zone layer is coupled to the fault separation as $4y_{fs} = y_{id}$, so $y_{id} = 28, 56, 84$ m. The initial shear stress varies between 64 MPa and 72 MPa and the nucleation locations are $y_{nuc} = \{\text{fault 1, fault 2, \ldots , fault 9}\}$. In all cases we use the following two sets of velocity contrasts: $\Delta v_1 = \{v_{y2}/v_{y1}, v_{x2}/v_{x1}\} = \{0.85, 0.94\}$ and $\Delta v_2 = \{v_{y2}/v_{y1}, v_{x2}/v_{x1}\} = \{0.75, 0.90\}$. The range of the investigated parameters is summarized in Table 1.

We ensure that all results shown in this study are free of artificial reflections or wrap around from the model boundaries by always choosing the model big enough for the investigated time interval.
2.2 Nucleation procedure

To prevent a bias for rupture propagation direction by the nucleation procedure, we nucleate each event bilaterally and symmetrically by increasing the fluid pressure in two limited space–time regions. This generalizes the nucleation procedure of Andrews & Ben-Zion (1997) of a travelling drop of normal stress to the symmetrically expanding case, that is, two drops of normal stress that propagate within the nucleation region in the opposite directions. Using a syntax similar to the one of Andrews & Ben-Zion (1997), the coordinates for the two pulses travelling in the opposite directions are

\[ x = (|x| - v_{\text{nuc}} t) a, \quad \eta = (|x| + v_{\text{nuc}} t) b - \eta_0, \quad \eta_0 = \sqrt{\alpha^2 + \beta^2} b, \]

and the boundary of the two sources are the ellipses \( 1 - \xi^2 - \eta^2 = 0 \). Within the two elliptical sources, the fluid pressure is given as

\[ P_f = P_0 (1 - \xi^2 - \eta^2)^2, \]

while outside those regions it is zero. The width of the pulse and the overall size of the nucleation zone are denoted as \( v_{\text{nuc}} \leq 2a \) and \( \Omega_{\text{nuc}} = 2b \).

Rupture along a material interface tends to propagate at the generalized Rayleigh wave speed \( C_{GR} \), which maximizes the interaction between in-plane slip and dynamic changes of normal stress (e.g. Weertman 1980; Ben-Zion 2001; Ranjith & Rice 2001). The generalized Rayleigh wave is a phase that propagates along a material discontinuity interface and reduces to the regular Rayleigh wave when the two materials are the same (Weertman 1980; Weertman 1980). The nucleation velocity \( v_{\text{nuc}} \) in our study is taken to be close to \( C_{GR} \) of the strongest material contrast. We tested the sensitivity of the model response to different nucleation velocities and found that with nucleation velocities substantially less than \( C_{GR} \), self-sustaining pulses on the interfaces always start travelling with a speed close to the generalized Rayleigh velocity. Additional results related to this issue can be found in Shi & Ben-Zion (2006). Since the main focus of the study is to examine migration patterns of ruptures, we conduct a large number of simulations in which the nucleation zone, dictating the initial rupture position, is varied systematically across the structure.
In cases where the wrinkle-like rupture pulse is self-amplifying (Ben-Zion & Andrews 1998; Ben-Zion & Huang 2002), this convergence is limited due to the growing instability beyond a certain propagation distance. However, increasing the viscous component of deformation can always suppress the self-amplification and lead to dying rupture pulses only. The existence of either self-amplifying or dying pulses, and required fine-tuning for a given propagation distance, is similar to the behaviour encountered with material interface governed by the Prakash–Clifton friction (Ben-Zion & Huang 2002). Exploratory calculations for sets of cases with Coulomb frictional interfaces in the general model configuration of Fig. 1 show that general aspects of the solution (self-sharpening, approximate rupture velocity, overall tendencies of rupture migration) are the same with and without viscosity in the bulk. To reduce the number of parameters, we use in most subsequent calculations Coulomb frictional interfaces without viscosity in the bulk. Since the calculations have grid size dependency, we focus only on general tendencies and relative (rather than absolute) amplitudes of effects.

3.2 Effects of multiple faults

For most of our simulations we use a fixed amount of nine equidistant parallel faults. The simultaneous sliding on multiple faults produces distributed slip that mimics viscosity in the bulk and leads to a weaker rupture behaviour. This is illustrated here with results for three simulations with 9, 17 and 33 parallel faults (Fig. 5). Each frictional interface has a Coulomb friction coefficient of 0.75 and initial stress components $\tau^\infty = 70$ MPa, $\sigma^\infty = 100$ MPa. As in all our simulations with multiple faults, each frictional surface is able to rupture spontaneously, independently of the neighbouring faults. Fig. 5 shows how the number of faults affects the rupture ability to migrate between the different interfaces and propagate along the material interfaces in a self-sustaining manner. In general, as the number of faults increases, the likelihood of ruptures to migrate and develop self-sustaining pulses decreases due to dissipation of elastic energy.

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strain energy associated with the slip on the multiple faults. This is similar to the influence of the Kelvin-Voigt viscosity, which also leads to energy dissipation and suppresses the ability of ruptures to develop to self-sustaining pulses. A model configuration with nine faults (Fig. 5 top) produces two rupture pulses travelling in the opposite positive directions along the two material interfaces. We note that such a case would be seen in a low-resolution imaging as a bilateral rupture. Keeping all the model parameters the same but increasing the number of possible faults to 17 (Fig. 5 centre) produces only a single rupture pulse propagating in the positive direction on the stronger material contrast. A system with 33 faults (Fig. 5 bottom) does not produce self-sustaining rupture pulses due to the dissipation of the initial stored elastic strain energy at the multiple faults.

3.3 Prakash–Clifton friction

In this section we present example results with purely elastic media and frictional sliding governed by the Prakash–Clifton friction (Prakash & Clifton 1993; Prakash 1998) with a gradual response to changes of normal stress (Fig. 6). A simplified version of the Prakash–Clifton friction that can regularize (Cochard & Rice 2000; Ranjith & Rice 2001; Ben-Zion & Huang 2002) the grid-size dependency associated with the Adams (1995) instability can be expressed as
\[
\dot{\tau}_{s}^{x} = -((|V| + V^*) - f \max(0, -\sigma_{yy})/L,
\]
where \(\tau_{s}^{x}\) is the frictional strength, \(V\) is the slip velocity, \(f\) is the friction coefficient, \(V^*\) is a characteristic slip velocity for friction evolution and \(L\) is a corresponding characteristic slip distance.

Fig. 7 shows migration patterns associated with different nucleation positions (red rectangular) across a fault zone structure having three materials and nine possible faults (Fig. 1). Each panel gives the spatial distribution of the maximum slip velocity in a given simulation. The employed conditions are material contrast \(\Delta v_{2}\), initial shear stress \(\tau_{\infty} = 70\ \text{MPa}\), fault separation \(y_{fs} = 21\ \text{m}\), characteristic slip distance \(L = 0.003\ \text{m}\), and characteristic slip velocity \(V^* = 1\ \text{m s}^{-1}\). In this and other calculated examples, self-sustained pulses are produced for nucleation locations that are within or at the boundaries of the low-velocity fault zone layer. The ruptures tend to migrate spontaneously to the material interfaces (fault 3 or fault 7) and become there self-sustained wrinkle-like pulses. Both material interfaces act as attractors for the ruptures, which become wrinkle-like pulses in the (opposite) positive directions associated with the local velocity contrasts across the fault zone layer, and can produce jointly apparent bilateral ruptures. The general aspects of the migration and propagation patterns of ruptures are similar to those that are produced with the classical Coulomb friction (Fig. 5 and Section 4).

3.4 Choice of parameters for subsequent simulations

As discussed in the previous sections, the use of the Prakash–Clifton friction and Kelvin-Voigt viscosity can regularize the grid-size dependency associated with the Adams (1995) instability by suppressing the development of sharp dynamical features. However, the parameters of both the Prakash–Clifton friction and Kelvin-Voigt viscosity have to be fine-tuned to provide apparent stability over a
given propagation distance (for a fixed set of model parameters). This renders the use of the Prakash–Clifton friction and viscosity unsuitable for the present work, where the main focus is to examine systematically tendencies of rupture migration for different cases of velocity models and multiple parallel faults. Simulations with material interfaces governed by Coulomb friction and purely elastic media are subjected to grid-size dependency. However, as pointed out by Ben-Zion (2001) and illustrated by Ben-Zion & Huang (2002) and above, such results can be used to obtain a general understanding of rupture behaviour by performing a careful comparative study. In the next section we use a model with three purely elastic materials and Coulomb frictional interfaces, and examine the overall large-scale properties of ruptures for various cases of velocity contrasts, initial shear stress and fault separation.

4 RESULTS ON RUPTURE MIGRATION

The parameter-space associated with a model consisting of nine Coulomb frictional faults and three elastic materials (Fig. 1) is quite complex. For this reason we first investigate in Section 4.1 a model of three materials and two frictional interfaces only. Results associated with the more general case of nine faults are discussed in Section 4.2. When a rupture pulse propagates throughout the analysed model region (850 m from nucleation), we refer to the initiated rupture as self-sustained. We distinguish between triggered rupture on parallel faults and migrated self-sustained rupture pulses by examining the maximum amplitude of slip velocity on the faults. A triggered rupture has much smaller slip velocity than that of the self-sustained rupture, although the amount of triggered slip close to the driving fault can be high.

4.1 Two-fault system

Here we consider the response of a model with three elastic media and two Coulomb frictional faults located on the two material interfaces. This is an extension of the studies of Ben-Zion & Huang (2002) and Harris & Day (1997) on the influence of a low-velocity fault zone layer on rupture dynamics. In contrast to those previous studies, our model allows simultaneous sliding on both sides of the low-velocity layer.

We use the set of material contrasts $\Delta v = \{v_{\text{sl}}/v_{\text{sa}}, v_{\text{sy}}/v_{\text{sa}}\} = \{0.85, 0.94\}$. These are realistic contrasts for natural faults and in the range for which $C_{\text{GR}}$ exists (Weertman 1963; Achenbach & Epstein 1967; Ben-Zion 2001; Ranjith & Rice 2001). In such cases an increase of velocity contrast leads to an increase in the strength of the wrinkle-like pulse on the material interface (Ben-Zion & Andrews 1998; Ben-Zion & Huang 2002). The friction coefficient is $f_c = 0.75$, the initial normal stress is $\sigma_0 = 100$ MPa and the initial shear stress is $\tau_0 = 70$ MPa. The nucleation procedure is as described in Section 2.2 and is applied to both faults in two sets of simulations. Below we discuss results of twelve simulations associated with six different widths of the low-velocity layer: $y_{\text{hl}} = 1.75, 3.5, 7, 14, 28, 56$ m, and two different nucleation locations (fault 1 associated with the stronger contrast, and fault 2 associated with the lesser contrast).

Fig. 8 shows the maximum slip velocities on the two faults for the twelve simulations. The slip velocities are unrealistically high since the calculations are done for purely elastic media. An incorporation of plastic yielding off the fault (Ben-Zion & Shi 2005) or viscous component of deformation (Section 3) will limit the slip velocities and stabilize the divergent behaviour of the wrinkle-like pulses. The results exhibit the following competing effects:

(i) A low-velocity layer with width on the order of the nucleation size makes it easier for ruptures to migrate from one fault to the other.

(ii) When the width of the low-velocity layer becomes considerably smaller, the overall material contrast of the two surrounding half-spaces dominates. In this case the fault on the weaker material contrast cannot sustain dynamic ruptures, while the fault on the stronger material contrast has a pulse with reduced strength. In some such cases, a small amount of slip on the interface with the weaker contrast suppresses the development of self-sustained slip on the other interface.

(iii) Once the width of the low-velocity layer becomes large enough, each material contrast between a bounding medium and the low-velocity layer acts like a single material interface, and the existence of the other frictional material interface has little effect.

The differences in amplitude and the dying versus amplifying behaviour in all simulations are consistent with the competing effects (i)–(iii) above. As examples, for a very narrow low-velocity layer $y_{\text{hl}} = 1.75$ m (Fig. 8a), the opposing influences of the two material contrasts inhibit rupture propagation on both faults. The overall contrast of 6 per cent between the two half-spaces would in general support rupture propagation on a single fault (e.g. Ben-Zion & Huang 2002; Shi & Ben-Zion 2006). However, in the simulated case with simultaneous sliding on two faults, self-sustaining rupture does not propagate even on fault 1 having a 15 per cent material contrast. Comparing the results for the two different nucleation locations (fault 1 (top), fault 2 (bottom) of $y_{\text{hl}} = 7$ m (Fig. 8c), we see that nucleating on fault 1 can initiate dynamically propagating rupture on fault 2, but nucleating on fault 1 cannot initiate dynamically propagating rupture on fault 2. This is consistent with the fact that fault 2 is a weaker material contrast and (as already mentioned) we are in the range in which the instability increases with increasing contrast. Comparing cases of nucleating on fault 1 for widths of the low-velocity layer of $y_{\text{hl}} = 7$ m and $y_{\text{hl}} = 14$ m (Figs 8c and d), it is seen that for $y_{\text{hl}} = 14$ m there are stronger ruptures, propagating in the (opposite) positive directions on both faults, than for $y_{\text{hl}} = 7$ m. Comparing the amplitudes of the triggered slip velocity in Fig. 8(e) top/bottom and 8f top/bottom, it is seen that the amplitude of triggered motion is smaller for the wider low-velocity layer.

4.2 A nine-fault system

In this section we examine the combined effects of sliding on many parallel faults, discussed previously in Section 3.2, and the competing effects associated with properties of the low-velocity layer discussed in Section 4.1. The complexity of the system with three media and nine faults produces a high richness of dynamic phenomena. To obtain a general understanding of overall properties of dynamic ruptures in this model we performed a large number (over 250) of simulations. Since it is not practical to show the results of all cases, we plot and discuss below details of several simulation examples and then summarize the key results of all the cases in Section 4.2.2.

4.2.1 Simulation examples

Fixing all simulation parameters other than the nucleation location (fault 1, fault 2, . . . , fault 9), we get a set of nine simulations for which we show results in Fig. 9. The set of employed model param-
etters are marked in Fig. 10(c). Starting from the nucleation region on a given fault (marked by a red rectangular), the slip may migrate to the neighbouring faults. In the examples shown in Fig. 9, self-sustained pulses are produced for the nucleation locations fault 3 to fault 7 that are within or at the boundaries of the low-velocity fault zone layer. The self-sustained pulses always localize on fault 3 or fault 7, which are the faults located on the material interfaces. For nucleation location on fault 3 associated with the stronger velocity contrast, the initiated rupture produces a self-sustained slip pulse that propagates on that fault in the positive direction. When the nucleation is on fault 4, the pulse migrates onto fault 3 and propagates there in a self-sustaining manner in the positive direction. When nucleating on fault 5, the rupture migrates onto both material interfaces (stronger velocity contrast on fault 3 and weaker contrast on fault 7). Both rupture pulses propagate in a self-sustained manner in the two opposite positive directions associated with the local velocity contrast and sense of loading. As mentioned earlier, a low-resolution ‘macroscopic’ view of this and other such simulated cases

Figure 8. Slip velocity on the two faults for each of the twelve simulations associated with two different nucleation locations \(y_{nuc} = (\text{fault 1, fault 2})\) and six different widths of the low-velocity layer \(y_{vel} = 1.75, 3.5, 7, 14, 28, 56\) \(\text{m}\). Fault 1 is associated with the stronger material contrast. For \(y_{vel} = 1.75\) \(\text{m}\) (Fig. 8a) the initiated rupture pulse is decaying for both nucleation locations. For \(y_{vel} = 3.5\) \(\text{m}\) (Fig. 8b) both nucleation locations produce self-sustaining ruptures on fault 1 in the positive direction, and only triggered slip is produced on fault 2 in the opposite positive direction. This case is classified as rupture migration, overall unilateral event. For \(y_{vel} = 7\) \(\text{m}\) (Fig. 8c) self-sustained pulses propagate on both faults for nucleation location 2 (classified as migration of rupture, overall bilateral event) but only on fault 1 for nucleation location 1 (overall unilateral event). For \(y_{vel} = 14\) \(\text{m}\) (Fig. 8d) self-sustained pulses exist on both faults for both nucleation locations (rupture migration, overall bilateral event). For \(y_{vel} = 28\) \(\text{m}\) (Fig. 8e) self-sustained pulse on fault 1 and triggered slip on fault 2 (overall unilateral event), while nucleation on fault 2 produces self-sustained pulses on both faults (migration fault ruptures, overall bilateral event). For \(y_{vel} = 56\) \(\text{m}\) (Fig. 8f) the results are similar to the case with \(y_{vel} = 28\) \(\text{m}\) (Fig. 8e). In all cases (b–f) with a self-sustained pulse, the primary pulses produce triggered (but not self-sustained) slip on the opposite side of the low-velocity layer.
Rupture migration to material interfaces

4.2.2 Overview of nine-fault system simulations

Fig. 10 summarizes the main results of 252 simulations with three elastic media and nine Coulomb frictional faults. The figure consists of six phase-diagrams associated with different values of initial shear stress, nucleation location, fault separation, and velocity contrast. Each symbol (cross, arrow or double arrow) specifies the overall macroscopic behaviour of rupture in a simulation associated with a given parameters set. The main features and implications of the simulated results are discussed in the next section.

5 DISCUSSION AND CONCLUSIONS

We performed a large numerical parameter-space study of 2-D in-plane ruptures in a structure consisting of three media and possible simultaneous slip on multiple frictional faults, two of which are material interfaces. Some cases with Kelvin-Voigt viscosity in the bulk and Prakash-Clifton friction on the material interfaces were used to explore effects of those rheologies and choose parameters for a systematic study of tendencies of rupture migration in a structure with purely elastic media and Coulomb frictional faults. Our model with two faults (among several others) on the boundaries of a low-velocity layer has two opposite positive propagation directions of wrinkle-like pulses. With the employed right-lateral loading, the positive direction of rupture propagation on the fault between the low-velocity layer and the faster bounding block (fault 3) is to the right, while the positive direction on the fault (number 7) between the low-velocity layer and the slower bounding block is to the left.

The main results of our study are as follows:

Migration of dynamic ruptures: A general observation of this study is that ruptures that are nucleated in the bulk tend to migrate spontaneously to the material interfaces associated with faults number 3 and 7. The ruptures continue to propagate in the positive directions on those faults in a self-sustained manner for a large number of parameter combinations (see Fig. 10). As might be expected, more cases of ruptures propagating to the right on the stronger velocity contrast fault 3 are produced, compared to ruptures propagating to the left on the weaker velocity contrast fault 7.

Apparent bilateral rupture due to low-velocity layer: When the low-velocity layer is not too thin, many sets of parameters produce simultaneous propagation of unilateral rupture pulses on faults 3 and 7 in the opposite positive directions (see e.g. Fig. 9: nucleation location 5, and Fig. 10). From a distant low-resolution perspective, such ruptures would look like bilateral events. For a model with very thin or no low-velocity layer, the model produces only single unilateral ruptures, in agreement with the analytical solution of Weertman (1980) and previous numerical simulations (e.g. Ben-Zion & Huang 2002; Shi & Ben-Zion 2006).

In a structure with three media, the eventual rupture plane is not necessarily the one with the largest material contrast: Depending on the nucleation location, ruptures can migrate onto either material interface. Rupture migration to the fault with the weaker velocity contrast (fault 7 in our model), can lead to self-sustained pulses that propagate unilaterally in the positive direction of that interface. Such examples can be seen, for example, in Figs 10(c) and (e). Since this direction is opposite to that associated with the overall velocity contrast across the bounding half-spaces, a low-resolution view would lead to the erroneous conclusion that the rupture propagated in the discussed features are summarized in Fig. 10(c), along with the main results of many other cases.
Figure 10. Main properties of rupture behaviour in 252 simulations with three elastic materials and nine parallel faults governed by Coulomb friction. The diagrams show the initial shear stress on the x-axes and the location of nucleation in terms of fault numbering on the y-axes. The result of each simulation is reduced to four symbols: An arrow to the right means generation of a self-sustained pulse travelling on fault number 3 (associated with the stronger material contrast $v_s^3/v_s^1$) in the positive (right) direction. An arrow to the left denotes a self-sustained pulse on fault number 7 (associated with the weaker material contrast $v_s^7/v_s^1$) travelling in the positive (left) direction. A double arrow means two self-sustained pulses propagating on both material interfaces in the opposite positive directions (i.e. generation of apparent bilateral rupture). A cross is shown when no self-sustained pulses were generated. The set of employed velocity contrasts are: $\Delta v_1 = \{v_s^2/v_s^1, v_s^3/v_s^1\} = \{0.85, 0.94\}$ and $\Delta v_2 = \{v_s^2/v_s^1, v_s^3/v_s^1\} = \{0.75, 0.90\}$. Panels (a–f) correspond to the following parameter sets: (a) $\Delta v_1, y_f = 7$ m; (b) $\Delta v_2, y_f = 7$ m; (c) $\Delta v_1, y_f = 14$ m; (d) $\Delta v_2, y_f = 14$ m; (e) $\Delta v_1, y_f = 21$ m; (f) $\Delta v_2, y_f = 21$ m.

opposite (negative) direction from that predicted for the wrinkle-like pulse.

Many parallel faults lead to ‘effective viscous’ behaviour: The existence of multiple faults dissipates elastic strain energy and reduces the divergent behaviour of the wrinkle-like pulses. As the number of faults increases the deformation become more distributed and the model response becomes similar to that of a viscous material. In contrast, self-sustained rupture pulses, leading to effective brittle behaviour, are associated with localization of the energy change on a small number of faults (in our case material interfaces).

Higher initial stress favours rupture migration and self-sustained pulses: Increasing initial stress leads in general to a higher likelihood that nucleated ruptures will migrate to the two material interfaces. With higher initial stress, such migrated ruptures have overall also a higher likelihood of becoming self-sustained pulses. Nevertheless, in some cases the latter does not hold. Specifically, when a relatively thin low-velocity layer has a relatively large number of faults (in our parameter-space 7 m fault separation, 28-m-wide layer), the effectiveness of migration and generation of self-sustained ruptures reduces for a range of increasing initial shear stress (see Fig. 10a). This is probably produced by higher dissipation of strain energy.
in such cases due to simultaneous low-level slip on several faults. However, increasing the initial shear stress further toward the shear strength always produces self-sustained ruptures on the material interfaces.

Wider fault separation, easier generation of self-sustained pulses: When the faults separation increases, once a rupture can migrate onto one of the material interfaces it is more likely to become self-sustained. This can be understood in terms of the features discussed above, since larger fault separation makes it more difficult for the energy to be transferred to one of the neighbouring faults.

Nucleation on slower side leads to stronger migration onto the interface: Using always the same nucleation procedure as described in Section 2.2, we found that ruptures that were initiated on the slower side of the material contrast could more often succeed in migrating onto the material interface. This is intuitive since a slower wave propagation velocity leads to higher wave amplitudes (e.g. as for fault-zone trapped waves or waves in sedimentary basins). This is also manifested in Fig. 10 where it is seen that more self-sustained ruptures are generated by nucleation locations within the low-velocity layer than from outside the layer.

Nucleation closer to the material interface can lead to less migration: In some cases we observe that ruptures nucleated close to a material interface could not migrate onto the material interface, while ruptures nucleated at some larger distance could (see, e.g. Fig. 10(a) for initial shear stress of 68 MPa). This is related to the fact that the radiation pattern of the shear waves has a nodal plane on the continuation of the rupture. Increasing the strength of the nucleation procedure, for example, by increasing the source size or stress drop, would produce migration also in such cases.

The results may have important implications to a number of issues of earthquake and fault physics associated with large structures that have well-developed material interfaces. The common spontaneous migration of ruptures to the material interfaces implies that the dynamic phenomena associated with the wrinkle-like pulses are not limited to the set of hypocentres located directly on the material interfaces. The migration of ruptures to material interfaces provides a mechanism for a positive feedback between structure and rupture properties that can lead to progressive regularization of geometrical heterogeneities with cumulative slip and suppression of dynamic branching from large fault zone structures (Ben-Zion & Andrews 1998). The dynamic reduction of normal stress at the tip propagating along a material interfaces in the positive direction (e.g. Fig. 4) increases the mechanical efficiency of such ruptures, and has fundamental implications for the effective constitutive laws and energy partition in structures with material interfaces.

The simulations provide a simple explanation for recent observations and inferences on rupture propagation directions along sections of the SAF. Rubin & Gillard (2000) relocated earthquakes in the Bear Valley section of the SAF and found that the number of immediate aftershocks near the edges of prior ruptures to the NW is more than double the number to the SE. They interpreted this asymmetry as resulting from the dynamic changes of normal stress associated with the material contrast across the SAF. Our results explain the ability of ruptures to propagate in both directions, with an elevated probability for propagation in the positive direction of the overall contrast across the fault (which is the same direction associated with ruptures on the stronger velocity contrast between the fault zone layer and the stiffer half-space). McGuire (2004) inverted directly seismic data for rupture directivity of two small (M2.7) earthquakes on the Bear valley section of the SAF. One of these earthquakes had clear unilateral rupture propagation to the SE, as predicted by the overall material contrast across the SAF, while the other had overall bilateral ‘macroscopic’ properties.

Dor et al. (2006a,b) performed multi-signal multiscale geological mapping in the structure of several faults of the San Andreas system in southern CA. Their results show strong asymmetry of rock damage across the faults, compatible with a preferred propagation direction and related generation of damage asymmetry across a bimaterial interface (Ben-Zion & Shi 2005). Similar asymmetric damage zones, which correlate with the velocity structure as predicted for wrinkle-like ruptures, were observed in seismic imaging studies using fault zone trapped and head waves at sections of the San Andrées and San Jacinto faults (Lewis et al. 2005, 2006). The possible relation between the observed asymmetric rock damage and preferred propagation direction of earthquake ruptures is supported by our general result that ruptures tend to migrate to material interfaces and become self-sustained wrinkle like pulses for wide range of conditions.

The Parkfield region of the SAF resembles overall the model configuration of Fig. 1, with two large faults—the main SAF and the southwest fracture zone—separated by 1.5 km wide deformation/damage zone (e.g. Rymer et al. 2006). As noted by Ben-Zion (2006b), both faults are highly active on the scales of small to moderate events, and the M6 2004 Parkfield event along with many of its aftershocks appear to be located on the southwest fracture zone rather than the main SAF. Rymer et al. (2006) found that the surface fractures generated by the M6 2004 event are concentrated on the SW side of the southwest fracture zone and on the NE side of the SAF. These observations and near-fault seismic data (Shakal et al. 2005) suggest that the M6 2004 Parkfield event consisted of two separate pulses, one propagating on the southwest fracture zone to the SE and the other propagating on the SAF to the NW. In that case, both pulses propagated (as occurred commonly in our simulations) in the two positive directions associated with the SAF and southwest fracture zone. This should be clarified in future observational analysis of the structure and rupture properties associated with the M6 2004 Parkfield event.

Harris & Day (2005) concluded from inferred propagation directions of several earthquakes on the Parkfield sections of the SAF that the prediction of a preferred rupture propagation direction does not hold for natural faults. The simulated richness of propagation behaviour in our model with two blocks separated by a low-velocity layer highlights the lack of decisive information in the results considered by Harris & Day (2005). More generally, our results emphasize the need to base tests of a preferred propagation direction along a material interface (and other features of wrinkle-like pulses) on detailed high-resolution observations associated with large data sets.

The calculations of this work were done for situations (in-plane strain, slip-independent friction) chosen to focus on effects associated with the assumed structure (many possible faults in a three-media configuration) and dynamic changes of normal stress along material interfaces. The generality of the results should be tested in future simulations incorporating additional levels of realism in the assumed structure (e.g. dimensionality) and rheology (e.g. slip- and rate-dependent friction).

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