Nordic Hydrology, 12, 1981, 167-184

Application of Soil Water Flow Theory in Field Simulation

Karsten Høgh Jensen

Technical University of Denmark, Copenhagen
Danish Hydraulic Institute, Hørsholm

A numerical model based on a finite difference approximation of the basic differential equation for soil water flow is described. The equation includes a sink term representing moisture extraction by root systems, and allowance for evaporation from the soil surface is made too. Estimation of these terms is based on a potential transpiration calculated from Penman-Monteith's equation, on a potential evaporation calculated from Ritchie's equation and in both cases on relations to soil moisture status over various soil depths. The model simulations are compared with field observations of moisture content and soil moisture tension. Further, the model has been applied in evaluating how irrigation treatment effects the increases in evapotranspiration and deep percolation.

Introduction

The state and movement of water in the soil is a complex phenomenon since it is affected by a variety of physical, biological and chemical properties and processes interacting with each other, which moreover may vary both in time and space. Especially complex are the conditions in the top-most soil layer, or the crop root zone, where soil, plant and atmospheric factors are interfering with each other. A current approach in simulating the soil water cycles in the root zone is to consider the field environment as a physically unified and dynamic system, which Philip (1966) has called the »SPAC« (»soil-plant-atmosphere continuum«). In this sys-

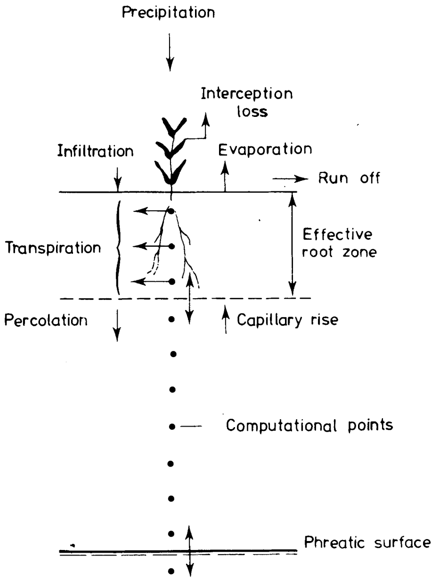


Fig. 1. Vertical flow components in the model.

tem, water flow takes place down a gradient of potential energy from soil to root to atmosphere, with the concept of a hydraulic gradient equally valid and applicable in soil, plant, and atmosphere alike.

In this model the description of the transport of water both as liquid in the soil and as vapour in the atmosphere relies on the fundamental governing equations for transport in these surroundings. The two descriptions are coupled through the root zone, where the transpired water is extracted through the root system. This soil water uptake is modelled according to the macroscopic-scale approach (Molz and Remson, 1970), which regards the root system in its entirety as a diffuse sink permeating the soil continuously. This sink term is added to the continuity equation for soil water flow, and it will under most circumstances vary throughout the root zone. The intergral of the diffuse sink over the root zone describes the actual amount of water that the plant has transpired. This amount plus the direct evaporation from the soil surface and the interception storage is always less than or equal to the evaporative potential of the atmosphere. The actual amount of water vapour lost to the atmosphere is determined by a large number of interacting physical and physiological factors, which are difficult, if not impossible, to predict from unique, theoretically based, mathematical expressions. This model therefore applies empirical relationships both as to the prediction of actual values of transpiration and evaporation, both based on the potential values, and of soil moisture status, and as to the distribution of the diffusive sink term.

The model only considers flow in the vertical direction both in the soil and in the atmosphere. With the input of meteorological data, precipitation, irrigation, soil properties, soil layering, and crop characteristics the model predicts for each time step infiltration, soil moisture content and tension at specified depths, soil water flux in each depth increment, root extraction, surface evaporation and interception loss. The model components are described below, and the various water fluxes in the model are illustrated in Fig. 1.

Model Components

Interception

Part of the precipitation and irrigation is retained on the crop surface from where it may either evaporate directly or, after some time, drain to the soil surface. The process is modelled as a simple accounting procedure for an interception storage, which has a maximum capacity I_m calculated as (Jensen 1979)

$$I_m = 0.05\,LAI \qquad (1)$$

I_m – maximum storage capacity (mm)
LAI – leaf area index (m^2 leaf/m^2 ground)

Evaporation from interception storage takes place with a rate determined by potential transpiration (Eq. (2)) and is asumed to be fulfilled in the first place. A surplus evaporation potential, if any, is then applied in the transpiration calculations.

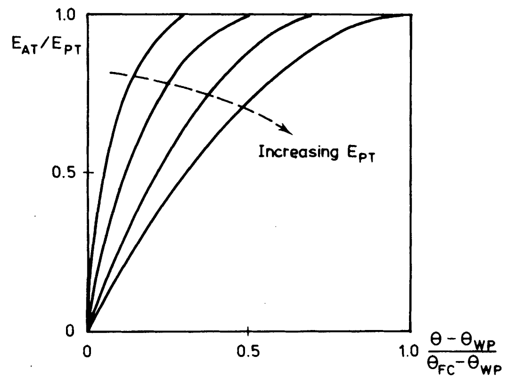
Transpiration

The model of Monteith (1965) is used for determining potential transpiration E_{pt}

$$E_{pt} = f\,\frac{\Delta R_{nc} + \rho C_p\,\delta e/r_a}{\lambda(\Delta+\gamma)} \qquad (2)$$

E_{pt} – potential transpiration rate (mm/time interval)
f – unit conversion factor
R_{nc} – net radiation absorbed by the crop (W/m^2)
Δ – slope of saturation vapour pressure curve (mb/°C)
ϱ – air density (kg/m^3)
c_p – specific heat capacity (J/kg/°C)
δ_e – vapour pressure deficit (mb)
r_a – aerodynamic resistance (s/m)
λ – latent heat of water (J/kg)
γ – psychrometer constant (mb/°C)

As the equation is written here, it has implicitly been assumed that the crop



E_{at} = actual transpiration
E_{pt} = potential transpiration
θ = actual soil moisture content
θ_{fc} = soil moisture content at field capacity
θ_{wc} = soil moisture content at permanent wilting point

Fig. 2. Relative transpiration as a function of soil moisture content and potential transpiration.

resistance, which appears in the original Monteith equation, is negligible for the crops discussed in this study.

Net radiation absorbed by the crop is calculated according to Beer's law (Ritchie 1972)

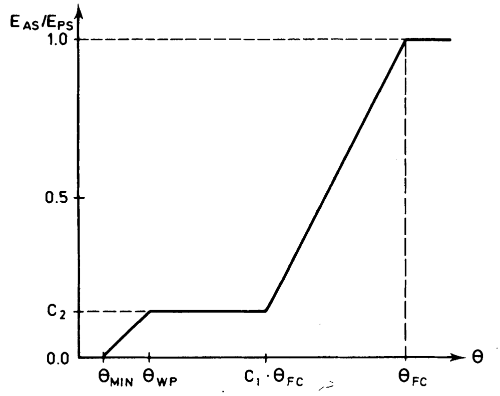
$$R_{nc} = R_n(1-\exp(-0.4\,LAI)) \tag{3}$$

R_{nc} – net radiation absorbed by the crop (W/m^2)
R_n – net radiation measured at standard height (W/m^2)
LAI – leaf area index (m^2 leaf/m^2 ground)

The external or aerodynamic resistance between crop surface and standard height is calculated from the turbulent transfer equations and under the assumption of a logarithmic wind profile it is implicitly assumed that effects of stability are either compensating or negligible.

With decreasing soil moisture content in the root zone the crop transpiration will decrease. This decrease could be modelled by the Monteith equation, if the stomatal response could be predicted. However, this requires further input data on the microscale, and even then serious problems seem to exist in modelling the stomatal response. In consequence of this a more traditional approach has been taken, which is to calculate the relative transpiration as a function of soil water status. A relationship, taken from Kristensen and Jensen (1975), has been adopted for this purpose, and a qualitative representation is given in Fig. 2. As can be seen, the function possesses the following fundamental characteristics: the decrease in transpiration starts earlier and is more pronounced the higher the potential transpiration.

The functional relationship illustrated in Fig. 2 is applied at all computational points over the root zone depth, and the derived values are subsequently multiplied by a distribution function to account for the extraction pattern of the root system. The obtained values act as sink terms in the solution of the flow equation.



E_{as} = actual soil evaporation
E_{ps} = potential soil evaporation
θ = actual soil moisture content
θ_{fc} = soil moisture content at field capacity
θ_{wp} = soil moisture content at permanent wilting point
θ_{min} = irreducible soil moisture content
C_1, C_2 = parameters

Fig. 3. Relative soil evaporation versus soil moisture content.

Soil Evaporation

Maximum evaporation from the soil surface, E_{ps}, is separately calculated from the formula suggested by Ritchie (1972), which assumes that the aerodynamic term in Eq. (2) is negligible

$$E_{ps} = f \frac{\Delta}{\lambda\,(\Delta+\gamma)}(R_{ns} - Q) \qquad (4)$$

E_{ps} – potential soil evaporation (mm/time interval)
f – unit conversion factor
R_{ns} – net radiation at the soil surface (W/m^2)
Q – soil heat flux (W/m^2)
Δ – slope of saturation vapour pressure curve (mb/°C)
λ – latent heat of water (J/kg)
γ – psychrometer constant (mb/°C)

Eq. (4) is only applicable for a surface covered by vegetation, since it is assumed that wind speed and vapour pressure deficit are considerably less than the values measured at standard height, which justifies that the aerodynamic term in the Penman-Monteith equation is neglected. For a soil only very sparsely covered by vegetation or a bare soil this assumption is no longer valid, and the full Penman-Monteith equation is used for predicting potential soil evaporation. Also for this case the crop resistance is set to zero.

As the soil near the surface becomes drier, soil evaporation is reduced to below the potential value. An empirical relationship (Kristensen and Jensen, 1975) is used for the prediction of actual soil evaporation, and the relationship is illustrated in Fig. 3. This relationship essentially consists of four stages: a constant rate stage, controlled by external evaporativity, i.e. potential evaporation; a falling-rate stage, controlled by the water transmission capability of the soil profile to the evaporation surface; a vapour diffusion stage, where the evaporation continues at a very slow and relatively constant rate controlled by the vapour diffusivity in the dried surface zone (indicated by the level C_2); and finally another falling rate stage, which reduces the soil evaporation to zero. The soil moisture content in the figure refers to the soil moisture content near the surface, here represented by the upper compartment in the space discretization.

The soil evaporation is applied as a flux component in the numerical solution of the flow equation.

Soil Water Flow

One form of the basic differential equation for soil water flow in the vertical direction reads

$$C(\psi)\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial z}\left(K(\psi)\frac{\partial\psi}{\partial z}\right) + \frac{\partial K(\psi)}{\partial z} - S(z) \qquad (5)$$

ψ – capillary pressure head (m) (the absolute value of capillary pressure will in the following be termed tension)
θ – volumetric soil moisture content (m^3/m^3)
t – time (h)
z – vertical coordinate, positive downwards (m)
K – hydraulic conductivity as a function of ψ (m/h)
C – soil water capacity as a function of ψ (m^{-1})
S – sink term representing water uptake by roots (h^{-1})

The governing equation as written above forms the basis for the description of vertical soil water flow in the model. It is derived under the assumption that the largest contributor to the total water flow in the soil is the bulk flow of liquid water, i.e. water vapour movement both as diffusion and as bulk flow is omitted. Further, in the numerical solution of the flow equation it is assumed that the relationship between moisture content and capillary pressure head does not display hysteresis.

Numerical Formulation and Computational Procedure

The flow equation (5) is solved by finite difference techniques in an iterative implicit scheme. The predicted evapotranspiration is introduced explicitly through the sink term (transpiration) and as a component of the water flux across the soil surface (evaporation).

To solve the equation, initial and boundary conditions must be specified. Initial conditions are in the form of the capillary pressure variation through the soil profile (subsidiarily the soil moisture variation), which can be evaluated from measurements.

The upper boundary condition can be a prescribed flux across the soil surface, the flux being a resultant of net precipitation (precipitation and irrigation minus interception loss) and soil evaporation. If this flux is larger than the infiltration capacity of the soil profile, the solution is repeated with a zero capillary pressure head at the surface. However, the latter situation is not occurring in the field simulation described below, since the soil profile is very permeable.

The lower boundary condition is defined by prescribed fluctuations of the water table.

The basic timestep in the numerical solution is 1 hour during periods with no rainfall or irrigation. During other periods the timestep is reduced both in proportion to the rainfall/irrigation intensity and as a function of the soil dryness. At every timestep the solution is checked for a proper mass balance and if this is not fulfilled to a satisfactory degreee, the solution is repeated with a smaller timestep. Under extreme conditions the timestep may be reduced to a value of 2-3 minutes.

The distance steps in the vertical are chosen according to the required resolution, but they are likely to be in the range of 5-50 cm.

Input/Output Description

Input data to the model consist of precipitation data and meteorological data for the evapotranspiration calculations, both on an hourly basis. In addition, information about irrigation treatment, variation of leaf area index and root zone depth, root extraction pattern and fluctuations of the water table has to be supplied.

Required model parameters are moisture content-capillary pressure relationship and hydraulic conductivity-capillary pressure relationship for each soil layer in the profile. In the computer program both relationships are represented by functional expressions.

Output from the model is simulated time variation in soil moisture content, capillary pressure and soil water flow at different levels.

Description of Field Characteristics

The performance of the soil water flow model is validated against extensive data collected from field studies of grass and barley at an agricultural experimental station in Southern Denmark. The data material is placed at the author's disposal for the use in the present study. The field studies are carried out for a period of 6-7 years, and the two crops are grown both with and without irrigation treatment. At various depths on the four experimental plots measurements of soil moisture content and capillary pressure are taken on nearly weekly intervals, and these measurements are used as verification data. Four years of simulation are made on all plots. The precipitation data and the necessary meteorological data are supplied from a nearby meteorological station.

Crop characteristics, as to variation in leaf area index, root zone depth and root extraction pattern, are all given a generalized seasonal variation based on measurements at the location. The variation in leaf area index and root zone depth is illustrated in some of the following figures.

The experimental station is situated in a outwash plain shaped during the latest glacial age. The soil type can be characterized as sandy. It is found appropriate to introduce two different types of soil properties with a dividing line at 50 cm below the surface.

The defined moisture content-capillary pressure relationships for the two soil horizons are based on measurements on several soil samples taken from the experimental plots. The relationships are illustrated in Fig. 4, and it should be pointed out that the curves are modified at small tensions, since the moisture content in field situations never attains full porosity because of entrapped air.

Hydraulic conductivity is only measured at complete saturation. For obtaining a first estimate of the whole range of conductivities the calculation method of Kunze et.al. (1968) is used. The calculated values are, as recommended, adjusted by the use of a matching factor, defined as measured to calculated conductivity at complete saturation. In the computer program the relationships are represented by Averjanov's empirical formula (Averjanov 1950); however, to obtain better simulations of the moisture contents as well as the tensions, the conductivities have been modified in relation to the calculated values for the smaller moisture contents, Fig. 5.

Discussion

As mentioned above, simulations have been carried out for all four plots over a period of four years; however, in this context only selected results will be presented. These will include simulations with a barley crop, both with and without irrigation treatment, for the period 1976-1977.

In Fig. 6 various input data and calculated quantities are illustrated on a daily basis. As can be seen, both leaf area index and root zone depth have a generalized seasonal variation, which is shifted only slightly in time according to information about times for germination and harvest. Fig. 7 illustrates a comparison between predicted and measured soil moisture contents for various depths for the 2-year period, and in Fig. 6 some of these results are shown on an integrated basis. Generally one must say that predicted moisture contents and field measurements are in close agreement. Inevitably, some deviations will occur, because of the complexities of the processes involved and the simplifications made. For instance

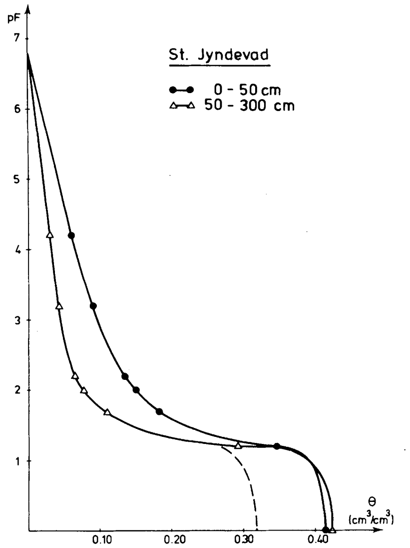


Fig. 4. Soil moisture content-capillary pressure relationships (Bennetzen 1978).

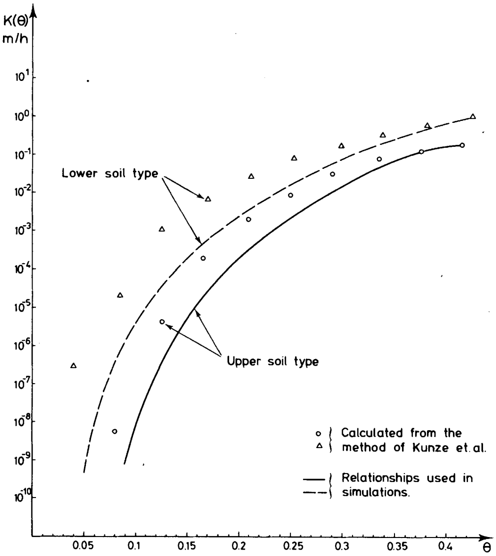


Fig. 5. Hydraulic conductivity-soil moisture content relationships.

are deviations evident in the depth of 40 cm. However, this is due to the restriction of two soil types in the vertical. Obviously, at this level the soil properties lie between the two defined ones, but it is believed that the model performance, as regards the prediction of actual evapotranspiration and deep percolation, will not improve considerably, if an additional soil type is introduced.

Fig. 10 is similar to Fig. 6 and illustrates the model performance for irrigated barley. The same model parameters are used in this simulation and since the model performance is very satisfactory in both cases this indicates that the dynamics of the model are fairly good.

Fig. 8 is a comparison between predicted and measured soil water tensions. It should be pointed out that tensiometers only are applicable for tensions up to about 8 metres. When the measurements are 8 metres or above, this is indicated with arrows in the figure, and correspondingly the simulated values are not showen when above 8 metres. The accordance in Fig. 8 is not quite as satisfactory as in Fig. 7, and this is despite the fact that the model actually solves for tension, and moisture content can be regarded as a derived quantity. Since the moisture content is of most concern, this discrepancy is not serious, and it can be explained by a slight uncertainty in the retention curves.

Fig. 9 shows how the input from infiltration travels through the unsaturated zone to the water table. During the summer periods no infiltration water reaches the lower layers because of the deficit in the root zone. Deep percolation starts in

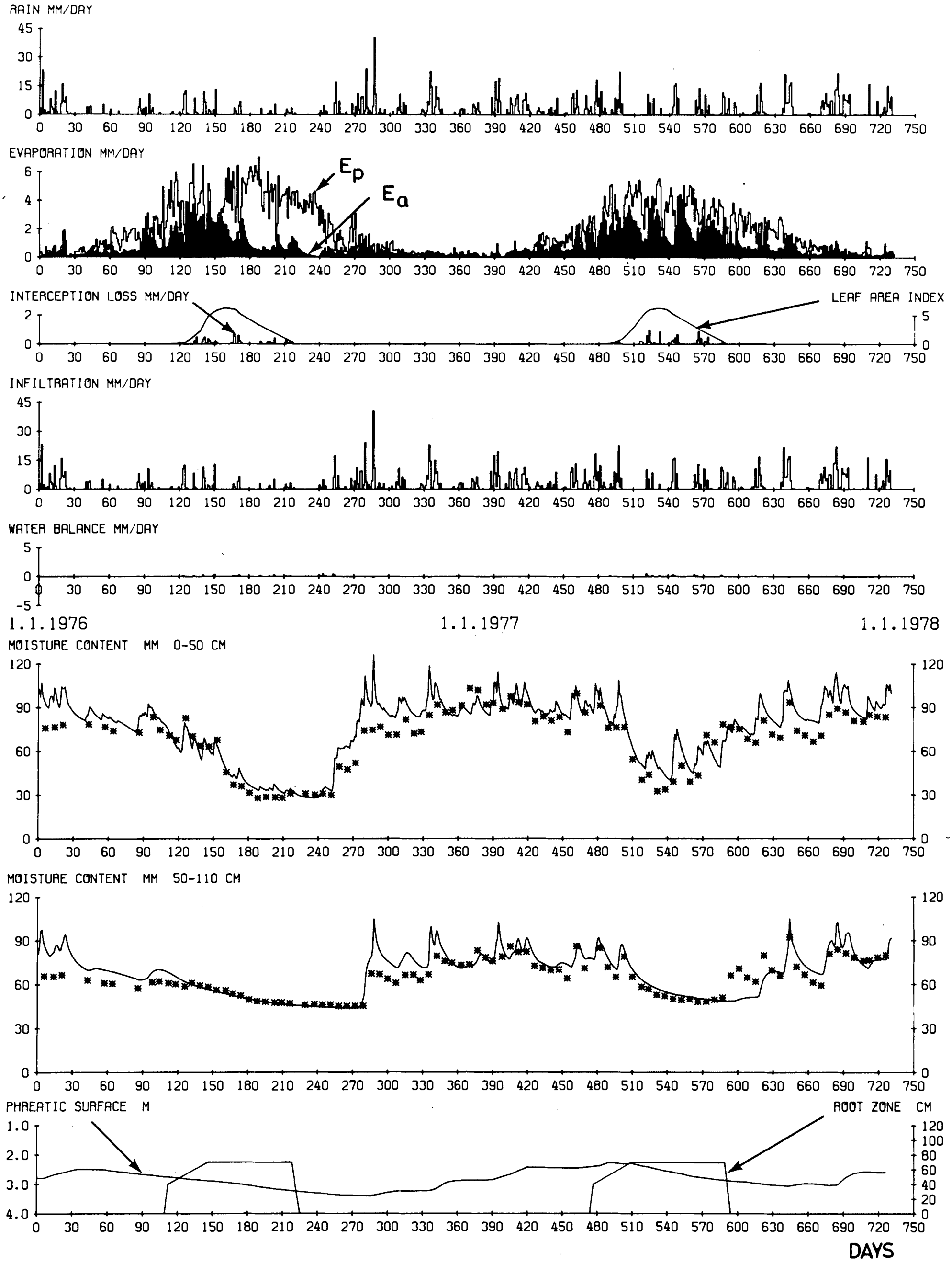


Fig. 6. Barley 1976-1977.

Input data, boundary conditions, interception, evapotranspiration and integrated soil moisture content. * measured (Bennetzen 1978) – calculated.

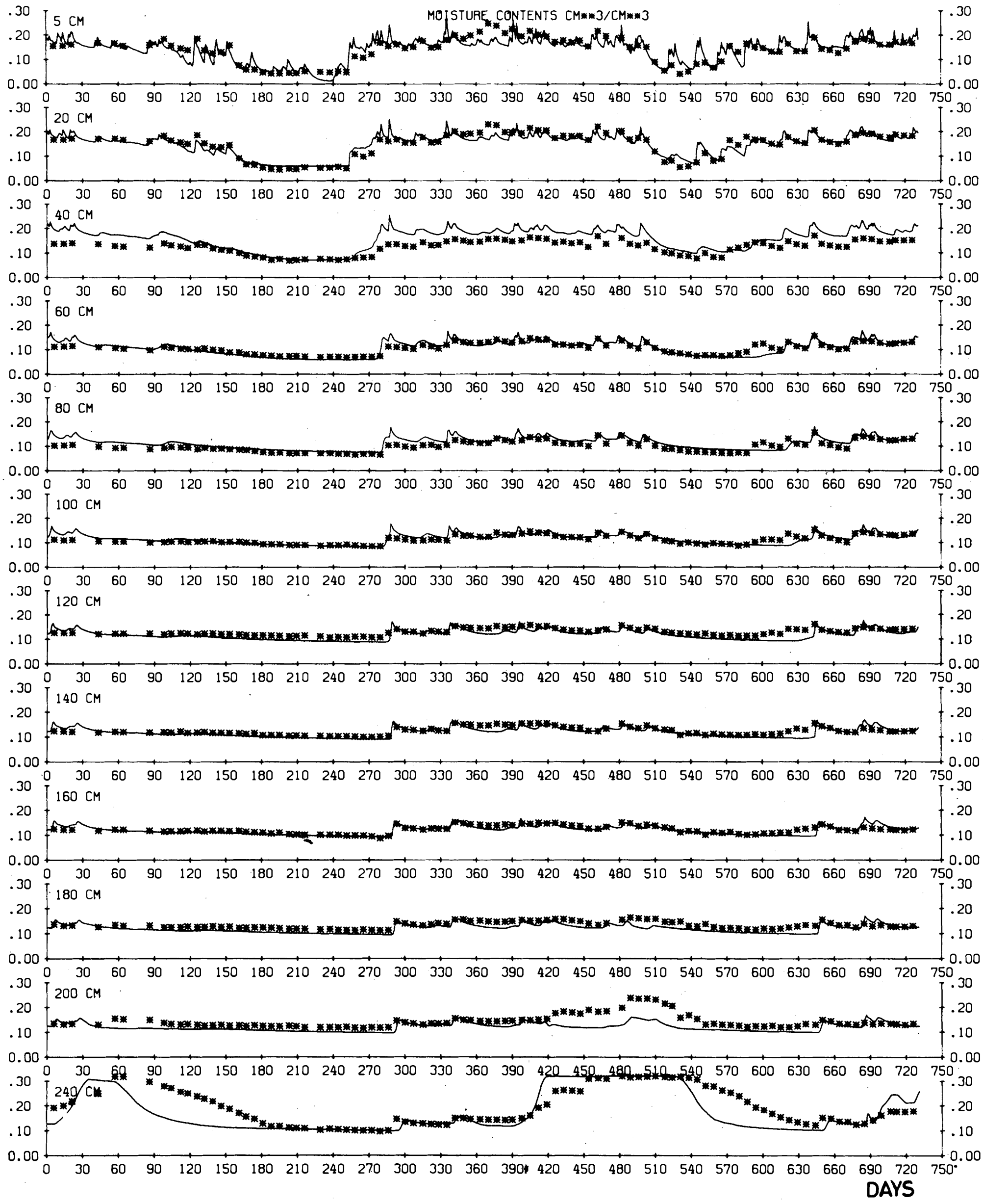


Fig. 7. Barley 1976-1977.
Measured and calculated soil moisture content.
* measured (Bennetzen, 1978)
– calculated.

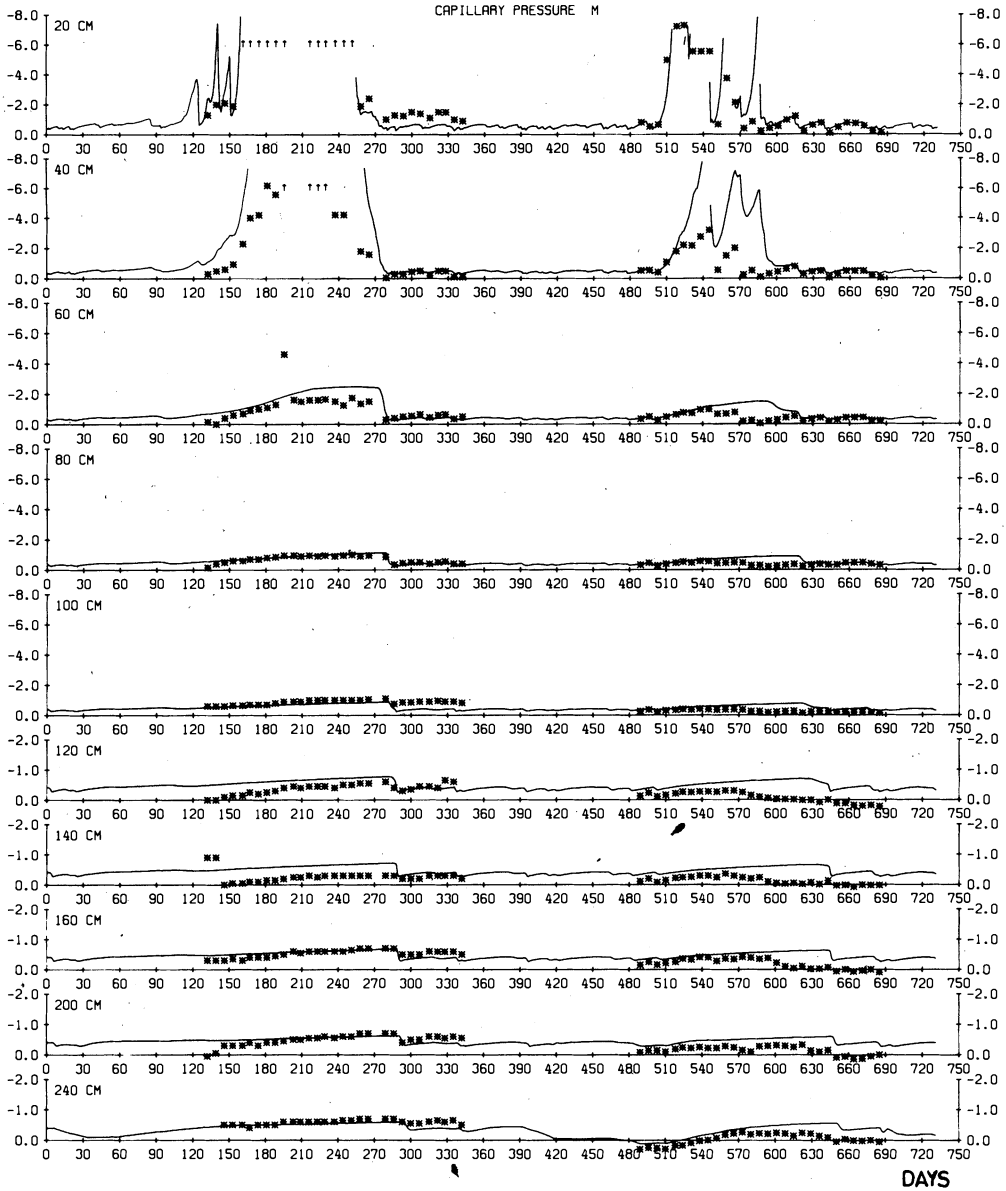


Fig. 8. Barley 1976-1977.

Measued and calculated capillary pressure.

* measured (Bennetzen, 1978)

↑ measured value less than -8 metres

– calculated (interrupted when less than -8 metres).

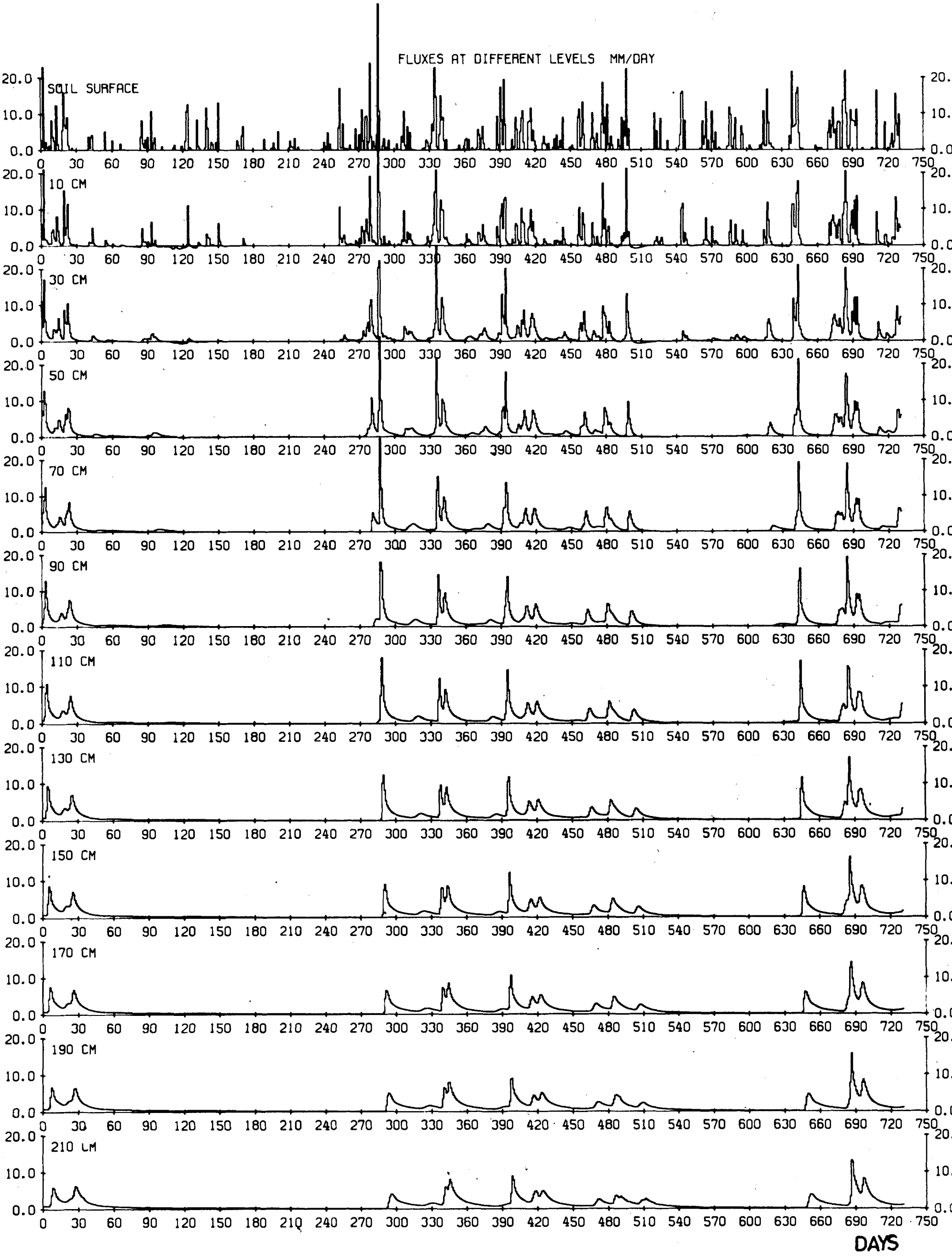


Fig. 9. Calculated water fluxes at various levels.

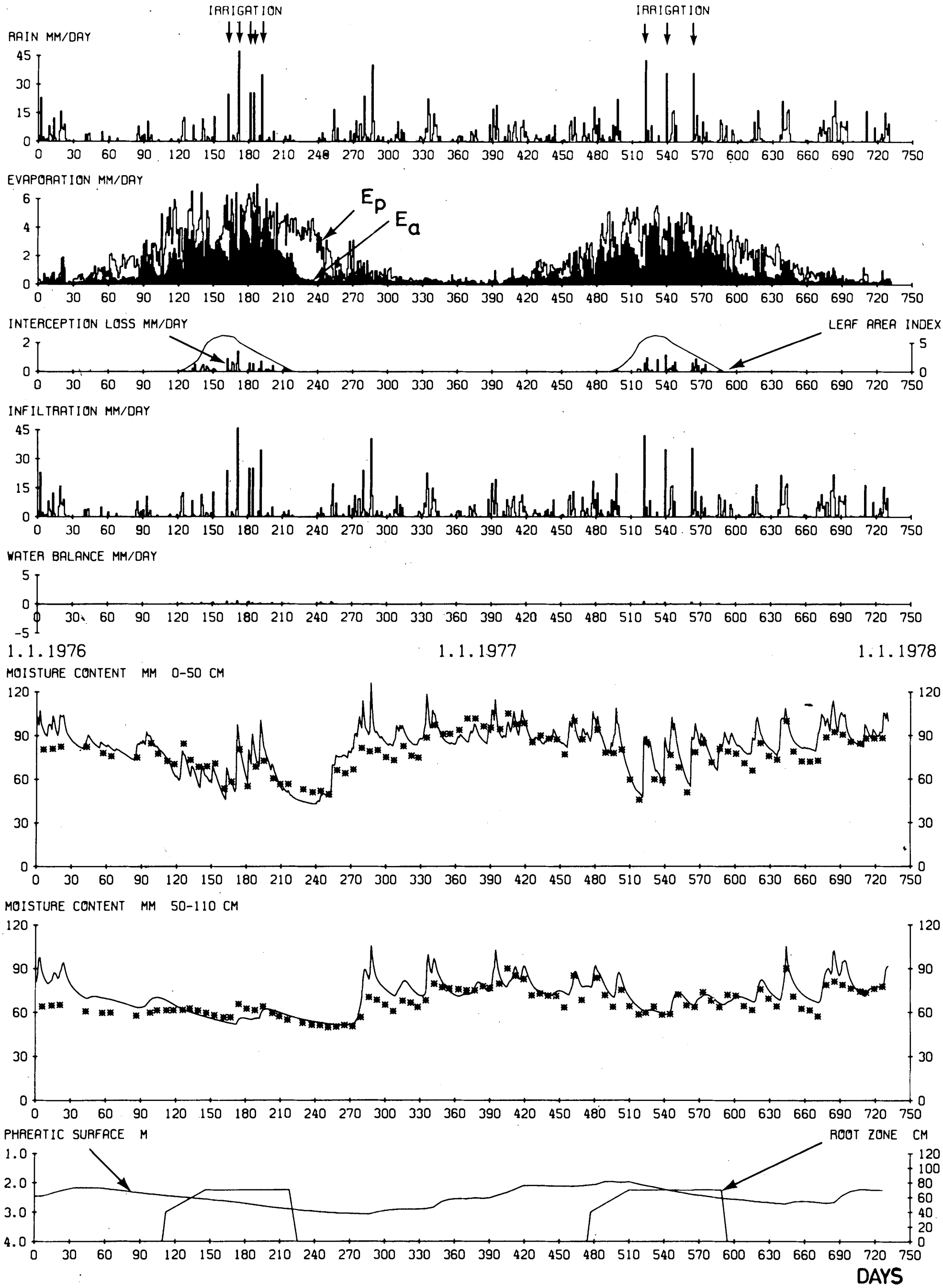


Fig. 10. Irrigated barley 1976-1977.

Input data, boundary conditions, interception, evapotranspiration and integrated soil moisture content. * measured (Bennetzen, 1978) – calculated.

September-October and continues until April-May. In the beginning the front velocity is rather slow (5-7 metres per month), but later, when the moisture content and with that the hydraulic conductivity has increased, the infiltration front moves downwards with a smaller phase shift with depth (10-15 metres per month).

The influence of irrigation treatment on various hydrologic subprocesses is shown in Fig. 11 and Fig. 12, which both demonstrate the differences between the simulations with the barley crop with and without irrigation for the same period. As seen in Fig. 11 irrigation results in an increased water content in the root zone, mostly in the upper layers, and this again enhances the evapotranspiration. Fig. 12 illustrates the effects on the soil moisture fluxes at various levels. One recognizes the general feature that no major increase in deeper percolation occurs at the time of irrigation, because the water is withheld in the root zone. Increased percolation is not of significance until in the late growing season or after. The simulated hydrologic effects from irrigation for both barley and grass for a four-year period are given in tabular form in Table 1. As can be read from the table the percentage increases in evapotranspiration, and percolation for the various irrigation treatments shows considerable variation.

Table 1 – Influence from irrigation on hydrologic variables for both grass and barley for the period 1974-1977. Numbers are yearly sums in mm.

Grass	1974	1975	1976	1977
Irrigation	117	265	349	159
Increased evapotranspiration	60(51%)	170(65%)	216(62%)	62(41%)
Increased percolation	57(49%)	95(36%)	133(38%)	91(59%)
Barley	**1974**	**1975**	**1976**	**1977**
Irrigation	117	93	153	96
Increased evapotranspiration	62(53%)	85(91%)	127(83%)	48(50%)
Increased percolation	55(47%)	8(9%)	26(17%)	48(50%)

Conclusions

A soil moisture model based on the basic differential equation for soil water flow is developed. The model includes descriptions for interception, evapotranspiration and root extraction to provide a mathematical soil-plant-atmospheric continuum. Since many interacting sub-processes occur for which no unique

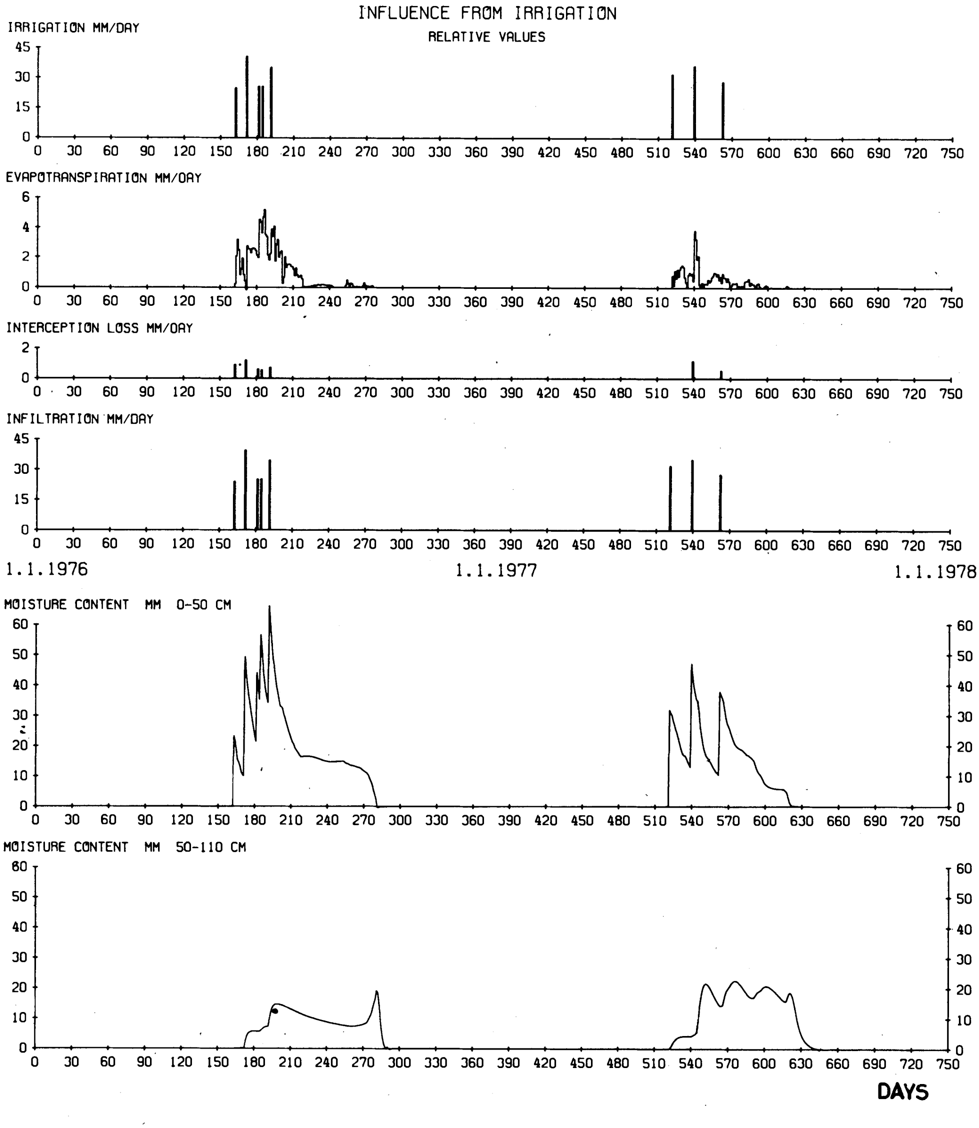


Fig. 11. Barley 1976-1977.
Influence from irrigation on hydrologic variables as differences between simulation with and without irrigation.

mathematical description is available, it is necessary to introduce empirical relationships published in literature. The model is an attempt to simulate natural physical processes in the unsaturated zone as they affect soil water movement and root uptake.

The model is tested against observed moisture contents and capillary pressures

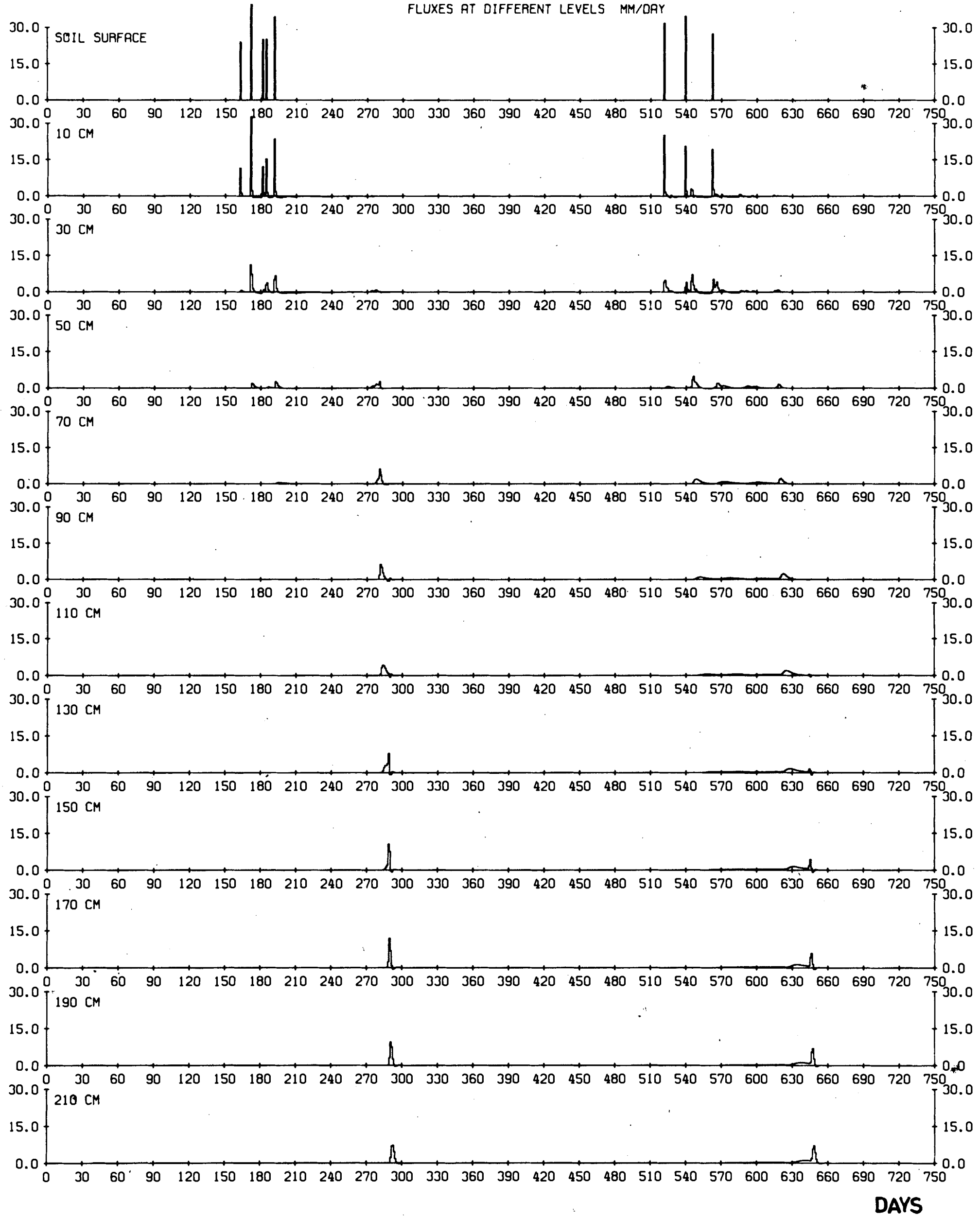


Fig. 12. Barley 1976-1977.
Influence from irrigation on soil moisture flows at different levels as differences between simulations with and without irrigation.

in various layers in Southern Denmark, and it performs fairly well. Although some deviations occur, they are considered acceptable in view of the complexities of the processes involved.

Acknowledgements

The author is indebted to the Danish Agricultural Experimental Stations in Højer and Jyndevad for providing the field data used in this study.

The study is partially supported by funds for the development of the European Hydrologic System (SHE).

References

Averjanov, S.F. (1950) About permeability of subsurface soils in case of incomplete saturation. Eng. Collect. 7.

Bennetzen, F. (1978) Water-balance and nitrogen-balance by optimal plant-production. III. Models and results. *Tidsskrift for Planteavl, 82,* p. 191-220. (In Danish).

Jensen, S.E. (1979) Model ETFOREST for calculating actual evapotranspiration. In: S. Halldin (Ed.). Comparison of forest water and energy exchange models. International Society for Ecological Modelling, Copenhagen, p. 165-172.

Kristensen, K.J., and Jensen, S.E. (1975) A model for estimating actual evapotranspiration from potential evapotranspiration. *Nordic Hydrology, 6,* p. 170-188.

Kunze, R.J., Uehara, G., and Graham, K. (1968) Factors important in the calculations of hydraulic conductivity. *Soil Sci. Soc. Amer. Proc. vol. 32,* p. 760-765.

Molz, F.J., and Remson, I. (1970) Extraction term models of soil moisture use by transpiring plants. *Water Resources Research, 6,* No. 5, p. 1346-1356.

Monteith, J.L. (1965) Evaporation and environment. In: G.E. Fogg (Ed.), *The state and movement of water in living organism,* p. 205-234. Acad. Press, New York, London.

Philip, J.R. (1966) Plant water relations: Some physical aspects. Ann. Rev. Plant Phys. 17, p. 245-268.

Ritchie, J.T. (1972) Model for predicting evaporation from a row crop with incomplete cover. *Water Resources Research, 8,* p. 1204-1213.

Received: 10 June, 1981

Address:
Institute of Hydrodynamics and Hydraulic Engineering,
ISVA, Technical University of Denmark,
Building 115, DK-2800 Lyngby,
Denmark.

Danish Hydraulic Institute
Agern Allé 5,
DK-2970 Hørsholm,
Denmark.