Nordic Hydrology, 18, 1987, 167-184

On the Applicability of Theoretical Expressions for Flow Rate into Perforated Draintubes

U.S. Panu

Civil. Eng. Dept., Lakehead Univ.,
Thunder Bay, Ont., Canada

An assessment of the existing theoretical expressions of flow into perforated draintubes has been made in regard to their applicability in the estimation of flow rate into perforated drains under field conditions. A method based on the experimental results is proposed for extending the application of existing expressions to curved watertable conditions.

Introduction

The tile drains made of clay and concrete have extensively been used in subsurface drainage systems. However, the advent of plastics technology has led to the introduction of long plastic draintubes in subsurface drainage systems. Such draintubes must be provided with perforations to permit entry of water into them. The number, shape, size, and location of perforations not only determine the rate of water inflow but also to some extent influence the strength of the draintubes. Thus, for an optimal design of draintubes for a drainage system the smallest but still sufficient number of perforations should be used. The influence of draintube geometry in relation to shape, size, location, and number of circular perforations on draintube inflow has been reported by Panu and Stammers (1977). For the design of subsurface drainage systems involving long perforated draintubes, there is a need for theoretical expressions to be used in the estimation of flow rate into the draintubes.

As early as 1951 significant attempts have been made by Kirkham and Schwab and in 1953 by Engelund to describe water flow into perforated draintubes. They developed theoretical expressions for inflow rate into perforated smooth draintubes under ponded watertable conditions. However, for practical drainage situations the ponded watertable condition has limited application, since this condition exists for only a small percentage of the total duration of flow. Further, the most common situation in the field is the transient condition of watertable drawdown due to drainage. It is in this regard that information is needed in relation to the effects of size and location of perforations on draintube flow rate and hence indirectly on the characteristics associated with the watertable drawdown. It should be stressed here that draintube flow rate is inversely proportional to draintube spacing and hence an inaccurate estimate of draintube flow rate could easily lead to inadequate (i.e., over or under) estimation of draintube spacing in the design of drainage systems, which may well lead to unwarranted economical losses. This paper attempts to assess the applicability of the theoretical expressions of Kirkham and Schwab (1951) and Engelund (1953) in the estimation of flow rate from perforated smooth draintubes. It also examines the possibility of the extension of these flow expressions to watertable drawdown condition.

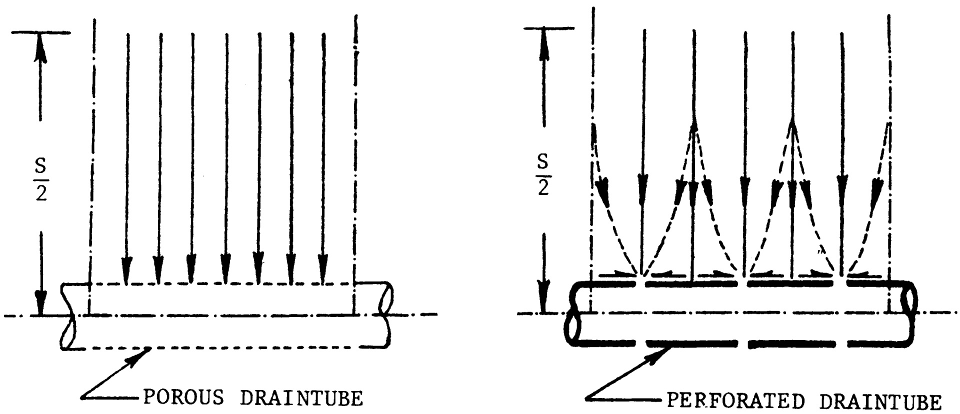
Theory

In a drainage flow system of parallel draintubes spaced at a distance, S, apart in an infinite isotropic soil, the rate of draintube inflow per unit length of draintube for a hypothetical completely porous draintube, Q_0, is directly proportional to the difference in head, Δh, between the watertable in mid-spacing and in the draintube, and is inversely proportional to the resistance to flow in the soil. The flow region between two such hypothetical draintubes is predominantly governed by a two-dimensional flow field (Fig. 1a). In the first case, resistance to flow is offered only by the soil media and can be quantitatively characterized by a multiple of the draintube spacing S and the soil resistance R_s. The units of draintube spacing S is in meters and the term R_s which is per unit length of draintube spacing, has units of sec/meter^2. Thus, the quantity SR_s results in units of sec/meter. Following Ernst (1962) therefore the drain discharge for a completely porous draintube may be written as

$$Q_0 = \frac{\Delta h}{SR_s} \tag{1}$$

Similarly, the resistance to flow toward a perforated draintube may be represented by two resistances, R_c and SR_s, arranged in series. The additional resitance R_c in sec/meter is in the three-dimensional flow region near the perforated draintube (Fig. 1b). Thus, the drain discharge for a perforated draintube may be expressed as

$$Q \equiv \frac{\Delta h}{R_c + SR_s} \tag{2}$$



a) two-dimensional flow b) three-dimensional flow

Fig. 1. Geometry of drainage flow.

In Eq. (2) the term R_c is the resistance to flow associated with the conversion of the flow field from a two-dimensional field to a three-dimensional field around the perforted draintube. Further, the resistance term, R_c is a convergence resistance due to the three-dimensional curvature of the flow field toward draintube perforations.

Theoretical relationships for flow rate under ponded watertable conditions into a draintube with circular perforations have been reported jointly by Kirkham and Schwab (1951) and independently by Engelund (1953). The expressions of these investigators can be stated as

$$Q \equiv \frac{2\pi K(d+t-r_w)}{\lambda+\ln\left(\frac{2d}{r_w}\right)} \tag{3}$$

The term λ in Eq. (3) is a function of size, spacing, and radial location of circular perforations in the draintube and has been expressed by Kirkham and Schwab (1951) as

$$\lambda = \frac{1}{m}\{2\sum_{n=1}^{\infty} k_0(2n\pi\rho_p) + 2\sum_{i=1}^{m-1}[\sum_{n=1}^{\infty} k_0(4n\pi\rho_w \sin\frac{\theta_i}{2})] + \ln(\rho_w+\rho_p) - \sum_{i=1}^{m-1}\ln(2\sin\frac{\theta_i}{2})\} \tag{4}$$

while Engelund (1953) has expressed the term λ in Eq. (3) as

$$\lambda = \frac{\varepsilon_1\varepsilon_2}{4r_w r_p}\{1-\frac{2}{\pi}[3.91-2\ln(\frac{\varepsilon_2}{\varepsilon_1})]\frac{r_p}{\varepsilon_1}\} \tag{5}$$

(See: notations page 181)

The theoretical expression for flow rate into a perforated smooth draintube given by Engelund (1953) is equal to the one expressed by Kirkham and Schwab (1951) if the term λ in Eq. (4) and Eq. (5) is equal. Engelund (1953), however, reported that his theoretical expression for flow rate into a perforated smooth draintube agreed well with the results determined from the electrical analogue of Schwab and Kirkham (1951). Schwab *et al.* (1969) on the other hand have reported that the values of discharge rate determined from the expression of Engelund (1953) for a particular case of 51 mm (2 inch) diameter draintube were considerably lower than those determined from the expression of Kirkham and Schwab (1951).

Kirkham and Schwab (1951) have also developed a theoretical expression for flow rate under ponded watertable conditions for a hypothetical completely porous draintube as

$$Q_0 = \frac{2\pi K(d+t-r_w)}{\ln(\frac{2d}{r_w})} \tag{6}$$

The theoretical expressions in Eq. (6) and Eq. (3) are equivalent to the Eqs. (1) and (2), respectively, provided that the terms $\lambda/2\pi K$ and $\ln(2d/r_w)/2\pi K$ are considered analogous to R_c and SR_s respectively in these equations. In Eqs. (1) and (2), Q_0, Q and Δh would be time dependent when representing a transient watertable situation. This simple series-resistance-flow model provides a convenient means for assessing the applicability of theoretical expressions of Kirkham and Schwab (1951) and Engelund (1953) for estimation of flow rate into perforated smooth draintubes.

Towards achieving the objective of this paper, the convergence resistance, R_c, may be expressed explicitly in terms of R_s by combining Eqs. (1) and (2) as follows

$$\frac{R_c}{R_s} = S\left(\frac{Q_0}{Q} - 1\right) \tag{7}$$

The term R_c/R_s has the dimension of length and the units of meters. It is the equivalent length of drain spacing due to the presence of the real draintube. The ratio is called, because of this, the Convergence-Resistance-Equivalent, i.e. the equivalent length of drain spacing due to flow convergence (Panu and Stammers 1977). It is determined from flow measurements only. Eq. (7) is employed to examine the experimental results in relation to theoretical expressions for flow rates. When a perforated draintube behaves like a completely porous draintube, Q is equal to Q_0 and thus ratio R_c/R_s becomes equal to zero. This implies that there exists only a two-dimensional flow situation and that there is no additional resistance to flow due to the draintube. The ratio R_c/R_s, as is apparent from Eq. (7), is independent of the hydraulic conductivity of the media through which flow takes place.

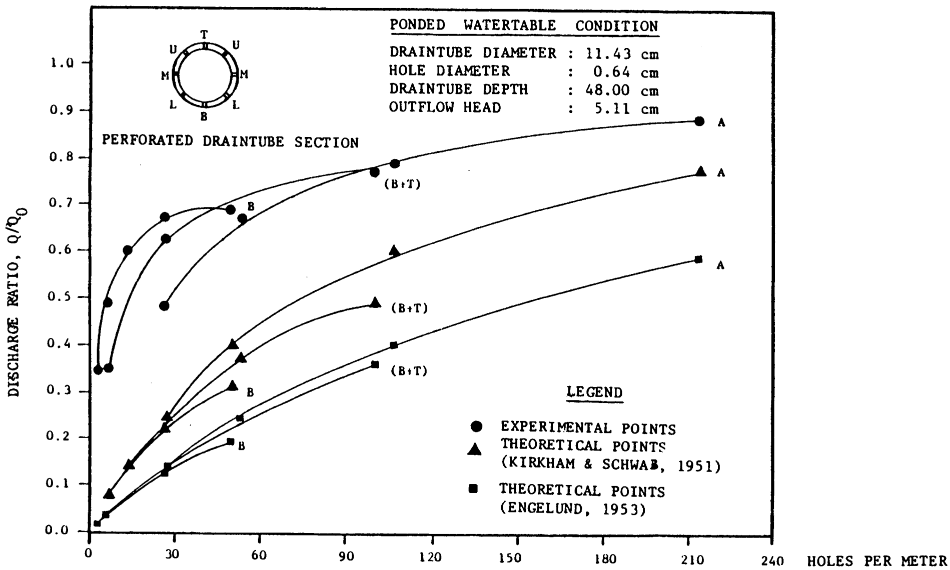


Fig. 2. Comparison of experimental and theoretical discharge ratios for various radial locations of holes.

Experimental Methods

Description of the Apparatus

A physical model suitable for the simulation of steady state ponded watertable as well as curved watertable conditions was developed. It consisted of a sand tank of inside dimensions 3.3 × 0.305 × 1.22 m (130 × 12 × 48 in) The other features of the physical model were end-water reservoirs, constant-head-tank, outflow-control device, draintubes, and measuring devices. Further details about the experimental apparatus are given by Panu and Stammers (1977).

Experimental and Analytical Procedure

The experimental procedures were designed to simulate, as closely as possible, field conditions for flow into a newly installed draintube. A summary of the experimental tests conducted on perforated smooth draintubes is provided in Table 1. In this table a perforated draintube refers to a 11.43 cm outer diameter (O.D.) smooth draintube of wall thickness 6.4 mm (0.25 in). The draintube had eight rows of circular holes radially spaced at 45 degrees as shown in Fig. 2. Where B, L, M, U, and T stand for bottom row, lower row, middle row, upper row and top row of holes, respectively. Further, 2L, 2M and 2U in the ensuing text are used to represent both lower rows, middle rows, and upper rows of holes, respectively. For example, a combination, B + 2M, represents a case where holes are located in the

Table 1 – Summary of drainboundary conditions of the experimental tests.

Watertable condition	Δh, cm	Perforated draintube						Porous draintube	
		Outflow head							
		(0.505 cm)			(5.105 cm)			(0.505 cm)	(5.105 cm)
		Diameter of holes, cm							
		0.16	0.32	0.64	0.16	0.32	0.64		
Curved	9.30	–	–	–	–	B, 2L, B + 2L	–	–	*
	10.30	–	–	–	–	–	–	–	*
	11.30	–	–	–	A, B, 2L, B + 2L	B, 2L, B + 2L	B, 2L, B + 2L	–	*
	12.30	–	–	–	–	–	–	–	*
	13.30	–	–	–	–	B, 2L, B + 2L	–	–	*
	13.90	–	B, 2L	–	–	–	–	*	–
	14.90	–	–	–	–	–	–	*	–
	15.90	A, B, 2L, B + 2L	B, 2L	B, 2L	–	–	–	*	–
	16.90	–	–	–	–	–	–	*	–
	17.90	–	B, 2L	–	–	–	–	*	–
Ponded	49.50	–	–	–	–	–	A, B, T, B + T, 2M	–	*

Note – All perforated draintubes were tested for hole spacing of 1.91, 3.81, 7.62, 15.24 and 30.5 cm in a row.
A = B + 2L + 2M + 2U + T
Δh = Difference in head between the piezometric head at midspacing and the draintube (see: text).
* ≡ Test carried out for porous draintube.

bottom (B) row and two rows at the middle (M) location in a perforated draintube. Each row extended over a distance of 0.305 m (12 in) with 15 circular holes at 0.019 m spacing. This facilitated a symmetrical hole spacing of 0.019, 0.038, 0.076, 0.15 and 0.30 m (0.75, 1.5, 3, 6 and 12 in.) to a row. The spacing alteration between holes in a row and among rows was achieved by using wooden plugs to block and deactivate certain holes in the draintube.

The porous draintube referred to in Table 1 was simulated by a cylinder of 0.416 mm (0.018 in) thickness copper screen (6,997 holes/m^2 of 0.41 mm diameter). The outer diameter of the porous draintube was made equal to that of the perforated draintube with circular holes.

The perforated and porous draintubes were tested under curved and ponded watertable conditions. The tests were initiated after steady-state flow conditions had been achieved. Tests on perforated draintubes with 0.16, 0.32 and 0.64 cm diameter holes were followed by tests with the porous draintube for both watertable conditions (Table 1). Tests for curved watertable condition for two outflow heads were also performed using the porous draintube as detailed in Table 1. The results of these trials provided information for developing Q_0 vs. Δh relationship for porous draintube. Tests were also carried out for a ponded water depth of 0.025 m on the sand surface. The combinations of draintube conditions and outflow head arrangement were randomized during any one series of draintube condition tests in order to minimize possible systematic errors. The same sand media was used throughout all experimental runs.

Computation of the Convergence-Resistance-Equivalent

The computation of Convergence-Resistance-Equivalent (CRE) for each perforated draintube was performed in the following manner. For each test on a perforated draintube the value of Q_0 required for the computation of the CRE was determined from the Q_0-Δh relationship. For the perforated draintube under the curved watertable condition, Δh is defined as the difference in water elevation between the piezometric head at the interface of water and sand media at midspacing and the maximum of the elevation of the highest discharging row of holes in the draintube and the outflow head in the draintube, that is:

Δh = {depth of watertable at midspacing – maximum (Elevation of the highest discharging row or depth of water in the draintube)}.

The value of Δh under the ponded watertable condition is equal to the elevation difference between water level at the top surface of sand media and the water level in the draintube. The value of CRE is then computed using Eq. (7) in which S in the physical flow model equals to 305 cm (120 in). The three-dimensional flow region does not extend beyond 1.22 m (Jones 1960) on either side of a draintube. Hence, for different perforated draintube conditions, the values of CRE reported in this paper may be considered valid for any draintube spacing.

Assumptions and Boundary Conditions in Theoretical Expressions and Experimental Investigations

The theoretical expressions of flow rate into smooth draintubes with perforations given by Kirkham and Schwab (1951) and Engelund (1953) are based on the assumption that such parallel draintubes are spaced at a distance, S, apart in an infinite isotropic soil media. Further, in the development of these expressions, the drainboundary is approximated by specifying that the circumference of the draintube is an equipotential surface. For a porous draintube such a drainboundary is assumed to be completely open and where as for a perforated draintube the entry of water is only permitted through the perforations. Such an assumption implicitly implies that draintube is flowing full with no back pressure. With this assumption, the head difference (Δh) causing the flow through perforations in a given row (B, L, M, U or T) location does not change in theoretical equations for a given flow condition (i.e., the curved watertable or the ponded watertable). For example, under ponded watertable condition, the head difference (Δh) causing flow into perforated draintube for any location of row of holes is obtained as $\Delta h = (d + t - r_w)$ in theoretical expressions (Eqs. (3) and (6)).

In the experimental investigations, the draintube has a spacing of 3.05 m and is located 0.61 m above an impervious boundary in an isotropic soil media. To minimize the effect of these physical dimensions on drain inflow in the sand tank model, the spacing of draintube is based on electrical analog studies by Jones (1960) and the depth of impervious boundary after the work of Kirkham (1949) and Luthin (1957). The drainboundary is represented by the physical draintube itself and the head difference (Δh) causing flow into perforated draintube is obtained as described earlier (see section: Computation of Convergence-Resistance-Equivalent). Thus, for a given flow condition (i.e., the curved watertable or the ponded watertable) the head difference (Δh) causing the flow through the perforations is highest for the holes in the bottom (B) row and is the lowest for the holes in the top (T) row.

Applicability of Theoretical Expressions in View of Experimental Results

The experimental results indicate that the Convergence-Resistance-Equivalent (CRE) is a function of radial location, size, and number of holes in the draintube (Panu and Stammers 1977). However, it is appropriate to make a relevant comparision of experimental results with the results obtained by numerically evaluating the theoretical expressions for the same boundary conditions as were maintained during the experimental investigations.

The results pertaining to ratios of flow rates (Q/Q_0) as a function of number of circular perforations for the author's experimental tests and the theoretical expressions of Kirkham and Schwab (1951) and Engelund (1953) are exhibited in Fig. 2. In this figure the curves for the experimental tests are higher, generally, than the theoretical curves of both Kirkham and Schwab (1951) and Engelund (1953).

For the case A (i.e., holes on all eight rows), the ratios (Q/Q_0) pertaining to theoretical expressions in Fig. 2 are expressed below in terms of percent decrease when compared to the ratios (Q/Q_0) for the experimental results.

Holes per Meter	Kirkham and Schwab (1951)	Engelund (1952)
30	47	72
60	39	59
90	29	53
120	23	46
150	19	41
180	17	39
210	13	34

From the above table it is apparent that the percent decease in ratios (Q/Q_0) is greater when there are 30 holes per meter. The percentages in both cases steadily decrease as the number of holes per meter increases. For example, in case of 210 holes per meter the decrease in ratios (Q/Q_0) is 13 percent in the case of Kirkham and Schwab (1951) and 34 percent in the case of Engelund (1953). It should be emphasized that the theoretical ratios (Q/Q_0) curve of Engelund (1953) is lower than the curve of Kirkham and Schwab (1951). A similar effect for a particular case of 51 mm (2 in) diameter draintube was observed by Schwab *et al.* (1951).

An explanation in regard to why the ratios (Q/Q_0) are generally higher for the experimental results is provided by examining the results of experimental tests carried out for ponded watertable condition as well as the curved watertable condition. A summary of such test results is provided in the Appendix. From the results presented in the Appendix, it is apparent that if the flow rate (Q_0) was measured with the same ponded watertable condition for the case of full flowing draintube (i.e., when the depth of water in the draintube was raised from 5.105 to 11.430 cm), the flow rate (Q_0) would have been reduced at the most by 43 percent. Similarly, such a decrease in flow rate (Q) for perforated draintube would have also been observed as the depth of water in the draintube was raised from 5.105 to 11.430 cm (i.e., full flowing draintube). Thus, it is reasonable to conclude that the ratios (Q/Q_0) for the experimental test results in case A (i.e., holes on all eight rows) will not change.

A comparison of the theoretical expressions of Kirkham and Schwab (1951) and Engelund (1953) with the author's experimental results for the effect of radial location of circular perforations on flow rate is also shown in Fig. 2. In this figure B, B + T and A represent bottom row, bottom and top rows, and all eight rows, respectively. It is interesting to note that the positions of the theoretical curves of Kirkham and Schwab (1951) and Engelund (1953) fall below the author's experimental curves. Further, for the cases when bottom (B) row, bottom and top (B+T) rows, and all of eight rows (A) are operative, the theoretical curves fall consistently in reverse order i.e. (A, B+T, and B) of positions than that of the order of positions (B, B+T, and A) as obtained by the author through experimental investigations.

In the case of holes in bottom (B) row, the inflow should decrease as the depth of water in the draintube is raised to full flowing condition (i.e., from 5.105 to 11.430 cm). Such a decrease in flow rate for bottom (B) row under ponded watertable condition is caused by the increase in the length of flow path for the bottom (B) row as compared to the top (T) row because the head difference (Δh) causing the flow through perforations in both rows is the same. Likewise, it is possible to conceive that the inflow from other locations of rows (i.e., L, M, and U) should decrease but such a decrease in flow would be less as the location of row is raised from bottom (B) row to the top (T) row. With such considerations in decrease of inflow for various location of rows of perforations, the ratios (Q/Q_0) for bottom (B) row may decrease such that the order of curves may conform to that of theoretical curves. Further, experimental tests are desired to investigate such an anomaly. It is stressed that the ratios (Q/Q_0) for experimental results will still be higher for all cases af A, B and B+T. However, it is noted that the order of positions of the experimental curves for any radial location of circular perforations is consistent with the experimental curves for similar situation reported by Panu and Stammers (1977).

The effect of radial location of circular perforations on the ratio of flow rates (Q/Q_0) for the distribution of circular perforations in the draintube is presented in Fig. 3. It is evident from the figure that, for an increase in the value of ζ, there is an increase in the theoretical ratio of flow rates irrespective of the radial location of circular perforations in the draintube. The ratio of flow rates for the author's experimental results is dependent upon the radial location of circular perforations (Fig. 3). As noted in earlier discussions, it is conceivable that as the depth of water in draintube is raised from 5.105 to 11.430 cm (i.e., full flowing draintube) the order of experimental curves for different locations of rows of holes may conform to the order exhibited by the theoretical expressions. However, the magnitude of ratios (Q/Q_0) for the experimental results will not change for reasons presented in Appendix. It is noted that the order of difference in ratios (Q/Q_0) for the experimental curves and theoretical curves for both cases of 26 holes and 52 holes per meter is approximately the same.

Further, since the term λ and the term R_c (Eqs. (2) and (3)) are similar and are related as follows

$$R_c = \alpha\lambda$$

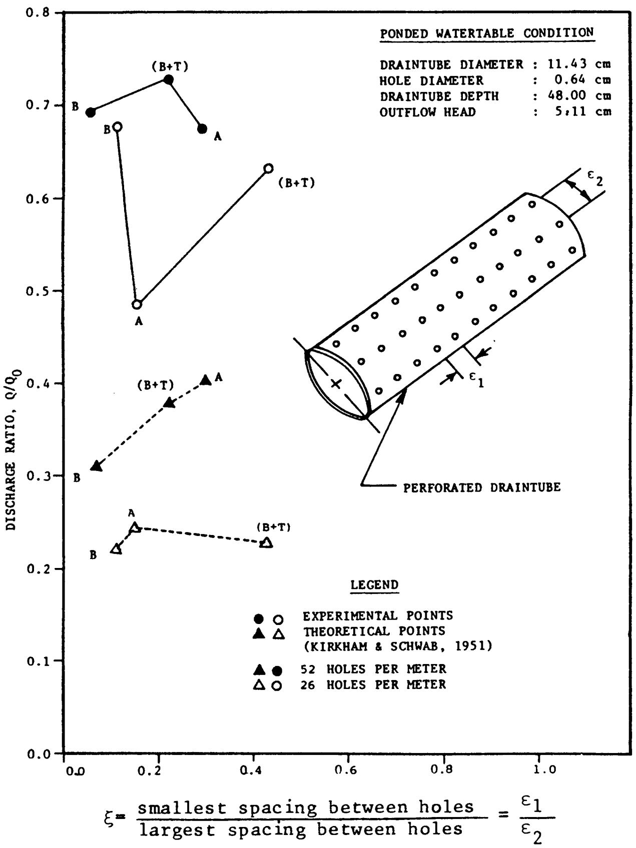


Fig. 3. Effect of distribution of perforations on relative discharge.

where, α (= $1/2\pi K$) is a constant of proportionality, R_c is the convergence resistance due to the presence of the draintube, and λ is as expressed in Eqs. (4) and (5). Hence for the ponded watertable condition R_c and λ should be linearly related irrespective of the radial location of circular perforations. However, an examination of Figs. 4 and 5 indicates that in case of experimental test results a separate straight-line relationship exists for each radial location (i.e., B, B+T, and A rows) of circular perforations in the draintube. It is also apparent from these figures that the theoretical results calculated from the expressions of Kirkham and Schwab (1951) (Fig. 4) and Engelund (1953) (Fig. 5) only exhibit a single straight line relationship. On the basis of earlier discussions, the ratios (Q/Q_0) for the perforations in bottom (B) row would decrease in experimental results as the depth of water is raised from 5.105 to 11.430 cm (i.e. full flowing draintube). Such a decrease in ratios (Q/Q_0) for perforations in bottom (B) row should cause corresponding increase in the values of (R_c/R_s) calculated using Eq. (7). As a result the

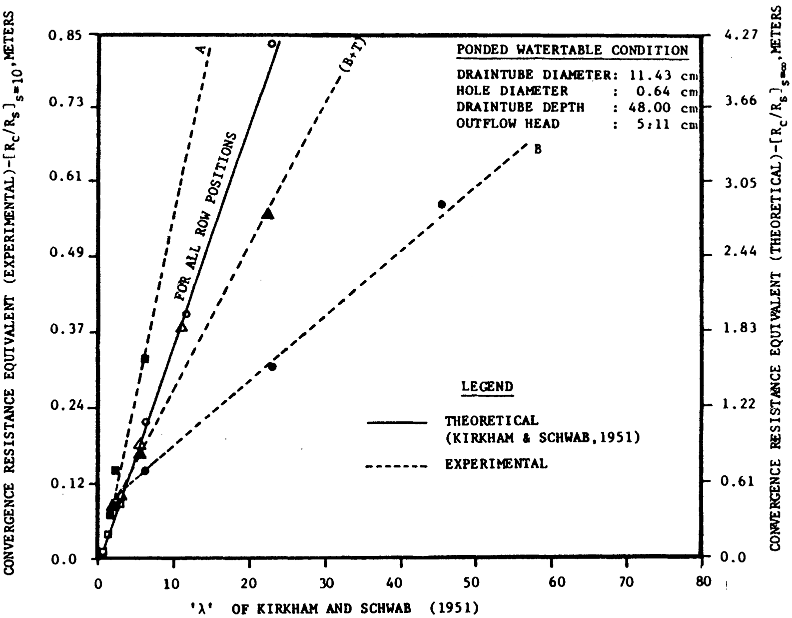


Fig. 4. Comparison of experimental and theoretical convergence resistance equivalent. λ of Kirkham and Schwab (1951).

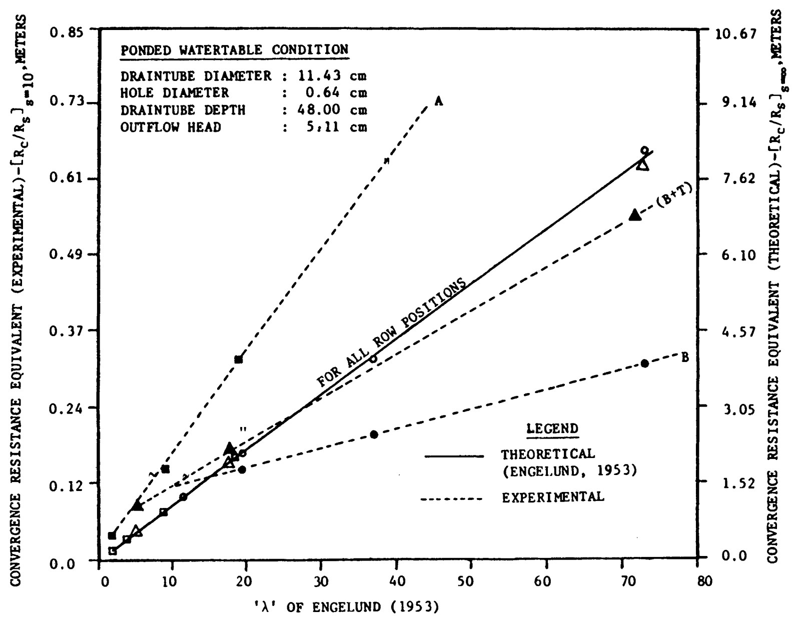


Fig. 5. Comparison of experimental and theoretical convergence resistance equivalent. λ of Engelund (1953).

experimental curves for B and (B+T) should exhibit a tendency to shift towards the curve represented by theoretical expressions in Figs. 4 and 5. However, such a shift in the curve for the case A (i.e., holes on all eight rows) is not apparent because the ratios (Q/Q_0) for the case A, as discussed earlier, will not change.

Based on above discussion, it appears that there happens, at least, some effect of the radial location of circular perforation on drain flow rate, which is indeed not represented by the theoretical expressions of Kirkham and Schwab (1951) and Engelund (1953).

To overcome the difficulty associated with theoretical expressions in estimation of flow rates into a partial flowing perforated draintube under ponded watertable and curved watertable conditions, there is a need to develop a suitable procedure for relating experimental results with the theoretical expressions. A theoretical understanding of the behaviour of water inflow into partially flowing draintube under curved watertable is generally lacking. However, in the following a tentative procedure is suggested to relate experimental results with the theoretical expressions under curved watertable condition.

Approximation of the Convergence Resistance

From Eq. (2) the convergence resitance, R_c is a function of radial location, number, size, and spacing of circular perforations. The approximation of the convergence resistance, R_c, therefore, has been sought by the use of the term λ, in order to extend the applicability of the theoretical expressions of Kirkham and Schwab (1951) and Engelund (1953) for curved watertable conditions. It should be re-emphasized here that the curved watertable conditions are a more realistic representation of the watertable conditions in the field.

Through experimental investigations it was observed that the relationships for the effect of the radial location of circular perforations in the draintube for the curved watertable condition (Fig. 6) are similar to those for the ponded watertable conditions (Figs. 4 and 5). However, it may be noted from Fig. 6 that although the curves are similar, they are not straight lines. It is obvious that the curved watertable system does not behave like the ponded watertable system, because both the flow systems are different.

Thus, from the plot of Convergence-Resistance-Equivalent (CRE) versus λ in Fig. 6 no definite statement can be made about the relationship between the CRE and λ. It may be argued that the linear relationship is a first order of approximation. On this basis, one might tentatively use the term λ, suitably adjusting it for the radial location of rows, as an estimate of R_c, thus permitting the estimation of the Convergence-Resistance-Equivalent (CRE) for design purposes.

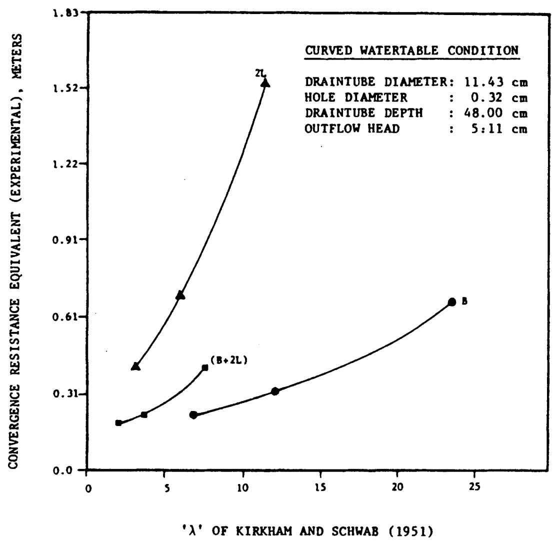


Fig. 6. Relationship between λ of Kirkham and Schwab (1951) and convergence resistance equivalent.

Conclusions

The experimental results show the pronounced effect of radial location of circular perforations on flow rate from smooth draintube, which is not adequately described by the theoretical expressions of Kirkham and Schwab (1951) and Engelund (1953). Further studies including experiments both in field and in laboratory need be carried out to better define the relationship between Convergence-Resistance-Equivalent and λ under watertable drawdown conditions.

Acknowledgement

The research reported herein was partially supported by a grant awarded by Natural Sciences and Engineering Research Council of Canada. The author is thankful to one of the anonymous reviewers for his valuable suggetions in enhancing the interpretation of the results presented in this paper.

Notation

The following symbols are used in this paper:

d – depth of the centre of draintube from soil surface
i – an integer having values 1,2,3,..., (m-l)
k_0 – Bessel function of second kind and zero order
K – hydraulic conductivity of flow medium
m – number of rows of perforations
n – an integer having values 1,2,3,...,
Q – flow rate per unit length of the perforated draintube
Q_0 – flow rate per unit length of the porous draintube
r_p – radius of circular perforations
r_w – outer radius of draintube
R_c – flow resistance in sec/meter in the three-dimensional flow region near the draintube
R_s – unit soil resistance in sec/square-meter in the mainly two-dimensional flow region between draintubes
S – draintubes spacing
t – thickness of the layer of surface water
θ_i – angle between the reference plane and plane passing through the draintube axis and line of perforations
ε – spacing between perforations in draintube
ε_1&ε_2 – respectively are the smallest and the largest spacings between perforations in draintube
ζ – ratio of $\varepsilon_1/\varepsilon_2$
ϱ_p&ϱ_w – respectively are r_p/ε, and r_w/ε
Δh – difference in head between piezometric head at midspacing and the draintube (see: text).

References

Engelund, F. (1953) On the Laminar and Turbulent Flows of Groundwater through Homogeneous Sand, Trans., Danish Acad. Techn. Sci. No. 3.

Ernst, L.F. (1962) (Abstract from Van Schilfgaarde, J., Theory of Flow to Drains, Advances in Hydroscience No. 6. pp. 43-103, Academic Press, New York, 1970).

Jones, B.A., Jr. (1960) Effect of Crack Width at Tile Joints on Soil Movements into Draintile Lines, *Trans. of the ASAE, Vol. 3(1),* pp. 33-51.

Kirkham, D. (1949) Flow of Ponded Water in Drain Tubes in Soil Overlying on Impervious Layer, *Trans., Am. Geophy. Union, Vol. 30,* pp. 369-385.

Kirkham, D., and Schwab, G.O. (1951) The Effect of Circular Perforations on Flow into Subsurface Draintubes, *Agril. Engineering, Vol. 32(1),* pp. 211-214.

Luthin, J.N. (1957) Drainage of Agricultural Lands, Am. Soc. Agric. Mono. 7, Madison, Wisconsin.

Panu, U.S., and Stammers, W.N. (1977) Effect of Drainboundary on Subsurface Drain Design, ASAE Publ. 1-77, pp. 22-26 and 30.

Schwab, G.O., and Kirkham, D. (1951) The Effect of Circular Perforations on Flow into Subsurface Draintubes, *Agril. Engineering, Vol. 32(5),* pp. 270-274.

Schwab, G.O., DeBoer, D.W., and Johnson, H.P. (1969) Effect of Openings on Design of Subsurface Drains, *Jour. of the Irrigation and Drainage Div., ASCE, Vol. 96,* No. IR 1, pp. 199-209.

First Received: 21 February, 1987
Revised version received: 12 June, 1987

Address:
School of Engineering,
Lakehead University,
Thunder Bay, Ont.,
P7B 5E1 Canada.

Appendix – Analysis of Experimental Flow Rates into Porous Draintube

Although the experimental tests were carried out for the partial flowing draintube under ponded watertable and curved watertable conditions, in the following an attempt is made to present an analysis of the experimental flow rates into porous draintube such that an estimate of flow rate into porous draintube can be made for the full flowing draintube condition.

The flow measurements were made corresponding to one water level (i.e., 5.105 cm) in the draintube under the ponded watertable condition. With this information, it is difficult to draw a definite conclusion on the behaviour of flow rate in response to changes in the depth of water in the draintube. However, for a fixed depth of ponded watertable condition in the physical model, the flow into the porous draintube is occurring through its entire surface area and thus flow rate will decrease as a function of the depth of water in the draintubes or in other words the head difference (Δh) causing the flow into the draintube. In order to estimate the flow corresponding to full flowing draintube for the experimental conditions, one needs to develop a relationship between the head difference (Δh) causing the flow into the draintube and the depth of water in the draintube. As indicated earlier, such tests were not carried out for the ponded watertable condition. However, tests were carried out for the curved watertable condition between (Δh) and the two different depths of water in the draintube. The results of these tests are now analyzed such that a reasonable estimate of the magnitude of decrease in flow rate (Q_0) under the ponded watertable condition can be made for the case when the draintube is considered as flowing full.

Depth of Water in draintube (cm)	Surface area of Porous draintube (cm^2)	Δh (cm)	Q_0 (CC/min)	Type of Watertable Condition
0.505	147.6	17.9	1256	Curved
		16.9	1180	Curved
		15.9	1116	Curved
		14.9	1038	Curved
		13.9	967	Curved
5.105	510.0	13.3	999	Curved
		12.3	920	Curved
		11.3	840	Curved
		10.3	766	Curved
		9.3	690	Curved
5.105	1094.5	49.50	9423	Ponded
(11.430)	(1094.5)	(43.81)	(5360)	Ponded

In the above table, the flow rate (Q_0) under the curved watertable condition only changes less than 3.2% when depth of water in the draintube is raised from 0.505 to 5.105 cm (or in other words the surface area of porous draintube is increased from 147.6 to 510.0 cm^2, i.e. an increase of 246% in the surface area) for a similar value of Δh. It is reasonable to assume that an appreciable (i.e., 246%) increase in surface area of the porous draintube does not appear to effect the flow rate (Q_0) into the porous draintube. On the other hand a decrease in one cm of head difference (Δh) causing the flow into porous draintube, on the average, decreases the flow rate (Q_0) by 7.58%. This decrease in flow rate (Q_0) appears to be less for higher depth of water in the draintube.

In the above table, the flow rate (Q_0) under ponded watertable condition was observed to be 9,423 cc/min corresponding to the 5.105 cm depth of water in porous draintube and a value of 49.50 cm for the head difference (Δh) causing the flow into the draintube. Since the flow occurs through the entire surface area (1,094.5 cm^2) of porous draintube for any depth of water in the draintube, therefore the flow rate (Q_0) decreases only as a function of Δh under the ponded watertable condition. For an increase in depth of water in draintube from 5.105 to 11.430 cm to correspond to the full flowing draintube with no back pressure, the head difference (Δh) decreases by 5.69 cm (i.e., from 49.50 to 43.81 cm). One can approximate the flow rate (Q_0) to be 5,360 cc/min (i.e., a decrease of 43%) for the full flowing draintube on the assumption that the rate of decrease of flow rate by 7.58% per cm decrease in head difference (Δh) under the curved watertable condition when considered to be applicable to the ponded watertable condition. Values in parenthesis in the above table are the calculated values for the full flowing draintube under ponded watertable condition.

From the above discussion, it is apparent that the flow rate would decrease as the depth of water in the draintube is increased. Such a decrease in flow rate would occur for both perforated draintube as well as the porous draintube. Futher, such a decrease in flow rate in Q as well as Q_0 results in no appreciable changes in the ratios (Q/Q_0) for the experimental test results.