

THE PARAMETRIC APPROACH TO WATERSHED MODELING

WILLARD M. SNYDER

The parametric approach to watershed modeling is shown to lie between the stochastic and deterministic approaches. Essentially, the parametric approach attempts to find functional relationships between numerically defined hydrologic and physical characteristics of a drainage area.

Successful parametric modeling requires progression through sequential stages of data processing, model formulation, optimization of parameters, examination of results, association of characteristics, and conversion to prediction procedures. Selected examples of the various stages are discussed.

“Parametric Hydrology is defined as the development and analysis of relationships among the physical parameters involved in hydrologic events and the use of these relationships to generate, or synthesize hydrologic events. Historical hydrologic data and known physical data generally are utilized to develop the relationships” (Committee on Surface Water Hydrology 1965).

It is desirable to modify this definition slightly to describe the parametric approach to watershed modeling. For purposes of this paper it is proposed that the parametric approach to watershed modeling be the development and analysis of relationships among the hydrologic and physical characteristics of the drainage area contributing streamflow. Utilization of historical hydrologic data and known physical data remain part of the definition.

The change of wording from “physical parameters” to “physical charac-

Contribution from the Southern Branch, Soil and Water Conservation Research Division, Agricultural Research Service, USDA, in cooperation with the University of Georgia and the Georgia Institute of Technology.

teristics” is deliberate. If the word “parameter” is limited to a mathematical connotation, the development of a fuller meaning of “a parametric approach” is made easier.

Parameters are defined to be mathematical terms in a functional relationship between variables. These terms operate to scale, shift, or shape the relationship. Consider the very elementary general function shown in Eq. (1):

$$y = c(x - a)^b \quad (1)$$

In this equation, c is a scaling parameter, a is a shifting parameter, and b is a shaping parameter. In Eq. 1 the variable x could represent some physical property of a drainage area and y could represent some hydrologic characteristic. For example, equations of this general form are often used to relate flood peak, y , to drainage size, x .

Eq. (1) can be called an explicit algebraic relationship. Not all hydrologic or physical characteristics are so explicitly expressed. A unit hydrograph can be represented as a set of tabular values instead of an equation. The cumulative area-elevation characteristics of a watershed may be represented as a graph – the hypsometric curve – instead of an equation. Whether the hydrologic and physical characteristics are given in explicit algebraic form or whether they are expressed as tables or curves, the parametric approach still means the development of relationships between characteristics. The parametric approach must be made up of two major phases: first, a concept of the desired relationship of characteristics must be developed, and, second, the parameters of the relationship must be quantified.

The parametric approach to watershed modeling must be placed in proper relationship to the other approaches, stochastic and deterministic. One comparison results from consideration of the different approaches as giving the position of a researcher on some scale of information. If the researcher is in a position near zero information of cause-and-effect, then he must regard the hydrologic characteristics as stochastic events. Without any information on flood-causing mechanisms, floods are regarded as chance results of a toss of hydrologic dice. If, on the other hand, the researcher is in a position near perfect information of cause-and-effect, then he regards the hydrologic characteristics as unique consequences of the situation. The flood peak now results from the solution of hydrodynamic equations describing flow into and through a reach of channel.

Following the idea of an information scale, parametric hydrology lies between stochastic hydrology and deterministic hydrology. Frequently we have some notion of cause-and-effect, so we are not in a purely stochastic position. However, we frequently do not have the required perfection of information

for the rigorous mathematics of determinism. The compromise position is the domain of parametric hydrology. If we postulate that man has a natural desire to attain better understanding of what he observes, then we may state that the goal of parametric hydrology is to retreat from stochasticism and advance toward determinism.

Since the goal of parametric hydrology is to advance toward determinism, one could ask, "Why not proceed directly to construction of deterministic models of a watershed?" For situations where it is presently possible, a deterministic approach should be taken. But, a close scrutiny of our system reveals roadblocks to this direct approach. These roadblocks are revealed by comparing man-made and natural systems. Man-made systems follow man-made designs. Such designs are, whenever feasible, a deterministic expression of system characteristics that are necessary to attain some objective. All the system components are rigorously expressed in the design. Such rigor does not normally carry over into natural systems. Particularly in watershed studies, the lack of spatial homogeneity and continuity in characteristics such as soil type, land use, geology, and topography results in a system which defies precise definition.

The complexity of the detail of natural systems requires that analytical approaches alternative to classical mathematics be developed. At the risk of oversimplification, it might be said that a macro-scale of cause-and-effect is necessary to substitute for the micro-scale of differential calculus. For example, it may be necessary to impute the gross action in streamflow generation of a significantly sized drainage area instead of deducing such action by areal integration starting from an infinitesimal control area. As another example, we may need to average the flood-attenuating effect of a reach of natural channel when the channel is so complex in configuration as to preclude solution of the simultaneous partial differential equations of continuity of mass and momentum.

The macro-scale concept of physical logic is, of course, an attempt to make respectable the so-called "black-box" procedures in input-output analysis. Black boxes have their purpose when they allow satisfactorily approximate solutions to problems not amenable to classic deductive procedures. After all, we can regard Manning's " n ", or a diffusion coefficient, or any other coefficient requiring empirical determination, as a black box. But far more important than this utilitarian aspect is the idea that black boxes may be stepping stones to a higher level of understanding. Again, postulate the natural inquisitiveness of man and we additionally postulate that natural systems, in all their complexity, are to some degree understandable. With these motivations we can construct experiments to test the consistency and adequacy of

black-box structure. Such experiments, with successful replication, serve to define our black boxes as macro-scale components and linkages of a hydrologic system. Successful experimentation allows us to move towards determinism on our information side.

PARAMETRIC APPROACH TO ANALYTICAL MODELS

Before proceeding further in a parametric approach to modeling, it is necessary to define our level of understanding of the various processes we wish to model. If we have a concept of the model and have it formalized and quantified, then we may use our model to predict, or synthesize, new data. If, on the other hand, we have not quantified our concept, we have only a hypothesis. It is necessary to verify the hypothesis against real-world data to move it from abstraction to reality. Verification of the hypothesis is a resultant rational quantification. An irrational quantification obviously prohibits acceptance of the concept.

When we proceed to verify a hypothetical concept, we are following an analytical approach. When we have arrived at a position of sufficient understanding to quantify our concept and then apply it, we are following a simulative approach.

The steps in the parametric approach presented and illustrated below have to do with analysis of data rather than simulation of data. The steps are listed sequentially, but obviously many of the steps overlap and are worked on simultaneously.

Data processing

Since analytical models presuppose data for verification, the organization of data into usable form is given as the first step. Following instrumental and manual recording, it is necessary to check the data for accuracy against reference information and to fill in missing data by estimation when feasible. In modern usage the above steps imply computer procedures and the end product is a magnetic tape library of edited data.

Model formulation

It is hardly conceivable that a research watershed would be established without a stated research objective. Therefore, it is necessary to regard the water-

shed as a means to provide data in the approach to the objective. The objective, until it is attained, must be in the nature of a hypothesis. The information forms into which the data will be cast to test the various hypotheses under the objective establish the conceptual analytical models. The information forms range from simple means and variances to mathematical components of a system to generate streamflow or other outflow.

Fitting routines

The verification and quantification of a hypothetical conceptual model require numerical procedures to optimize the correspondence between the concept and the data. We must fit the model to the data by getting "best" estimates of the numerical values of the parameters. The optimizing processes range over tremendous differences in complexity just as concepts range in structural complexity. The basic techniques of conventional statistics and operations research, as well as the more exotic techniques from such areas as multivariate statistics and systems theory, should all be considered potentially useful in optimization.

Examination of results

Following the quantification of the concept by fitting it to data, the numerical results are checked to establish whether the hypotheses have been verified. Three general modes cover most methods of examination. First, tests of significance from probability theory can be employed where the numerical elements have known statistical properties. Second, the physical rationality of the resultant model structure should conform to the investigator's prior knowledge based on experience and judgment. Thirdly, the methods of simulation can be used to test the sensitivity of the various parametric controls of the model.

Association of characteristics

The steps of parametric approach to modeling to this point have dealt almost exclusively with treatment of data from one research area. However, in the previous section (Examination of results) there is a strong implication that the investigator has access to additional information. Most research is not an isolated activity and most modern researchers are not isolated personages. The results of research form a total pool of information. From this pool of information it is necessary to develop cause-and-effect relationships which support the results of individual contributions to the pool.

The function of the pooled information can be stated in another manner. In any one research watershed it is possible to describe the physical situation – the watershed characteristics – in which the experiment is conducted. However, to apply the results of the research to some other watershed – implying a different set of physical characteristics – the variation of results with variation of characteristics must be established. The parameters of the various relationships among hydrologic and physical characteristics will vary because the models are not perfect. The satisfactory explanation of the parametric variation is the heart of the parametric approach.

Conversion to prediction procedures

The final goal of a parametric approach to analysis is prediction. It is at this step that our level of understanding has moved from hypothetical concept to quantified concept. Prediction has a remaining requirement that sampling and observation provide the data necessary to use in prediction. These data are input data to the model and describe the physical factors of any new – possibly ungauged – watershed for which we wish to establish levels of hydrologic characteristics needed for design purposes.

EXAMPLES OF WORK IN PROGRESS

Some examples from the various steps in a parametric approach are given below. The steps as listed are somewhat idealistic in their sequence. Practically, work in the Southeast Watershed Research Center is proceeding at several levels simultaneously. All of the examples given show the concentration upon computer-oriented methods. This concentration is based on three major considerations: (1) to capture the efficiency and time-saving of automated procedures on large masses of data; (2) to establish objectivity of the various analyses and reduce the bias of personal subjectivity; (3) to provide processing continuity, even with changeover of personnel.

Data processing

Data as observed or recorded are rarely in the form necessary for analysis. Instrumental failure and instrumental error must be anticipated. Algorithms for correcting data and estimating missing data are a necessary part of any approach involving experimental verification.

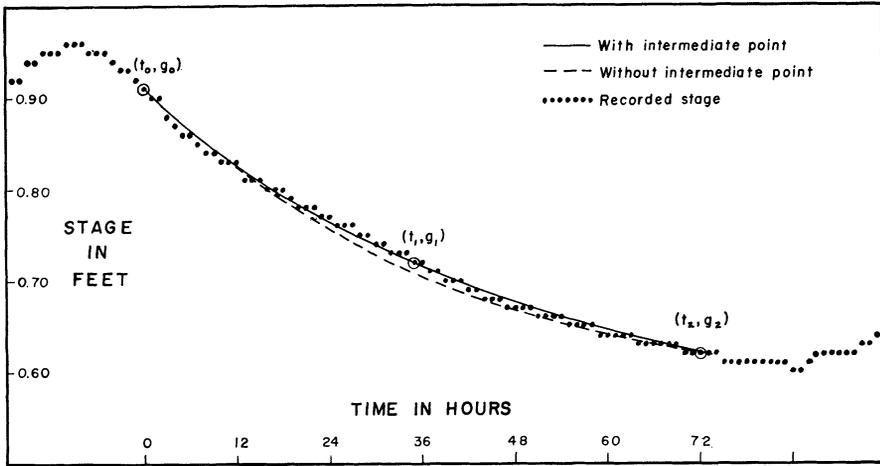


Fig. 1.
Stage estimates for hydrograph recession.

Gauge height estimation and correction. Continuous recording of stream stage with digital-punch gauges is hampered by two main troubles – an error in gauge height due to debris blocking the weir or control or else clock stoppage. We are presently developing an error and estimation procedure based on simple polynomials (Mills & Snyder 1970). Such polynomials can be structured

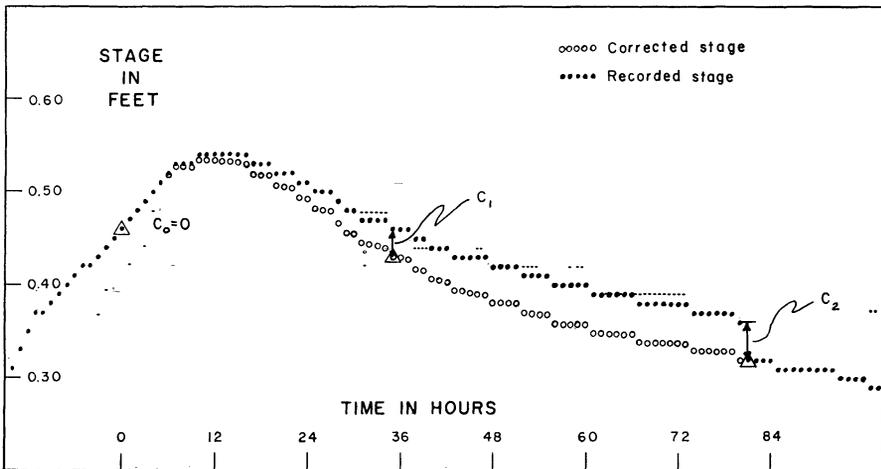


Fig. 2.
Stage correction with known intermediate point.

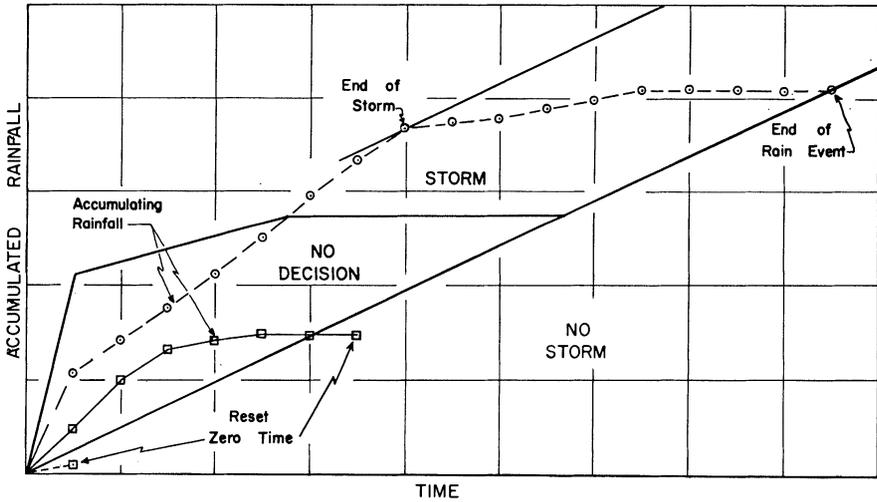


Fig. 3.
Decision algorithm for selection of storms.

to be common and tangent to good record before and after the record break. Additionally, polynomials of higher degree may be required to pass through intermediate points if a field check is made by the observer.

Figure 1 shows a 70-hour period during stream recession where the constrained polynomials were used to estimate a record. Since this is a test case, the true record is shown as a series of data under the estimation. Correspondence is excellent.

Figure 2 shows the correction of a record using constrained polynomials. The result is a smooth transition to eliminate an abrupt change such as might be caused by resetting a gauge or clearing a weir-notch.

Selection of storms. In selecting storm events for use in model verification, attention should be paid to the statistical requirements of the data sampling technique. Special attention is particularly necessary when a model contains a component expressing the volumetric reduction of storm rainfall to storm streamflow. The selection of storms should be based on the rainfall, not on the runoff, since a significant rain can conceivably have little runoff and, consequently, significant volumetric reduction.

An algorithm for selection of rainfall events above some threshold has been proposed previously (Snyder & Curlin 1969) and will be employed or modified for future studies. The algorithm is shown schematically in Figure 3. Starting from some arbitrary point of zero time, the accumulating amount of rainfall

= say, hourly amounts = is checked against previously set decision boundaries. For insignificant rains the event is rejected and the time zero point advanced through the record. For significant rains a storm event is selected and the amount and duration of the storm rainfall are given automatically. The advantage of such an algorithm is that all storms above the decision thresholds are selected from the record. There can be no subjective tendency to reject a storm because of analytical complexity of the consequent hydrograph.

Model formulation

Modeling of several hydrologic elements significant in watershed analysis is being attempted. Each of these models should be regarded as representative of subsystems. A model representative of the total watershed system will, it is hoped, evolve.

Water retention. A mathematical form is under development for quick and easy calculation of rainfall proportion that is effective in the generation of a storm hydrograph (Snyder 1971). This form is defined as watershed retention for two reasons. First, it is hoped that the model will be usable in urban as well as non-urban areas, and the infiltration approach is obviously a poor starting point for urban runoff estimation. Secondly, storm hydrographs are being analyzed without first separating so-called surface runoff from ground-water runoff. In other words, the total response of the stream to an input of effective rainfall is analyzed. This eliminates the subjective separation of flow prior to analysis. The definition that surface runoff is rainfall in excess of infiltration no longer applies. The rate of retention can be thought of as the rate at which dead storage on and in the soil profile is satisfied.

The mathematical form of the retention model is given in Eq. (2).

$$r_{t + \Delta t} = r_t - (a + br_t)AB(r_t - r_c) \Delta t \tag{2}$$

where $A = \frac{R_{\Delta t} + r_u - r_t}{R_{\Delta t} + r_u - r_c}$

$$B = \frac{R_{\Delta t} - r_c}{R_{\Delta t} + r_c}$$

- $r_{t + \Delta t}$ is rate of retention at time $t + \Delta t$
- r_t is rate of retention at time t
- a and b are shape parameters
- $R_{\Delta t}$ is rainfall rate during time increment Δt
- r_u is maximum rate of retention
- r_c is minimum rate of retention

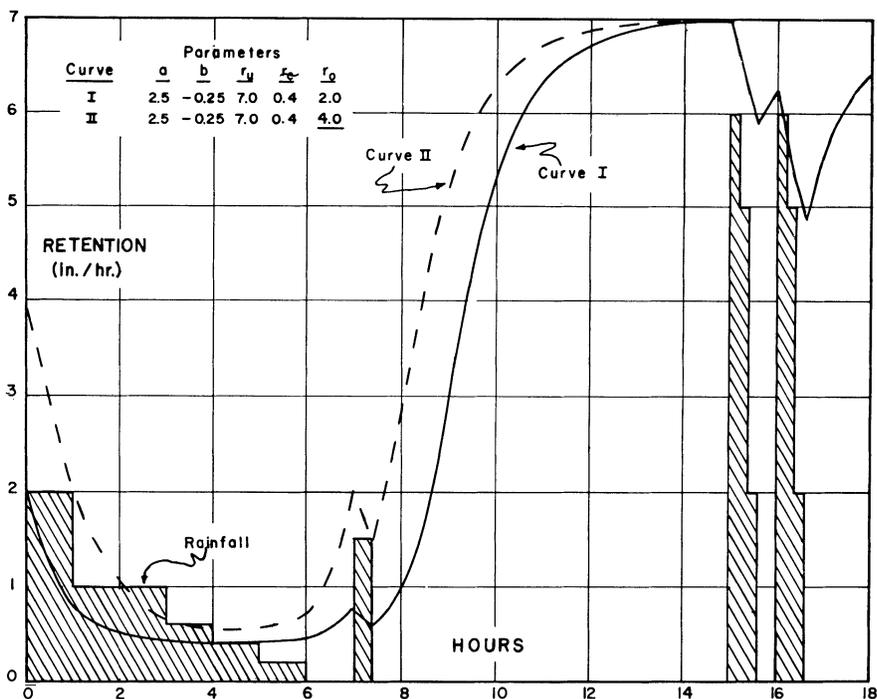


Fig. 4.

Example computations with watershed retention function.

Eq. (2) is a finite difference form of an initial value problem. Given the retention rate at time zero, the rate a short time later can be computed from rainfall and the equation parameters and site characteristics. More work needs to be done in calibration of this model against selected small watersheds and subwatersheds. Figure 4 shows the retention function evaluated for a set of parameters and two different starting values. Effective rain is any rain greater than the retention rate.

Time-separated hydrographs. It was mentioned above that an attempt is being made to analyze total storm streamflow and eliminate arbitrary separation of flow into categories. When information is needed about relative timing of varying proportions of the streamflow, the total hydrograph can be apportioned following analysis. Lines separating the hydrograph into various flow delay times are constructed. It is necessary to derive a unit hydrograph - or more appropriately here, a unit total hydrograph - from the storm or else utilize

The Parametric Approach to Watershed Modeling

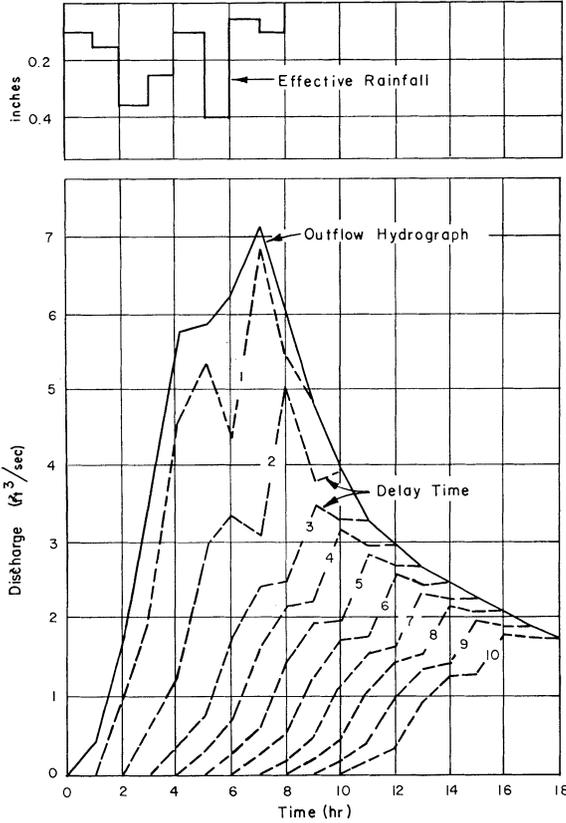


Fig. 5.
Example of time-separated hydrograph.

one already available. The method is probably best illustrated by using a discrete convolution example.

Consider a unit response function defined by its ordinates at 1-hour intervals. The area under the response function from time zero to the 1-hour ordinate represents the proportion of flow having between zero and one hour of travel time from runoff source to the gauge. The area between 1-hour and 2-hour ordinates represents the proportion of flow having from one to two hours of travel time, and so on. If now, one reconstructs the storm hydrograph using the 1-hour increments of effective rain and the unit response with the 1-hour ordinate deleted, the water with zero to one hour of travel time is

eliminated, and a partial storm hydrograph results. Deleting the 2-hour ordinate from the response function additionally eliminates from the total storm hydrograph the water with one to two hours' delay time.

Figure 5 shows a constructed example of time-separation of hydrographs (Snyder & Curlin 1969). The rising dashed lines are partial hydrographs which define the patterns of travel time given the effective rainfall indicated at the top of the figure. The areas in the subdivisions of the hydrograph represent volumes of flow in the different timing categories. More important, however, is the condition that at any elapsed hydrograph time the total discharge can be fractionated into proportions by delay time. This is an important feature if one were, for example, sampling the flow for suspended sediment or some chemical or biological constituent. If such constituent were related to some proportion of rapid flow rather than total flow at sampling time, such proportions are readily computed.

Two-stage convolution. Conventional procedures for outflow hydrograph construction, or for hydrologic flood routing, use a single stage of convolution. Under this procedure an input effective rain or an inflow hydrograph is transformed to an outflow function by operation of a unit response function — usually called in hydrology a unit hydrograph. Such procedures are relatively simple if the response function is assumed to be invariant during the progress of the storm.

It has been amply demonstrated by many researchers that the response function does not remain constant. However, the resulting nonlinear systems are difficult to express and solve mathematically. To overcome some of these difficulties, a discrete computational system involving two stages of convolution has been developed. A schematic comparison of one-stage and two-stage convolution is shown in Figure 6. In the first of two stages, a static characteristic function is convolved with a variable-state function to produce variable unit response functions. The characteristic functions may be regarded as a one-dimensional representation of the potential of the drainage area to produce runoff. The variable-state functions represent the capability of the area to translate this runoff potential to the stream gauge under varying states of “wetness” of the area. Each finite time period during a storm thus has a separate and distinct response function. The second stage of the procedure convolves these response functions, each with its respective rainfall increment.

This concept of two-stage convolution with the first-stage variable has been used in storm hydrograph analysis. A nonlinear yield model utilizing the same concept is under development. It is expected that the potential of the concept in flood routing in natural channels will also be explored. The concept may

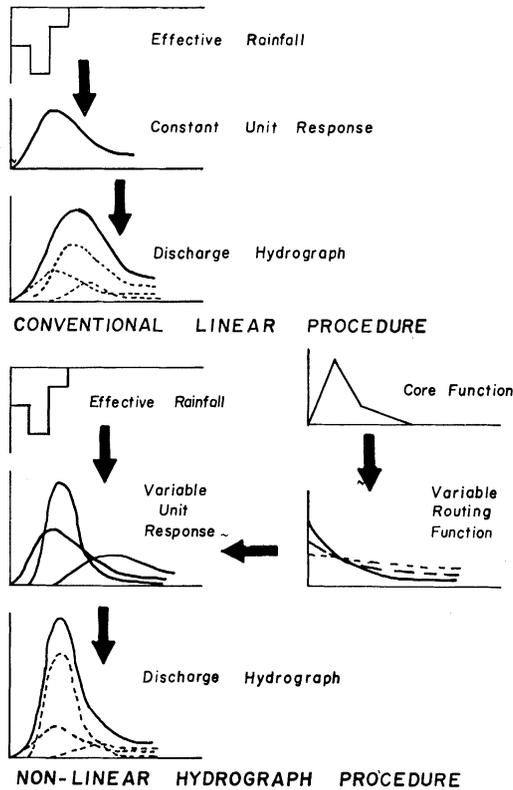


Fig. 6.

Schematic comparison of hydrograph procedures.

also find utility in predicting “waves” of sediment suspended in the stream during storm flow. Some results of quantification of the concept will be presented in the next section.

Fitting routines

In early hydrologic usage, curve fitting often meant no more than matching a straight line to a set of points on a graph. If the points did not line up along a straight line, one looked to see whether their logarithms did. Usually little thought was given to whether a straight line was proper because any other form was considered too complicated to fit. In modern usage, curve fitting means optimization. For analytical models optimization means achieving

a high level correspondence between predictions based on theory and the experimental data. The important modern characteristic is that the concept comes first, and from the concept comes the form, usually mathematical, that is quantified.

A discussion of optimization is not possible here. The methods are many but the fundamental principle is the same. The principle is expressed in the following quotation: "Whatever route a mountain climber takes, he recognizes the peak when he gets there" (Wilde & Beightler 1967).

The "path to the peak" that has been used quite successfully in our modeling work is the principle of least squares. This is expressible as multiple regression when optimization of only linear coefficients is required. Components regression is useful for linear coefficients involving multiple variables instead of the fixed variates of ordinary regression. And lastly, least-squares differential correction coupled with components regression has been found to be an extremely powerful and versatile numerical method (DeCoursey & Snyder 1969).

Watershed retention. It has not been possible to quantify the retention function, Eq. (2), as desired, since retention data of the type needed are not presently available. In order to test the flexibility of the equation, it was fitted to published plot infiltration data (Holtan 1961). Table 1 shows results of fitting

Table 1.
Optimized parameters based on plot infiltration.

Plot	Run date	Optimized values				r_c (in/hr)	Δt (hr)	Cor. coef.
		a	b	r_u (in/hr)	r_o (in/hr)			
1	17 June 40	1.576	-0.099	5.744	1.489	0.19	0.25	99.6
1	10 June 41	1.404	-0.085	5.202	2.064	0.01	0.25	99.7
6	13 Sept. 40	1.610	-0.034	7.328	1.849	0.25	0.25	99.1
6	23 July 40	1.465	-0.115	5.204	2.041	0.40	0.25	99.3
8	26 June 40	1.870	0.128	21.437	1.871	0.49	0.25	99.5
8	27 June 41	1.556	-0.052	6.650	1.874	0.53	0.25	99.8
16	26 Mar. 41	1.513	-0.025	8.984	1.677	0.17	0.25	99.9
16	12 July 40	1.737	-0.059	14.107	2.093	0.30	0.25	99.7

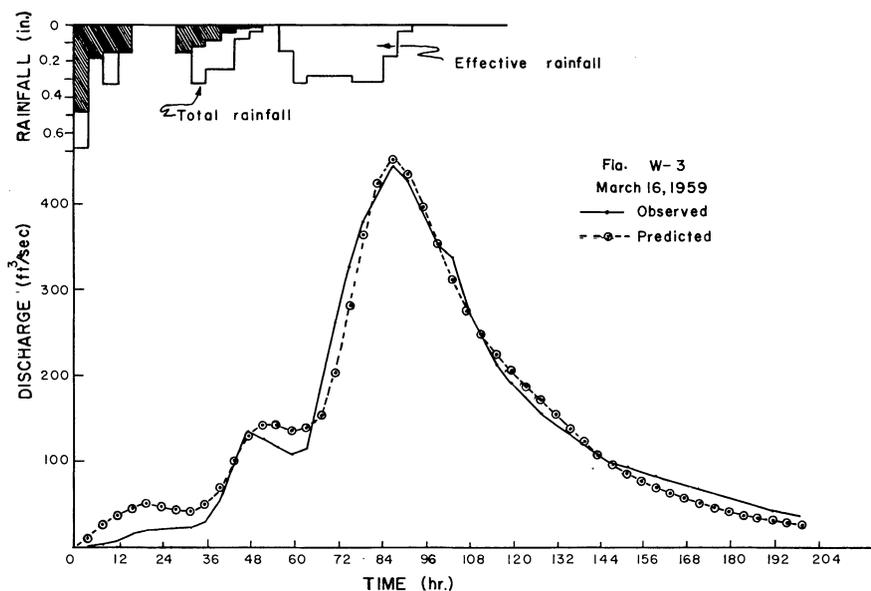


Fig. 7.
Comparison of predicted and observed hydrographs.

to certain of the Edwardsville, Illinois, plot data. Four elements, the two shape parameters, the maximum value, and the initial value, were optimized. The results are remarkably consistent except for one run on Plot 8, which arouses suspicion as to interaction between shape parameter "b" and the maximum value, r_u . The values of the correlation coefficients indicate almost exact correspondence between infiltration data and model.

Since the retention model to this time appears to be a satisfactory form, it is expected that attempts at quantification will be continued. It will be necessary either to incorporate the form in an input-output model or else first develop retention-rate data by input-output analyses.

Two-stage convolution. The numerical method of nonlinear least squares coupled with components regression has been used to evaluate two different mathematical models embodying the concept of two-stage convolution. A storm analysis model has been programmed to derive two components from data (Snyder et al. 1970). The first component is the characteristic function expressed in mathematically free form as a series of connected linear segments. The second component is the variable routing function, expressed as the simple

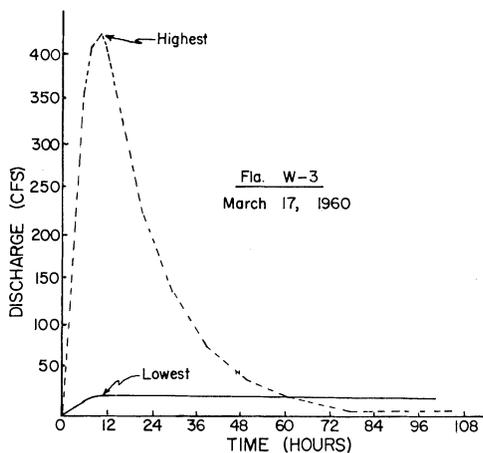
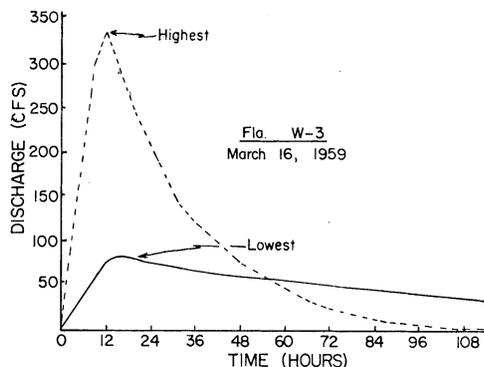


Fig. 8.
Derived variable unit response functions.

descending exponential. The shape parameter of the exponential curve is found by fitting, using the streamflow antecedent to each period of effective rainfall as a feedback variable. Both components are evaluated simultaneously by the fitting process.

Figure 7 shows the degree of correspondence between the model and the observed outflow hydrograph for a 16-square-mile watershed in the Taylor Creek basin just north of Lake Okeechobee in Florida. Figure 8 shows the extreme variability in unit response functions within two different storms on the Florida watershed. The lowest peak response and the highest peak response within each storm are shown. These unit response functions for 4-hour duration of effective rain are the result of the first stage of convolution.

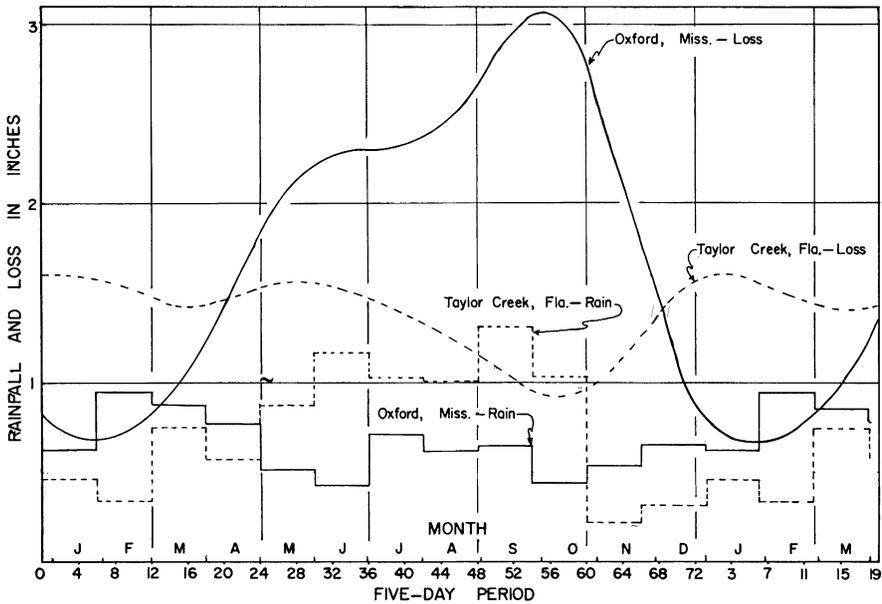


Fig. 9.
Derived seasonal loss curves.

A non-linear water yield model based on the concept of two-stage convolution is under development. This model differs only in mechanical detail from the storm hydrograph non-linear model. The unit of time of input and output is five days instead of a few hours. The input-output process is continuous through a long record, four years in our analysis, instead of starting with effective rain and ending during recession as in storm analysis. The water yield model differs in one other detail from the storm hydrograph model. It contains a characteristic function and a state function as in storm analysis, but additionally it contains a third component which allows simultaneous solution for non-effective rain. More work needs to be done on the mathematical detail of the characteristic and state functions of the yield model, but preliminary trials have shown that the seasonal rain reduction or loss function is not strongly dependent on the form of the other two model components.

Figure 9 shows the seasonal loss curves for two different drainage areas which were derived from rainfall and runoff data for two different watersheds. It might be thought at first inspection that the two loss curves shown in Figure 9 are so different as to cast doubt on the results of fitting. The differences have a rational explanation, however. The main difference can be explained

by the difference in the seasonal distribution of rainfall. Total rainfall for a five-day period was doubly averaged across six successive periods and across the four years of record used in the analysis. These averages are shown as bar diagrams in Figure 9. At Oxford the rainfall is almost uniformly distributed throughout the year. The soil profile begins to dry out in spring and early summer. Therefore, the capacity of the soil to store water – and hence the loss curve – increases rapidly at this time. A double peak of loss occurs – the first in June near the time of maximum solar energy, the second in October when the soil reaches its lowest moisture following a summer of drying. The period of little loss is comparatively short and is in the winter season.

For Taylor Creek in Florida, rainfall is highest during the summer months. The loss curve shows the same tendency toward a maximum in June as shown at Oxford. But then loss decreases rapidly and reaches a minimum at about the time Oxford is at a maximum. This is due to increasing water stored in the sandy Florida soils under the abundant summer rain. Under this condition the storage capacity of the soil is satisfied, little additional water can be infiltrated, and hence a minimum of loss occurs.

SUMMARY

The parametric approach to watershed modeling has been presented as “black-box” input-output analysis. Such input-output analysis cannot be done blindly. Each black box must have a rational, identifiable function in representation of watershed hydrologic processes. The mathematical form of the black box must be a formulation of the researcher’s concept of the process. The blind choice of a straight line or a simple parabola without regard to its suitability as rational black-box structure will usually produce only fitted curves. Such blind choice will not produce rational model structure leading to improved methods of hydrologic prediction for design purposes.

The parametric approach was outlined in sequential steps of procedure culminating in improved forecasting. Some of these steps were illustrated by examples taken from work in progress at the Southeast Watershed Research Center. There is no intent to imply that these examples represent the only attempts at the parametric approach to watershed modeling. But it is felt that examples drawn from one location do have cohesion and relationship as parts of a total research objective. The relatively recent emergence of parametric hydrology is a consequence of growing recognition that limited empirical

association of crudely expressed characteristics will produce limited improvement in prediction techniques. The rational development of the form of the relationship between sets of characteristics and the quantification of the mathematical parameters of these relationships using modern optimization techniques offers much more promise.

REFERENCES

- Committees on Surface Water Hydrology (1965) Parametric and stochastic hydrology. *Amer. Soc. Civil Eng. Trans.* 91 (HY 6).
- Mills, W. C. & Snyder, W. M. (1970) An algorithm for stream stage adjustment. Approved for publication.
- Snyder, W. M. & Curlin, J. W. (1969) *Walker Branch Watershed Project: Hydrologic analysis and data processing*. Oak Ridge Nat. Lab. Rept. ORNL-4392. April 1969.
- Snyder, W. M. (1971) A proposed watershed retention function. Manuscript in preparation.
- Wilde, D. J. & Beightler, C. S. (1967) *Foundations of optimization*. Prentice Hall, Inc., Englewood Cliffs, N.J., p. 10.
- DeCoursey, D. G. & Snyder, W. M. (1969) Computer-oriented method of optimizing hydrologic model parameters. *J. Hydrol.* 9, 34-56.
- Holtan, H. N. (1961) *A concept of infiltration estimates in watershed engineering*. USDA-ARS 41-51, Beltsville, Md.
- Snyder, W. M., Mills, W. C. & Stephens, J. C. (1970) A method for derivation of non-constant watershed response functions. *Water Resources Res.*, 6 (1).

Address:

Mr. Willard M. Snyder,
Principal Hydrologist,
Southeast Watershed Research Center,
ARS-SWC,
Athens, Georgia 30601,
U.S.A.

Received 1 May 1971.