Reduced order model monitoring and control of a membrane bioreactor system via delayed measurements

Venkatesh Kattigari Madyastha, Vijaysai Prasad and Venkatram Mahendraker

ABSTRACT

Activated sludge treatment is one of the most widely used processes for wastewater treatment (WWT). These systems are built with sufficient design margin to allow changes in loading and process conditions. This is necessary and prudent to overcome limitations in measurement, monitoring and controlling of WWT process parameters at the desired frequency. Online sensors for mixed liquor suspended solids, chemical oxygen demand (COD), nitrogen, phosphorus, and other parameters available today are limited in application due to high cost and low reliability. Hence, many of the parameters are measured off-line when needed. This paper provides a framework to estimate parameters on-line using limited and delayed measurements. The proposed approach is based on the design of a Bayesian filter such as an extended Kalman filter (EKF), which measures and controls membrane bioreactor system using limited and delayed measurements. The objective is to estimate the states and parameters with limited and delayed measurements. Simulations show the efficacy of the proposed approach.

Key words | Bayesian filter, delayed measurements, extended Kalman filter, Gaussian noise, membrane bioreactor system, model based estimation, soft-sensor for wastewater treatment

NOMENCLATURE

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
<th>Units</th>
<th>iXB</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>XCOD, XCODin</td>
<td>biodegradable organic matter</td>
<td>mg/litre</td>
<td>Xna</td>
<td>particulate biodegradable organic nitrogen mg/litre</td>
</tr>
<tr>
<td>SnH, SnHin</td>
<td>ammonia nitrogen concentration</td>
<td>mg/litre</td>
<td>Xd</td>
<td>particulate COD concentration due to biomass decay mg/litre</td>
</tr>
<tr>
<td>SnO, SnOin</td>
<td>nitrate nitrogen concentration</td>
<td>mg/litre</td>
<td>XS</td>
<td>slowly biodegradable COD concentration mg/litre</td>
</tr>
<tr>
<td>XbA, XbH, XbAin, XbHin</td>
<td>active autotrophic and heterotrophic biomass concentration</td>
<td>mg/litre</td>
<td>XI</td>
<td>particulate inert COD concentration mg/litre</td>
</tr>
<tr>
<td>rA, rH</td>
<td>autotrophic and heterotrophic reaction rates</td>
<td>litre/mg/day</td>
<td>SO, SO,in</td>
<td>DO concentration in the reactor and feed mg/litre</td>
</tr>
<tr>
<td>bA, bH</td>
<td>autotrophic and heterotrophic decay rates</td>
<td>/day</td>
<td>SO,sat</td>
<td>saturation concentration of DO mg/litre</td>
</tr>
<tr>
<td>YA and YH</td>
<td>autotrophic and heterotrophic yields</td>
<td>unitless</td>
<td>Kla</td>
<td>mass transfer coefficient /day</td>
</tr>
</tbody>
</table>

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**INTRODUCTION**

Conventional WWT methods use biological processes coupled with gravity settling for separation of solids from the treated water. The conventional activated sludge process has been in use for a century. However, growing environmental concerns and shrinking water resources are leading to adoption of advanced membrane bioreactor (MBR) technology that has become a ubiquitous choice globally for high quality treatment and reuse of WW. This technology is expected to play an important role in future WWT (Busch 2009). In a modified Ludzaack-Ettinger (MLE) MBR, denitrification is achieved in the anoxic zone utilizing the incoming COD and nitrification and additional COD removal is achieved in the aerobic zone. The membrane serves as an absolute physical barrier to solids, enabling separation between suspended solids and clear permeate. The application of MBR for treatment has been growing and GE ZeeWeed® is a well established polyvinylidene difluoride hollow fibre membrane in use over 15 years. The performance and operations of ZeeWeed® MBR system have been discussed extensively in the literature (Pearce 2008). Although MBR technology is sufficiently matured and has been proven successful for treating various types of municipal and industrial wastewaters, there is room for understanding and improving the controls strategy. Some of the controls used in MBR are presented in Table 1. More advanced controls can be implemented based on the design and level of automation desired for a plant.

It can be seen in Table 1 that a simple feedback strategy is the most extensively used control action, making the system response often reactive. Most WWT plants are designed in such a way that disturbances are typically attenuated due to high reactor volume. Although, a high conservative design can attenuate disturbances through buffering action and subsequently reduce loads on controls, they significantly contribute to capital cost of the system. Using advanced controls, the volume can be decreased and in addition, a better disturbance rejection can be achieved by taking operational constraints into account (Olsson & Newell 1999). The important benefits of advanced controls can be:

- Minimization of operating costs in terms of chemicals and energy.
- Reducing capital cost by minimizing conservatism in the design.

Economic benefits can be achieved through optimal aeration such that nutrient levels in discharge do not drop far below the norms prescribed by regulatory bodies. Many advanced controls strategies applied in chemical process industries have not been used in WWT processes including MBRs. This is due to high cost and poor reliability of instruments. Furthermore, high degree of uncertainty in biological processes and need for highly skilled manpower creates additional challenges. This paper is focused on addressing the aforementioned limitations. Activated Sludge Model (ASM) namely ASM1 (Henze et al. 1987), ASM2d (Henze et al. 1999) and ASM3 (Gujer et al. 1999) are some of the models used to capture complex dynamics of the activated sludge treatment. Many attempts to amend the ASM models have been proposed. However, the structure proposed in the ASMs still remains to be the backbone of activated sludge process. For effective monitoring and control of activated sludge process, it is important to exploit the benefits offered by high fidelity models. These models capture complex interactions between various parameters (Hahn & Edgav 2000). However the challenge in implementing the full order ASM models for online applications is the need to measure the required inputs such as slowly degradable COD, readily degradable COD,
particulate COD, soluble COD, inert COD etc. The goal of this paper is to propose a reduced order model derived from ASM1 to model the anoxic and aerobic (MLE) process. This approach essentially combines all CODs and nitrogen fractions into a single variable while retaining the fundamental structure necessary to estimate overall interaction between various parameters (Jeppson 1996). This paper proposes an approach to estimate the unknown states and parameters online via on-field sensors and lab provided delayed measurements.

Kalman Filtering (KF) is a popular estimation methodology proposed for linear systems (Kalman 1960), where the measured values are corrected for inaccuracies and noise in the measurement. The behaviour of the system is typically described by a single or a set of linear differential equations where the terms in the derivatives form are called as the states. The measured responses of the system that are driven by the states are called as the outputs/measurements. Thus a typical linear system can be represented as \( \dot{x} = Ax + Bu, \ z = Cx \), where \( x \) denotes the vector of states, \( u \) denotes the vector of inputs, \( z \) denotes the vector of outputs, \( A \) denotes the system state matrix, \( B \) denotes the system input matrix and \( C \) denotes the output/measurement matrix. One of the major advantages of using the KF is its ability to predict and update the states recursively, making it ideal for online application. It should be noted that all states of the system need not be measured in order to estimate them. The philosophy of KF is to use available measurements in order to estimate even those states that are not necessarily measured. This is essentially called as ‘soft sensing’ in contrast to ‘hard sensing’ where all the states to be estimated are available as measurements. However, when the process dynamics is nonlinear, the design of nonlinear filter becomes a challenging problem and this has received a considerable amount of attention in the literature. It has been noted that the direct implementation of the KF on nonlinear systems is not guaranteed to yield optimal estimates. Thus the need to effectively reconstruct the states of a nonlinear system, such as an MBR system, has promoted nonlinear filtering theory. A typical nonlinear system can be represented as \( \dot{x} = f(x, u), \ z = h(x) \), where \( x, u \) and \( z \) respectively denote the vector of states, inputs and outputs and \( f(\cdot), h(\cdot) \) respectively denote the nonlinear process dynamics and the measurement map. The Extended Kalman Filter (EKF) is the most popular approach of all the nonlinear filters, primarily due to its ease of implementation. The EKF design is based on a first order Taylor series approximation of the nonlinear system dynamics (Eykhoff 1974). Thus the partial derivative of the nonlinear system dynamics is computed with respect to the system states and evaluated at the state estimate for each time step. EKFs handle nonlinearities in dynamic processes and measurements and have been successfully applied to such areas as adaptive state and parameter estimation and missile-target tracking (Madyastha 2009; Madyastha & Calise 2005). To be consistent with the terminologies used in estimation theory, the known/measured inputs that impact the system’s response are called as measured disturbances and likewise the unknown variables that affect the system are called as unmeasured disturbances. The proposed approach overcomes the limitations of feedback strategies by predicting the impact of disturbances on the outputs. These disturbances could possibly vary in time. The key contributions of this paper are:

- Formulation of a joint state and parameter estimation algorithm via an EKF using a reduced order model. This helps to predict the future system response for any change in the inputs.

- Some of the process parameters such as DO and pH are measured very frequently online. However, many critical parameters such as MLSS, alkalinity, ammonia, nitrate, feed and effluent COD are measured in the lab less frequently. An online methodology is proposed to estimate unmeasured and less frequently measured variables with the help of a few variables, measured very frequently.

**PROBLEM FORMULATION**

This section presents the formulation of a full order aerobic bioreactor model using ASM1 with necessary additions. The anoxic, aerobic and membrane sections of the tank are in series with a part of the sludge from membrane tank coming back to anoxic as a recycle. The model assumes that the rest of the sludge is wasted continuously to maintain MLSS constant for the process. The aerobic and the anoxic sections are modelled using a full order ASM1 with an appropriate switching function based on DO. The model essentially uses the original 15 state equations along with a few more state additions to describe the growth conditions. Seven different types of CODs, 4 forms of nitrogen along with alkalinity and DO describe the growth kinetics. A few additions made to the model are necessary to describe other parameters, discussed below.
Mixed liquor suspended solids (MLSS)

ASM1 considers inert COD and bacterial concentration as a part of the total COD. As the MLSS is not directly modelled, this is estimated based on other parameters. MLSS is measured in mg/L and hence is modelled as the weighted average (Michaela 2006) 

\[ \text{MLSS} = 0.75(X_I + X_S) + 0.9(X_{bhH} + X_{bhA} + X_d + X_{nS}), \]

where \(X_{nS}\) is the particulate biodegradable organic nitrogen, \(X_d\) is the particulate COD concentration due to biomass decay, \(X_S\) is the slowly biodegradable COD concentration and \(X_I\) is the particulate inert COD concentration. The terms \(X_{bhH}\) and \(X_{bhA}\) are defined later.

MLSS correction

MLSS concentration offers resistance to the oxygen transfer. The oxygen transfer rate accounted for variation in MLSS while estimating change in oxygen transfer and power demand. It is given by

\[ \alpha = e^{-0.0771 \times \text{MLSS}} \] (Krampe & Krauth 2005).

Mass transfer coefficient (MTC)

A correlation based approach was used to capture the effect of MLSS, temperature \((T)\), diffuser density and superficial velocity of air to estimate the mass transfer coefficient. The mass transfer coefficient \((K_{la})\) (Gillot et al. 2005) is given by

\[ K_{la} = 1.024^{1-20} \text{Da}^k, \]

where \(D\) is the correlation for diffuser density, \(a\) is the mass transfer correction factor, \(U\) is the superficial gas velocity, \(T\) is the temperature in degree Celsius and \(k\) is a constant set to 0.8198. The diffuser density is defined as the ratio of the total area occupied by the diffusers to the bioreactor surface area. The mass transfer coefficient is a strong function of temperature and diffuser density. Figure 1(a) shows the response surface of the mass transfer coefficient as a function of the other 2 parameters.

Dissolved oxygen (DO)

It is assumed that the bioreactors do not have spatial variations and hence are modelled as continuous stirred tank reactors. The DO concentration can hence be modelled as

\[ \frac{dS_O}{dt} = \left(\frac{q}{V}\right)\left(S_{O, in} - S_O\right) + K_{la}\left(S_{O, sat} - S_O\right) + \text{OUR}, \]

where \(q\) denotes the feed rate of influent wastewater, \(V\) is the reactor volume, \(S_O\) is the DO concentration in the reactor, \(S_{O, sat}\) is the saturation concentration of dissolved oxygen, \(S_{O, in}\) is the dissolved oxygen concentration in the feed and \(\text{OUR}\) is the oxygen uptake rate. The effect of temperature on saturation oxygen concentration is shown in Figure 1(b).

pH calculation

1 additional state is added to the ASM1 model to estimate pH. The steps are:
1. Use alkalinity derived from the ASM1 and estimate equivalent bicarbonate alkalinity.
2. Calculate dissolved CO\(_2\) in equilibrium between atmosphere and reactors using Henry’s law.
3. Estimate pH using bicarbonate and dissolved CO\(_2\).

This model was applied to the anoxic and aerobic reactors of the process.

Air flow and blower

The simulation also incorporated the blower power calculations proportional to the air demand of the system. Depending on the DO level in the aerobic reactor, the blower rate was adjusted and the power requirement was recorded.

Temperature correction

Temperature has considerable impact on biological processes. Aerobic processes are inherently exothermic in nature. Dissipation of heat due to COD oxidation, effect...
of atmospheric and ambient conditions and radiations change the operating temperature of the process which in turn affects the kinetic parameters. The simulation accounted for the effect of all factors in estimating the impact on biological process.

The above discussed full order model serves as a reference/proxy for the process design. Performance of the reduced order model was compared against the outputs of the full order model. The formulation of the reduced order model is explained below.

Reduced order model

The reduced order model (Jeppson 1996) is derived from the ASM1 structure by lumping different CODs and ammonia into one state. To implement an EKF, it is necessary to describe the real system by a set of differential equations. The reduced order ASM model for the aerobic process to estimate the unknown system states and parameters is given as:

\[
\begin{align*}
\dot{X}_{COD} = & \left(-\frac{r_H}{Y_H} X_{COD} X_{bh} + b_H X_{bh} + b_A X_{ba}\right) - \frac{q}{V} X_{COD} + \frac{q}{V} X_{COD,in} - i_{XB} \left(r_H X_{COD} X_{bh} + b_H X_{bh} + b_A X_{ba}\right) \\
\dot{S}_{NH} = & \left(-i_{XB} \left(r_H X_{COD} X_{bh} + b_H X_{bh} + b_A X_{ba}\right)\right) - \left(i_{XB} + \frac{1}{Y_A}\right) r_A S_{nh} X_{ba} - \frac{q}{V} S_{nh} + \frac{q}{V} S_{nh,in} - r_A X_{COD} X_{bh} - b_H X_{bh} - \frac{q}{V} X_{bh} + \frac{q}{V} X_{bh,in} \\
\dot{X}_{bh} = & r_A S_{nh} X_{ba} - b_A X_{ba} - \frac{q}{V} X_{ba} + \frac{q}{V} X_{ba,in}\end{align*}
\]

(1)

where, \(X_{COD}\), \(S_{nh}\), \(S_{nh,in}\), \(X_{bh}\) and \(X_{ba}\), the unknown system states, are respectively the COD, ammonia nitrogen, nitrates nitrogen, active autotrophic and heterotrophic biomass concentrations. The unknown parameters of the system are \(r_H\), \(r_A\), \(b_H\), \(b_A\), which are respectively the reaction and decay rates for heterotrophs and autotrophs. \(Y_A\) and \(Y_H\) are the autotrophic and heterotrophic yields respectively and \(i_{XB}\) is the ratio of the mass of nitrogen to the mass of COD in the biomass. The RHS of (1) denoted as \(f\) represents the system nonlinear process dynamics. All parameters with the subscript ‘in’ denote concentration in the steady feed to the MBR system, i.e. known inputs to the system. The measurements for this process are performed by taking the output of the ASM1 model for composite variables such as COD, ammonia, nitrate and MLVSS, which are modelled as the sum of heterotrophic, autotrophic and inert particle products as a result of biomass decay. The unknown system parameters are augmented to the state Equation (1) as 0 derivative states. By ‘augmenting’ we mean that the dimension of the original state vector is increased by including the system parameters in the original state vector. This new state vector is called the augmented state vector. Thus the augmented state vector takes the form \(\dot{x} = [x^T \ \theta]^T\), where, \(x = [X_{COD} \ S_{nh} \ S_{nh,in} \ X_{bh} \ X_{ba}]^T\) is the original state vector and \(\theta = [r_H \ r_A \ b_H \ b_A]^T\) is the constant parameter vector which is included as a part of the original state vector to form \(\dot{x}\). Also, \(\theta = [0 \ 0 \ 0 \ 0]^T\), since \(\theta\) is a constant. Thus the new nonlinear process dynamics takes the form \(\dot{f} = [f^T \ 0 \ 0 \ 0]^T\). A list of measurements is shown in Table 2.

Remark 1. The parameters \(r_H\), \(r_A\), \(b_H\), \(b_A\) shown in the reduced model are not the same as in ASM1. These parameters vary based on the lumped value of the feed properties.

Remark 2. Like (1), the MBR anoxic section can be represented by the reduced order model.

Remark 3. In this approach, COD is treated as the only online measurement while ammonia, nitrate and MLVSS are assumed to be available from lab analysis. This leads to a delay in receiving all the 3 measurements required for EKF design, resulting in multi-rate estimation.

Remark 4. Studies in (Constable & McBean, 1979; Steyer et al. 1979) show relations between TOC, COD and biochemical oxygen demand (BOD) for a specific given wastewater. The TOC instruments have been used extensively for online measurements and in this work TOC has been used as a proxy for COD measurement of feed.

Table 2: List of parameters measured

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mode</th>
<th>Frequency</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Total Organic Carbon (TOC)</td>
<td>Instrument</td>
<td>Online</td>
<td>Proxy for COD</td>
</tr>
<tr>
<td>2 Organic nitrogen</td>
<td>Lab measurement</td>
<td>3 times a day</td>
<td>Estimation</td>
</tr>
<tr>
<td>3 Nitrate</td>
<td>Lab measurement</td>
<td>3 times a day</td>
<td>Estimation</td>
</tr>
<tr>
<td>4 MLVSS</td>
<td>Lab measurement</td>
<td>3 times a day</td>
<td>Estimate (X_{bh}, X_{ba}, X_D)</td>
</tr>
<tr>
<td>5 DO</td>
<td>Instrument</td>
<td>Online</td>
<td>Validate model</td>
</tr>
</tbody>
</table>
Remark 5. Although the DO in WWT is measured online, it has not been used here for parameter estimation. Instead, the DO is used to validate the effectiveness of the proposed estimation scheme by comparing the estimated profile with the measured using

\[ \dot{S}_O = \frac{1 + \tau_H a}{Y_H} r_H X_{COD} X_{bh} + \frac{4.57 - Y_A}{Y_A} \tau_A S_{NH} X_{ba} \]

\[ \frac{q S_O}{V} + \frac{q S_{O,in}}{V} \]

Remark 6. The dead-biomass concentration required for estimating MLVSS is derived from the ASM1 structure. Assuming concentration of particulate COD arising from dead-mass decay as 0, the dead-mass COD is estimated as \( \dot{X}_D = -(q X_D/V) + 0.16 (b_H X_{bh} + b_A X_{ba}) \). MLVSS is hence estimated as \( \text{MLVSS} = 0.9 (X_{bh} + X_{ba} + X_D) \).

EXTENDED KALMAN FILTER

Application of the EKF for nonlinear estimation has been discussed in earlier sections. The model for the design of the EKF has the same structure as shown in (1). The design of the EKF has 2 stages: (i) prediction and (ii) update. In the prediction stage, the EKF predicts the states of the system based on the nonlinear process dynamics \( \dot{f} \). In the update/correction stage, the EKF proceeds to compute a gain, called the Kalman gain, based on the linearized version of the nonlinear system for both the process dynamics and measurement. In the approach proposed here, the measurements are linear in nature. Thus, the computation of only the linearized process matrix is required. The 2 stages are described below:

**Prediction**

\[ \hat{x}_{n-1} = \dot{f} \hat{x}_{n-1} + \frac{q}{V} u_{n-1} \quad (2-A) \]

\[ P_n = \Phi_{n-1} P_{n-1} \Phi_{n-1}^T + Q_{n-1}, \quad Q_{n-1} = \int_0^{\Delta t} e^{\Phi_{n-1} t'} Q e^{\Phi_{n-1}^T t'} dt' \]

\[ F_{n-1} = \frac{\partial \dot{f}}{\partial x_{n-1}} \bigg|_{x_{n-1} = \hat{x}_{n-1}} \quad (2-B) \]

where, \( \hat{x}_{n-1} = \begin{bmatrix} \hat{X}_{n-1}^T & \hat{V}_{n-1}^T \end{bmatrix}^T \) denotes the estimate of the corrected state at \( n-1 \), \( \hat{x}_n \) denotes the predicted state at \( n \), \( \dot{f} = \begin{bmatrix} f^T & 0 & 0 & 0 \end{bmatrix}^T \) is the new nonlinear process vector, \( u_{n-1} \), a known constant. The matrix \( P_n = E [ (x_n - \hat{x}_n)(x_n - \hat{x}_n)^T] \) is the predicted state error covariance at \( n \). This matrix represents the square of the predicted deviation of the estimated value from the true value before a measurement is received. The matrix \( P_{n-1} = E [ (\hat{x}_n - \hat{x}_{n-1})(\hat{x}_n - \hat{x}_{n-1})^T] \) is the corrected state error covariance at \( n-1 \). This matrix is the square of the corrected deviation of the estimated value from the true value after a measurement is received. The matrix \( \Phi_{n-1} = I + F_{n-1} \Delta t \) is called as the state transition matrix and has the property that its product with the state vector of the system at an initial time gives the value of the state at the desired time and \( I \) is an identity matrix. The matrix \( Q_{n-1} \) is the discrete process noise covariance matrix and \( Q \) is the continuous process noise covariance matrix. The matrix \( Q \) represents the deviation between the true system and the mathematical model for the filter design. In other words, \( Q \) represents the modelling errors of the system. In simulations, the realistic implementation of such filters is always discrete and hence \( Q_{n-1} \) has been used (shown in (2-B)) (Brown & Hwang 1992). The matrix \( F_{n-1} \) represents the linearized dynamics of (1) at \( n-1 \). The linearized dynamics is obtained by computing the partial derivative of the nonlinear system dynamics with respect to the system state vector and evaluating this partial derivative at the estimated state. Finally \( \Delta t \) is the time step of simulation.

**Update**

\[ K_n = P_n H^T \left( H P_n H^T + \frac{R}{\Delta t} \right)^{-1} \quad (3-A) \]

\[ \hat{x}_n = \hat{x}_{n-1} + K_n (z_n - \hat{z}_n) \quad (3-B) \]

\[ P_n = (I - K_n H) P_{n-1} \quad (3-C) \]

where, \( K_n \) is the Kalman gain matrix that denotes the weighting/importance that should be associated with a processed measurement in correcting the predicted state estimate, \( H \) is the measurement matrix, \( R = \text{diag} \left[ \sigma_{COD}^2, \sigma_{S_{tot}}^2, \sigma_{S_{sol}}^2, \sigma_{\text{MLVSS}}^2 \right] \) is the measurement noise covariance matrix which represents the errors between the sensor and the mathematical model of the sensor, where the diagonal entries correspond to the variances of the measurements, \( \hat{x}_n \) is the corrected state at \( n \) after receiving a measurement, \( z_n \) is the actual sensor measurement and \( \hat{z}_n \) is the predicted estimate of the
actual measurement of the system. A block diagram showing the interconnection of the MBR system and the EKF is shown in Figure 2. In Figure 2, the block diagram marked as ‘I’, ‘II’ and ‘III’ in circles, respectively represent the MBR system with the EKF and the delayed measurements (DMs), the prediction and correction stages in a typical EKF implementation and the measurement delay process map. In the block diagram marked as ‘III’ in Figure 2, $z_n^D$ refers to the delayed measurement (via lab analysis), $z_n$ is the online measurement (online sensor) such that $z_n = [z_n^D z_{n,MLVSS}]^T$ and $T_D$ is the delay time. For the system in (1), the online measurement is $z_n = [z_{n,COD}]$ and DMs are $z_n^D = [z_{n,MLVSS}^D z_{n,MLVSS}^D]$. Thus $H = [I_{4 	imes 4} 0_{4 	imes 5}]$, $H(4,4) = H(4,5) = 0.9$. The term $H(4,4) = H(4,5) = 0.9$ is due to the structure of MLVSS (Remark 6) with $X_D = 0$.

**EKF design methodology**

$q, V, H$ and $u_{n-1}$, $n = 1, 2, 3, \ldots$ in (2-A) are known. The matrices $Q, R$ and time step $\Delta t$ are user defined. For this problem $\Delta t$ and the online measurement sampling rate are assumed the same, i.e. rate at which $z_n$ is received. The generic EKF design procedure is shown in Table 3 (Brown & Hwang 1992). Start by initializing the augmented state vector, $x_0$ and the state error covariance matrix, $P_0$ to user defined values. Next follow the steps shown in Table 3 till the end simulation time.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Generic EKF design methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>For $n = 1$, compute:</td>
<td>5 predicted state error covariance $P_1$ from (2-B), given $F_0, \Phi_0$</td>
</tr>
<tr>
<td>1 predicted state estimate $\hat{x}_1$ from (2-A)</td>
<td>6 Kalman gain $K_1$ from (3-A), given $P_1, H$ and $R$</td>
</tr>
<tr>
<td>2 linearized process dynamics of (1) as $F_0 = \frac{\partial f}{\partial x}</td>
<td>_{x=x_0, u=u_0}$</td>
</tr>
<tr>
<td>3 state transition matrix $\Phi_0 = I + F_0 \Delta t$</td>
<td>8 corrected state error covariance $P_1$ from (3-C)</td>
</tr>
<tr>
<td>4 discrete process noise covariance matrix $Q_0 = \int_0 e^{Hx} Q e^{Hx} dx$</td>
<td>9 Repeat steps 1 through 8 for $n = 2, 3, \ldots$ till the end of simulation</td>
</tr>
</tbody>
</table>

For the proposed approach, since DMs are used, a modification to Table 3 is required. First collect a sludge sample at time $\tau_1$ for lab analysis of $S_{S_{OH}}, S_{COD}, MLVSS$. Then steps 1 through 5 proceed as given in Table 3. Steps 6~9 of Table 3 are replaced with steps 6 and 7 of Table 4.

In other words, in Table 4, when DMs are not available, the EKF will rely only on the prediction and make negligible correction (because of very high variances). This is seen from (3-A) where, if, $R$ is large because of large variances, the Kalman gain $K$ approaches 0. But when a DM is
against which the reconstructed states of the estimator are computed.

(Proceed as though only $\hat{z}_{COD}$ is available)

6A Kalman gain $K_t$ from (3-A), given $P_{t}^{-1}$, $H$ and $[R = \sigma_{COD}^2, \kappa_2, \kappa_3]$ where $\kappa$ is a large number.

6B corrected state estimate $\hat{x}_t$ from (3-B), given $\hat{x}_{t-1}$, $z_t$, $\hat{z}_t = H\hat{x}_t$ and $K_t$

6C corrected state error covariance $P_{t}$ from (3-C)

else if sludge sample analysis is complete, at $t = t_2$, then first collect next sludge sample for lab analysis of $S_{sat}$, $S_{sat0}$, MLVSS at $t = t_2$. Execute for loop below:

6D For $t = t_1$ to $t_2$ (smoothen estimates from $t_1$ to $t_2$) With new delayed measurements, $(\hat{z}_{D}^{S_{sat}}, \hat{z}_{D}^{S_{sat0}}, \hat{z}_{D}^{MLVSS})$, go back in time and recompute steps 1 to 5 in Table 2.

6E At step 6, do (6A), (6B) and (6C) with $[R = \sigma_{COD}^2, \sigma_{S_{sat}}^2, \sigma_{S_{sat0}}^2, \sigma_{MLVSS}^2]$ end for loop (step-6D) end if loop (step-6)

7 From $t = t_2$ till the time the next sludge sample analysis is received, repeat steps 1 to 6 with (6A), (6B) and (6C) for $n = 2, 3, \ldots$. When sludge sample is received from lab, say at $t = t_3$, recompute steps 1 to 6, with (6D) and (6E) and $t_1 = t_2$, $t_2 = t_3$.
compared. In Figures 3 and 4, the thick-solid and dotted curves correspond to the true and estimated values respectively. The estimator shows acceptable convergence even in the presence of random noise corrupting the measurements. Specifically, the performance of the EKF in estimating the original state vector $x$, shown in Figure 3, displays superior tracking performance. The top four figures of Figure 4 show the EKF tracking for the parameter vector $\theta$. Figure 4 shows that even with a sufficient uncertainty in the initial estimate of the unknown parameters, convergence to within a neighbourhood of the true value occurs almost instantaneously. The bottom right sub figure of Figure 3 shows the response of DO for high dynamic loadings. The estimated DO closely matches the DO from the ASM1 model.

### EKF accuracy and precision

The estimator accuracy is dictated by its bias, $Bias = E[\hat{x}] - \bar{x}$, where $\bar{x}$ is the true augmented state and $\hat{x}$ is the state estimate. The estimator precision is given by its variance, $Var = E\left[ (\hat{x} - E[\hat{x}]) (\hat{x} - E[\hat{x}])^T \right]$. The mean square error (MSE) is given by $MSE = Var + (Bias)^2$. Thus by computing MSE, the estimator accuracy and precision can be evaluated.

To compute MSE, we performed for 1000 simulation runs, each run lasting 1000 hours. The plots of MSE across runs for every time instant are shown in Figure 5. For each sub plot in Figure 5, the mean and standard deviations of the MSE values have also been computed. For a linear system with Gaussian process and measurement noise, the KF is an unbiased estimator ($Bias = 0$). However, for nonlinear systems, the EKF, in general, cannot be assumed to be an unbiased estimator (Brown & Hwang 1992). For the MBR system shown in Equation (1), the nonlineairities are such that the Taylor terms beyond the third order are zero. Thus for the MBR system considered in this paper, the EKF more closely resembles that of an unbiased estimator, which is easily verifiable from Figure 5. This also means that the EKF for the MBR system in (1) is tolerably accurate in estimating the unknown states and parameters of (1). This can be seen in Figure 5, along with evidence from Figures 3 and 4.

### CONCLUSION

In this paper we have presented an approach for recursively estimating the unknown states and parameters of an MBR system via an EKF with a limited set of measurements. In addition, some of the measurements such as ammonia, nitrates and MLVSS are treated as delayed measurements, obtained in a laboratory, which are provided to the MBR system after 8 hours of delay. COD measurement is assumed available online. Our results show that despite high modelling uncertainty in the parameters, we are able to, within acceptable accuracy, estimate the states and parameters of the reduced order MBR system. The EKF was simulated in the presence of band-limited white noise. The estimated states and parameters are used to predict and control the concentration profile of DO.

### REFERENCES


Madyastha, V. 2009 Adaptive Neural Network Based Target Tracking. VDM Verlag, Saarbrücken, Germany.

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