6 "The Inelastic Behavior of Engineering Materials and Structures," by A. M. Freudenthal, John Wiley & Sons, Inc., New York, N. Y., 1950, 587 pages.

7 "The Effect of Speed and Feed on the Mechanics of Metal Cutting," by B. T. Chao and G. H. Bisacre, Proceedings of The Institution of Mechanical Engineers, London, England, vol. 165 (W.E.P. No. 63), 1951, pp. 1–13.

8 "Metal Cutting Principles," by M. C. Shaw, Massachusetts Institute of Technology Publication, Cambridge, Mass., third edition, 1954.

9 "Influence of Rate of Strain and Temperature on Yield Stress of Mild Steel," by M. J. Manjoine, Trans. ASME, vol. 66, 1944, pp. A-211–218.

10 "Measurement of Dynamic Stress and Strain in Tensile Test Specimens," by R. O. Fehr, E. R. Parker, and D. J. DeMichael, Trans. ASME, vol. 66, 1944, pp. A-65–71.

11 "Basic Mechanics of the Metal Cutting Process," by M. E. Merchant, Trans. ASME, vol. 66, 1944, pp. A-168–175.

12 "Force Components, Chip Geometry, and Specific Cutting Energy in Orthogonal and Oblique Machining of SAE 1015 Steel," by Dimitri Kececioglu, published in this issue, pp. 149–157.

13 "The Mechanics of Three-Dimensional Cutting Operations," by M. C. Shaw, N. H. Cook, and P. A. Smith, Trans. ASME, vol. 74, 1952, pp. 1055–1064.

14 "The Size Effect in Metal Cutting," by W. R. Backer, E. R. Marshall, and M. C. Shaw, Trans. ASME, vol. 74, 1952, pp. 61–72.

15 "The Shear Stress in Metal Cutting," by M. C. Shaw and Iain Finnie, Trans. ASME, vol. 77, 1955, pp. 115–125.

16 "The Rotary Cutting Tool," by M. C. Shaw, P. A. Smith, and N. H. Cook, Trans. ASME, vol. 74, 1952, pp. 1065–1076.

17 "Mechanics of the Metal Cutting Process, II—Plasticity Conditions in Orthogonal Cutting," by M. E. Merchant, *Journal of Applied Physics*, vol. 16, June, 1945, pp. 318–324.

18 "The Theory of Plasticity Applied to a Problem of Machining," by E. H. Lee and B. W. Shaffer, Technical Report No. 43, Graduate Division, Applied Mathematics, Brown University, Providence, R. I., September, 1949.

19 "Brittle Failure of Steel Structures," by M. E. Shank, *Metal Progress*, vol. 67, June, 1955, pp. 111–121.

20 "Resistance to Deformation of Metals at Velocities 10^{-6} to 100 m/sec.—II," (in Russian), by F. F. Vitman and N. H. Zlatin, *Zhurnal Tekhnicheskoi Fiziki*, vol. 19, March, 1949, pp. 315–326.

21 "Basic Reasons for Good Machinability of 'Free-Machining' Steels," by M. E. Merchant and N. Zlatin, Trans. ASM, vol. 41, 1949, pp. 647–672; discussion pp. 672–677.

Discussion

D. R. WALKER.[4] The author has made a valuable contribution to metal-cutting research by providing the first really quantitative estimates of strain rate in cutting. If metal-cutting data are ever to become of importance in the evaluation of the physical properties of materials, quantities such as shear-strain rate and shear-zone temperature most certainly will have to be adequately determinable in addition to the parameters now commonly measured.

The normal shear angle ϕ_n may be calculated from the author's data according to the relationship

$$\tan\phi_n = \frac{\frac{K}{2}\sin(2\alpha_n)\cos\eta_s \pm [\cos\alpha_n(K\cos\eta_s - \cos\alpha_n)]^{1/2}}{\cos\alpha_n - K\sin^2\alpha_n\cos\eta_s} \quad \ldots\ldots [18]$$

where $K = \dfrac{d_{cn}\dot{\gamma}}{0.2\,V}$. When $\tan\phi_n$ is known, the shear strain γ may be calculated from Equation [6].

Unfortunately, sufficient information is not included in Table 1 to permit calculation of ϕ_n and γ for other than orthogonal cutting conditions. However, the values of ϕ_n and γ in Table 2 may be added to the data of Table 1.

TABLE 2 VALUES OF ϕ_n AND γ

Item	ϕ_n (deg)	γ
1	11	5.6
2	13	4.7
3	22	3.2
4	0.7	82
11	13	4.7
12	14	4.1
20	20	2.8
21	26	2.2
22	30	1.9
23	20	2.8
30	39	1.3
31	46	1.2

The data in Table 2 illustrate strongly that the technique used by the author to estimate shear-strain rate is not sufficiently accurate to permit quantitative evaluation of the relative effect of various parameters on the cutting process. The values of ϕ_n and γ obtained for item 4 are, for example, not realistic quantities. Moreover, one would not expect the effect of a change of depth of cut on shear angle and shear strain to be as strong as suggested by comparison of items 1 and 3, 20 and 22, and 21 and 23; nor would one expect the reversal of the effect of depth of cut on ϕ_n and γ indicated by items 21 and 23 when compared to items 20 and 22, or 1 and 3. The high values of ϕ_n and low values of γ exhibited by items 30 and 31 are also unusual for SAE 1015 steel.

In evaluating the effect of strain rate on flow stress, the author admittedly has not considered such variables as normal compressive stress, shear strain, shear-zone temperature, preflow, and shear-zone size. The effect of shear strain and shear-zone size on flow stress may be expressed qualitatively in the oversimplified relation

$$\tau = \tau_0 + A\gamma + \frac{B}{d_c t/\sin\phi_n} \quad \ldots\ldots\ldots\ldots [19]$$

In this equation τ_0 is the shear stress required to initiate flow, A is the strain-hardening modulus, and B is a size-effect constant. In essence, this equation states that the stress required to achieve a given amount of plastic strain is equal to the stress required to produce initial plastic flow, plus a stress increment due to strain-hardening, plus a stress increment due to constraint of the strain within a small zone. This last increment may be considered as the increase in strain-hardening effect owing to a volumetric limitation of the number of deformation systems available for plastic flow.

Application of the oversimplified Equation [19] to the data of Table 1 as augmented by Table 2, gives the results shown in Fig. 12 of this discussion. The shear strain-hardening modulus A was arbitrarily taken as 2400 psi, a plausible value for SAE 1015 steel. The solid line drawn among the data of Fig. 12 is simply a typical curve of the family given by Equation [19] rearranged as

$$\tau - A\gamma = \tau_0 + \frac{B}{d_c t/\sin\phi_n} \quad \ldots\ldots\ldots\ldots\ldots [20]$$

The scatter of data in Fig. 12 is such that no conclusions may be drawn as to the validity of the oversimplified Equation [19]; while similarly the scatter in the author's Fig. 11 precludes proper evaluation of any strain-rate effect.

In conclusion, it must be said that all the important factors influencing shear-flow stress will have to be considered simultaneously if an adequate picture of the metal-cutting process is to be presented. The qualitative Equation [19] should thus contain additional terms to account for strain rate, temperature, compressive stress, and so on. An excellent attack on the strain-rate

---

[4] Senior Scientist, Materials Section, Research and Advanced Development Division, Avco Manufacturing Corporation, Lawrence, Mass.

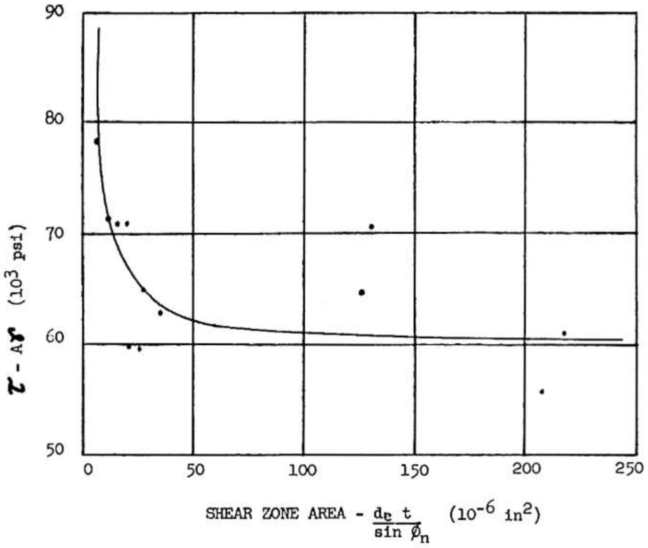

FIG. 12 SHEAR STRESS CORRECTED FOR STRAIN-HARDENING VERSUS SHEAR-ZONE AREA

factor has been made by the author. His promised discussion of the other parameters in future papers should clarify the influence of strain rate as presented here.

AUTHOR'S CLOSURE

The author thanks Mr. Walker for his discussion. The values of φ_n and γ given in Table 3 were used to arrive at the values of $\dot{\gamma}$ which correspond to the items in Table 2. The values in Table 3 are realistic and some differ substantially from those in Table 2. If Mr. Walker used the same equations to derive Equation [18] as were used to derive Equation [14] and the same values given in the paper to calculate φ_n as were used to calculate $\dot{\gamma}$, the values in Table 2 should be the same as in Table 3. The difference may be due to the high sensitivity of Equation [18] to small variations in α_n and to the number of significant figures used to calculate φ_n. The author agrees with Mr. Walker that only a simultaneous consideration of all factors significantly affecting τ would provide an adequate picture of the metal-cutting process.

TABLE 3 VALUES OF φ_n AND γ USED TO CALCULATE THE $\dot{\gamma}$ FOR THE ITEMS GIVEN IN TABLE 2

Item	φ_n (deg)	γ
1	13.1	4.70
2	13.7	4.50
3	13.3	4.65
4	15.0	4.16
11	13.8	4.25
12	13.8	4.25
20	13.5	2.91
21	19.0	2.88
22	20.5	2.69
23	24.9	2.24
30	25.2	1.98
31	26.9	1.79