The effect of cavity length on cascade performances is also of practical interest. Some of the theoretical calculations are depicted in Figs. 6 and 7. It is found in general, that the lift coefficient and cavitation number decrease with increasing cavity length and approach rather rapidly to certain asymptotic values. Also shown in Figs. 6(*a*) and 7(*a*) are experimental data of Wade and Acosta [13] for supercavitating flat plate cascades. The theoretical predictions are in reasonably good agreement with experimental findings.

Concluding Remarks

In the present study a linearized theory of supercavitating flow past a straight cascade with arbitrary blade shapes is developed. From the analysis, it is possible to determine the lift and drag coefficients, cavitation number, cavity shape, and exit flow conditions for any given specific cascade geometry, blade shape, cavity length, and initial inflow conditions. The cavitating performance of the cascade is, in general, found to depend strongly on stagger angle, solidity, blade shape, and cavity length.

It is needless to say that the present analysis is limited to cases in which the disturbance, caused by the presence of the blade, is small—an inherent restriction in the linear approximation. However, in practice the pump, turbine or propeller blades are quite thin and the linearized results obtained serve as a guide to the designer and aid in the interpretation of test results obtained for supercavitating pumps and turbines.

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DISCUSSION

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The linearized free-streamline theory may now be regarded as an established engineering calculation procedure much like its "thin wing" predecessor of aeronautics. Numerical examples of the type given in the paper are therefore welcome as they are useful to the designer just as are equivalent examples in fully wetted cascade flow theories. We should like to point out however that linearized cascade theory is probably not as accurate as for corresponding cases of isolated supercavitating hydrofoils, and it is regretable that comparisons of the present calculated results with those of an exact theory are not available. The comparisons with experiment in Figs. 6 and 7 are certainly encouraging. However, there do seem to be significant differences in the computed cavitation number (Fig. 7) with those actually measured. One wonders here if the incidence angle of 8 deg is not already too large for the linear theory to be valid.

Beyond this, the present work does not use the same conventions for angle of attack and stagger angle as in fully wetted cascade analysis. There the angle of attack referred to is the angle between the chord and the vector mean of the inlet and outlet velocities, and the relevant velocity about which perturbations are made is the vector mean of these two. In the present work the characteristic velocity is the inlet velocity, not the mean velocity. It would also appear that the correct stagger angle in the notation of the present work should be $\gamma + \alpha_1$ and not just γ. The difference is small for small stagger angle and angle α_1 but it could be important for large stagger angle as the angle, $\gamma + \alpha_1$, is a parameter of the mapping function. Although we cannot estimate what effects these differences might cause we would appreciate a comment by the author on them.

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Author's Closure

The author would like to thank the discussers for their interest and comments.

It is quite true that the linearized theory, even for isolated supercavitating hydrofoils, is limited to cases in which the disturbance is small. It is well-known that for flow past thin airfoils the thin wing theory yields force coefficients which are correct to the second-order. This fortunate circumstance does not present itself in the case of flow past supercavitating hydrofoils. The force coefficients, by applying the linearized theory, are only correct to first-order and generally overestimate, as found in a study by the present author [15],[4] by about $C_L/2 \times 100$ percent. The problem is probably more serious for flow past supercavitating cascades, especially for cascades with high solidities and stagger angles in which the interferences between blades and cavity wakes are considerable. However, it may be possible to extend the linearized results, at least for cascades with small or moderate solidities and stagger angles, to include higher order effects by a simple procedure similar to that developed in reference [15]. In fact, a pump inducer, based on such procedures, has been designed and tested [16]; the results seem encouraging.

In regard to the questions of angle of attack and stagger angle the definitions used in the present paper, from theoretical point of view, are probably more appropriate. According to these definitions the mathematical problem reduces directly to that for isolated supercavitating hydrofoils as both the stagger angle and solidity approach zero. The so-called angle of attack, conventionally used in the fully wetted cascade analysis, can of course be determined once the exit flow angle and velocity are obtained from the solution. As for the stagger angle the difference, within the framework of linearization, between the present and conventional definitions are not large since the angle α_1 is generally small; the effect of such difference, in any case, on the overall results is probably inappreciable.

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