Nordic Hydrology, 16, 1985, 169-176

A Comparison of Water Lag Times for Converging and Plane Surfaces

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In order to examine the influence of the degree of convergence for converging geometry on lag time, a comparison of surface water lag times for converging and plane surfaces is made using the Overton lag time concept. Assuming that four physical characteristics, watershed length, slope, roughness and rainfall excess are the same, an expression of the ratio of the lag time for converging surface to the lag time for plane surface is derived. The expression is solved numerically, and evaluated graphically for practical purposes. And the findings are compared with those of a previous study.

Introduction

Lag time for a watershed can be derived from the kinematic wave theory, which is applicable to most hydrologically significant cases of overland flow. According to this theory, the lag time is related to the representation of the watershed length, slope, roughness, and rainfall excess (Overton and Meadows 1976).

The complex geometry of a watershed has been modelled by various simplified geometric configurations. These configurations can be classified as: 1) Plane surface (Wooding 1965), 2) Converging surface (Woolhiser 1969; Campbell et al. 1984), and 3) Diverging surface (Ağiralioğlu and Singh 1981; Campbell et al. 1984).

Overton (1971) has investigated lag time for a plane surface while Singh (1975) has examined lag time for a converging surface. Singh and Ağiralioğlu (1982) considered lag time for a diverging surface. Overton (1971) also presented a lag time modulus as a ratio of the lag time for converging geometry to the lag time for plane geometry. This modulus was derived only for Manning resistance formula and was based on some simplifications. Besides, the expression of this modulus gives meaningless results for some values of the geometric parameter.

This study attempts to derive a general expression for the ratio of the lag time of a converging surface to the lag time of a plane surface in order to investigate the effect of watershed geometry on lag time. The solution of this expression would be used reliably for all ranges of the geometric parameter and for most friction formulas.

Concept of Lag Times

There exists a multitude of concepts of surface water lag time. One of them is the Overton concept which will be used in this study. Overton (1970) defined the lag time as the lapse between the time of occurrence of 50 % input and 50 % output.

On the Overton conceptual equilibrium hydrograph, $T/2$ and t_0 denote times of occurrence of 50 % of rainfall excess and 50 % runoff volume as shown in Fig. 1, in which T is the rainfall duration, T_L is the lag time and i is the rainfall excess.

The volume continuity can be written as

$$I + II = II + III \tag{1}$$

and therefore the volume

$$I = III \tag{2}$$

The volume I equals the watershed surface storage S_e at equilibrium condition, and will remain unchanged as long as i remains steady. Since volume III is approximately equal to width times the height of the rectangle, the lag time can be expressed as

$$t_L = \frac{S_e}{i} \tag{3}$$

Based on kinematic theory, solution of the lag time can be developed for converging surface and can be compared with that of plane surface.

Lag Time for Converging Surface

The converging and plane surfaces are shown in Fig. 2 in which R is the radius of flow region, c is degree of convergence for converging surface, and L is the length of watershed for both surfaces. For the converging surface with a given L, as the convergence parameter c approaches unity, which means R approaches infinity in

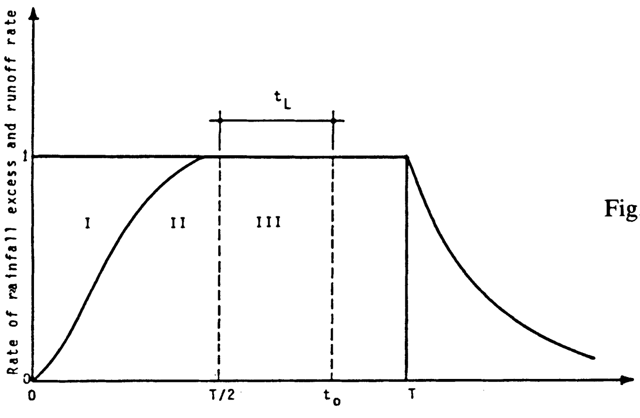

Fig. 1. Definition of Lag Time.

$R = L/(1-c)$, the converging geometry transforms to plane one. As c approaches zero, the converging geometry approaches the cone which is the extreme case of this configuration.

For a converging surface the basic equations of flow, based on kinematic wave theory, can be written on a unit width basis as (Woolhiser 1969)

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial r} = \frac{Q}{R-r} + i \tag{4}$$

$$Q = \alpha h^{\beta} \tag{5}$$

where h is the local depth of flow, Q is the rate of outflow per unit width, r is the space coodinate, t is the time coordinate, α is the parameter related to slope and roughness, and β is an exponent related to friction formula.

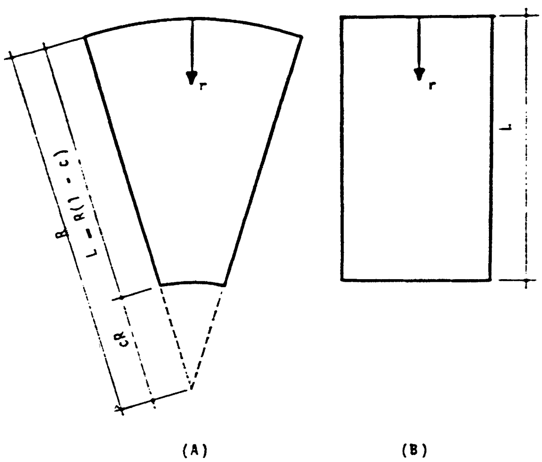

Fig. 2. Geometric Representations: (*A*) Converging Surface, (*B*) Plane Surface.

At equilibrium condition, the transient term in Eq. (4) will vanish. By eliminating Q from Eqs. (4) and (5) the solution for depth profile subject to the condition $Q = 0$ at $r = 0$ can be written as

$$h = \left[\frac{ir(R - \frac{r}{2})}{\alpha(R-r)} \right]^{1/\beta} \tag{6}$$

For the converging surface, the length of flow, L, is $R(1-c)$. The average depth at equilibrium, S_e, then follows

$$S_e = \frac{1}{R(1-c)} \int_0^{R(1-c)} h(r)\, dr \tag{7}$$

where $h(r)$ is the depth of flow as a function of space coordinate r. Upon substituting Eq. (6) into (7), inserting the new equation into Eq. (3) and using $R=L/(1\text{-}c)$, the lag time for converging surface can be obtained as

$$t_{LC} = \left(\frac{1}{2\alpha} \right)^{1/\beta} (i)^{(1-\beta)/\beta} \frac{1}{L} \int_0^L \left[\frac{2rL-(1-c)r^2}{L-(1-c)r} \right]^{1/\beta} dr \tag{8}$$

Comparison of Lag Times for Converging and Plane Surfaces

To make a comparison between lag times for converging and plane surfaces, a dimensionless solution is developed for lag time. In this producedure, it is assumed that these two representations have the same watershed length, slope, roughness, and rainfall excess. In other words, L, α, β, and i are the same for two representations.

For a rectangular plane surface, which is shown also in Fig. 2, the lag time, based on kinematic wave theory, can be written (Overton and Meadows 1976) as

$$t_{LP} = \frac{\beta}{\beta+1} \left(\frac{Li^{1-\beta}}{\alpha} \right)^{1/\beta} \tag{9}$$

The ratio of the lag time for converging surface to the lag time for plane surface defined as τ_L, i.e.,

$$\tau_L = \frac{t_{LC}}{t_{LP}} \tag{10}$$

Substituting Eqs. (8) and (9) into Eq. (10),

$$\tau_L = \left(\frac{1}{2}\right)^{1/\beta} \frac{\beta+1}{\beta} \left(\frac{1}{L}\right)^{(\beta+1)/\beta} \int_0^L \left[\frac{2rL-(1-c)r^2}{L-(1-c)r}\right]^{1/\beta} dr \tag{11}$$

In order to eliminate L in Eq. (11), a change of a variable is considered, i.e.,

$$\frac{r}{L} \equiv z \tag{12}$$

Then, Eq. (11) becomes

$$\tau_L \equiv \left(\frac{1}{2}\right)^{1/\beta} \frac{\beta+1}{\beta} \int_0^1 \left[\frac{2z-(1-c)z^2}{1-z(1-c)}\right]^{1/\beta} dz \tag{13}$$

As seen from Eq. (13), the ratio of the lag time of a converging surface to the lag time of a plane surface is the function of parameters β and c.

Effect of Friction Formula and Watershed Geometry

Eq. (13) can be solved numerically for different values of β and c. Since parameter β is related to only friction formula, some formulas widely used in practice are examined.

If Darcy-Weisbach friction formula is used, $\beta = 3$; when the Manning or Chezy formula is applied $\beta = 5/3$, $\beta = 3/2$, respectively.

Values of c vary from 0 to 1 depending on watershed geometry for a converging surface. If c approaches 1, which means that the converging representation transforms to the rectangular plane one, Eq. (13) yields that $\tau_L = 1.0$ for all values of β as expected.

If β equals 1, which means that the kinematic model is linear, Eq. (13) reduces to,

$$\tau_L = \int_0^1 \frac{2z-(1-c)z^2}{1-z(1-c)} dz \tag{14}$$

Using the change of variable as $1-z(1-c) = y$, Eq. (14) gives an exact analytical solution such as

$$\tau_L = \frac{1}{2}\left(\frac{1}{1-c}\right)^2 (c^2-2\ln c-1) \tag{15}$$

Remember that Eq. (15) is the solution of a special case of Eq. (13).

On the other hand, the general expression, Eq. (13), is solved numerically by the

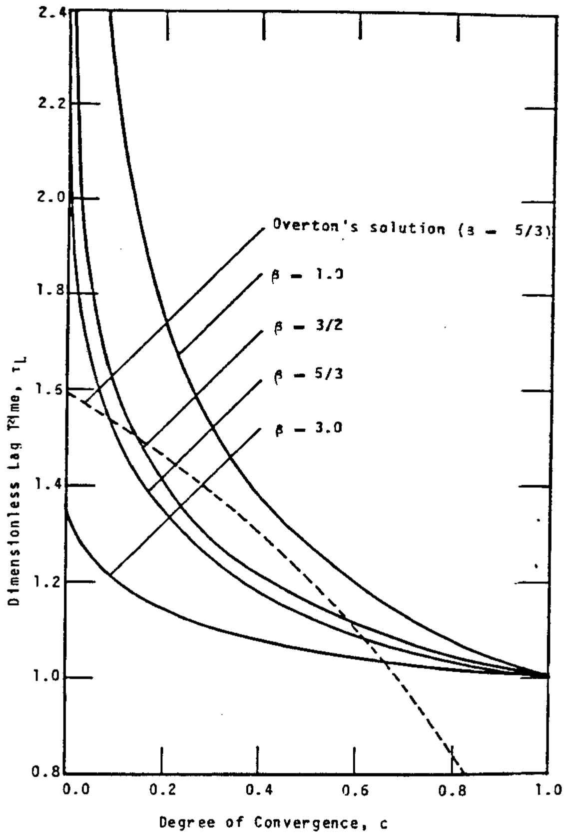

Fig. 3.
Comparison of Lag Times for Converging and Plane Surfaces.

Simpson's method for different values of c and β. The results of the solution are evaluated graphically in Fig. 3 for $\beta = 3.0$, $\beta = 5/3$, and $\beta = 3/2$. Also the results of Eq. (15) are illustrated in this figure ($\beta = 1.0$). As seen from this figure, the converging representation yields longer lag time than that of the rectangular plane one. Dimensionless lag time, τ_L, increases while parameter β decreases.

Comparison of Results with those of a Previous Work

Overton (1971) has derived an expression for the ratio of the lag time of the converging surface to the lag time of the plane surface using the Manning friction formula ($\beta = 5/3$) as

$$\tau_L = 1.6(1-c)^{0.4} \tag{16}$$

In his derivation, the kinematic theory is used and it is assumed that parameters

α, L, and i are the same for two representations, as in this study.

The solution of Eq. (16) is indicated also in Fig. 3. As seen from the figure, Eq. (16) gives some meaningless results for some geometric parameters. For instance, if $c = 1$, Eq. (16) yields that $\tau_L = 0$, which is not a meaningful result from the physical point of view. However, τ_L should become 1 for this example. Moreover, the Overton results are different from those of this study for all values of c because his expression was based on some simplifications. As an example, for $c = 0$, the Overton expression ($\beta = 5/3$) yields that $\tau_L = 1.6$ while this study gives a value as 2.31. On the other hand, although, in this study the dimensionless lag time, τ_L, becomes greater than 1 for all values of c, the Overton expression yields smaller results than 1 for the values of c which are greater than 0.69.

Example of Application

Fig. 3 can be used for determining the lag time for any converging watershed. If Chezy formula is used, $\beta = 3/2$ and $\alpha = CS_o^{0.5}$ where C is the Chezy roughness coefficient and S_o is the watershed slope. When Manning equation is used, $\beta = 5/3$ and $\alpha = S_o^{0.5}/n$ where n is the Manning roughness coefficient. If Darcy-Weisbach formula is applied $\beta = 3.0$ and $\alpha = 8gS_o/(a\nu)$ in which g is the gravitational acceleration, a is the Darcy-Weisbach roughness coefficient, and ν is the kinematic viscosity.

An illustrative example of application for determining the lag time is given. For example, a watershed can be approximated as the converging surface with $c = 0.1$. Estimate the lag time using the following physical data: Length of watershed, $L = 400$ m; watershed slope, $S_o = 0.02$; Chezy roughness coefficient, $C = 3$; and rainfall excess intensity, $i = 2.5$ cm/hr.

For $c = 0.1$, and $\beta = 1.5$, from Fig. 3, $\tau_L = 1.61$. Then, $i = 2.5$ cm/hr $= 2.5/(3{,}600 \times 100)$ m/s. And $\alpha = CS_o^{0.5} = 3 \times 0.02^{0.5} = 0.42$. Since $\tau_L = t_{LC}/t_{LP}$, as seen in Eq. (10), t_{LP} is found from Eq. (9) as 3,023.8 sec or 50.4 min. Then, the lag time for the converging surface will be $1.61 \times 50.4 = 81.1$ min.

Conclusions

To compare the lag times for converging and plane surfaces, a dimensionless expression of lag time is derived. The solution of this general expression can be used for determining the lag time for any converging watershed.

Findings of this study show that the converging surface yields longer lag time than the plane surface for all ranges of geometric parameter c.

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First version received: 14 January, 1985
Revised version received: 29 April, 1985

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