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Gravity flow water distribution system design

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ABSTRACT

Gravity flow water distribution systems are reliable and cost effective over pumping systems as no external power is required to maintain the flow. Generally, the gravitational networks are designed as branched systems and in order to maintain their looped configuration the missing links are joined by pipes of nominal diameters. This approach does not take full advantage of looped configuration for economy and reliability. Presented herein is an algorithm for the design of a gravity flow distribution system with looped configuration and multiple input points. A linear programming (LP) technique has been used to solve the discrete problem for the optimal pipe design, keeping the looped configuration intact. Also developed is a criterion for the choice between the pumping and gravity systems where the elevation difference is marginal. It is hoped that the algorithm will be useful to engineers engaged in the design of gravity flow water distribution networks.

Key words | economy, gravity system, linear programming, loops, material selection

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INTRODUCTION

Gravity flow water distribution systems are advantageous where sufficient elevation difference is available to permit the water to flow from an input point (source) to the terminal point of the distribution system in required quantity and pressure head. What is most important in such systems is that no external power is required to move the water, thus achieving greater economy and reliability.

In a gravity system the water flows from an input point, which is at the highest elevation, to all other points of the system. Such systems are generally possible in hilly areas where the source of water is a natural water course, which can be tapped by constructing an intake chamber. Normally depending upon the topography and availability of land, the water treatment plant and a reservoir are also constructed at source point to ensure water quality and quantity. The gravity flow water distribution systems have single or multiple input sources with branched and/or looped network configuration depending upon the local conditions, topography, availability of sources and the water requirements. The pumping and the gravity systems differ in their functional and constructional requirements as listed in Table 1.

Presented herein is an algorithm for the design of gravity flow water distribution systems. It is hoped that this algorithm will be useful to the engineers engaged in the design of gravity flow distribution networks.

LITERATURE REVIEW

Bhave (1983) presented an algorithm for the design of gravity flow looped water distribution networks. In this algorithm the looped network is first converted to a branched network and then the cost of the branched network is minimised using linear programming (LP). The pipe links, which are removed for the purpose of converting the looped network into a branched, are redundant in the optimal design. These links are restored with minimum pipe diameter and a minimum discharge specification. Conversion of a looped network into a branched network outweighs the advantages of the looped geometry.

Chiplunkar and Khanna (1983) developed a methodology for optimal design of a branched network. Instead of considering the network as a single entity,

Table 1 | Comparison of pumping and gravity systems

S. No.	Item	Pumping system	Gravity system
1	Energy requirements	External energy	Gravitational energy
2	Source of water	Tubewell, river or lake	Water springs, creek or lake
3	Intake	Pumping station	Intake chamber
4	Storage reservoir	Elevated or surface reservoir	Surface reservoir
5	Pressure corrector	Booster/variable speed pumps/PRV	Break pressure tank/PRV
6	Conveyance main	Pumping main	Gravity main

several distribution mains emerging from the input point to various terminal nodes were considered. These distribution mains were designed using Lagrange method. Since in a pipe network many pipe links are common in various paths, it was recommended to adopt the maximum computed diameter for these pipes. The criteria of distribution mains and the maximum diameter will not yield optimality.

Morgan and Goulter (1985) used a heuristic approach to define critical constraints involving multiple demand patterns. Lansey and Mays (1989) used nonlinear programming to obtain optimal design of looped water distribution systems. Ormsbee and Kessler (1990) gave an algorithm for the optimal design of topologically and hydraulically redundant water distribution networks for capacity expansion. The algorithm involves LP, which indicates choice between various alternative paths.

Young (1994) presented a design methodology for branched water supply networks on uneven terrain, which provides continuous pipe diameters. The solution is converted into commercially available diameters retaining the same head loss as in continuous diameter pipes without maintaining the telescopic nature of the pipe network. Moreover, the discrete problem would have a significantly different optimum than the converted solution. Varma *et al.* (1997) developed a procedure for the optimal design of water distribution systems using an NLP method. In this design process, network geometry is modified by introducing pseudolinks and pseudonodes. The solution gives the continuous pipe diameters, which are rationalised for commercially available pipe sizes. Such a rationalised solution would have a different solution from that of the discrete problem.

CHOICE BETWEEN GRAVITY AND PUMPING SYSTEMS

While the pumping system can be adopted in any type of topographic configuration, the gravity system is feasible only if the input point is at a higher elevation than all the withdrawal points. If the elevation difference between the input point and the withdrawal point is very small, the resulting pipe diameters will be considerably larger and it will not be economical compared with the corresponding pumping system. Thus there exists a critical elevation difference at which both gravity and pumping systems will have the same cost. If the elevation difference is greater than this critical difference, the gravity system will have an edge over the pumping alternative. For an arbitrary network it will be very difficult to obtain a criterion for adoption of a gravity system. However a criterion for adoption of a gravity main can be obtained.

The pipe cost C_{pipe} can be expressed as:

$$C_{\text{pipe}} = K' LD^m; \tag{1}$$

in which K' = a proportionality constant; m = an exponent; L = the length of the pipe main; and D = the pipe diameter. The cost of the pumping plant C_{pump} is expressed as:

$$C_{\text{pump}} = K'_{\text{p}} \frac{Q\rho g h_{\text{o}}}{\eta};\qquad(2)$$

in which K'_{p} = a proportionality constant; ρ = mass density of water; η = combined efficiency of pump and prime mover; Q = discharge; h_{o} = pumping head; g = gravitational acceleration. The capitalised cost of the power C_{power} can be written as:

$$C_{\text{power}} = \frac{8.76 F_{\text{A}} F_{\text{D}} R_{\text{E}}}{\eta r} \rho g Q h_{\text{o}};\qquad(3)$$

in which F_{A} and F_{D} are annual and daily averaging factors; R_{E} = the cost of power per kilowatt hour; and r = interest rate expressed as a fraction.

The initial investment C'_{o} of a component can be converted to the overall investment by the following relationship:

$$C_{\text{o}} = C'_{\text{o}}\{1 + (1 - \alpha)\ [(1 + r)^{T} - 1]^{-1} + \beta/r\};\qquad(4)$$

in which α = salvage fraction; T = life of the component; and β = maintenance fraction. Using Equation 4 the overall coefficients K and K_{p} are obtained as:

$$K = K'\{1 + (1 - \alpha_{\text{m}})[(1 + r)^{T\text{m}} - 1]^{-1} + \beta_{\text{m}}/r\};\ \text{and}\qquad(5a)$$

$$K_{\text{p}} = K'_{\text{p}}\{1 + (1 - \alpha_{\text{p}})[(1 + r)^{T\text{p}} - 1]^{-1} + \beta_{\text{p}}/r\}.\qquad(5b)$$

in which the subscripts m and p stand for the pipe main and the pumping plant respectively. Thus the overall cost F_{p} of the pumping main is expressed as:

$$F_{\text{p}} = KLD^{\text{m}} + K_{\text{T}}\rho g Q h_{\text{o}}\qquad(6)$$

in which

$$K_{\text{T}} = \frac{(1+s)K_{\text{p}}}{\eta} + \frac{8.76 F_{\text{A}} F_{\text{D}} R_{\text{E}}}{\eta r};\qquad(7)$$

where s = standby fraction.

The pumping main should be designed in such a way that the residual pressure head at the exit point is equal to the minimum terminal head H. If z_{o} = the entry point level and z_{L} = the exit point level, the pumping head h_{o} can be expressed as:

$$h_{\text{o}} = H + z_{\text{L}} - z_{\text{o}} + \frac{8fLQ^2}{\pi^2 g D^5};\qquad(8)$$

in which f = the coefficient of surface resistance; popularly known as the *friction factor*. Swamee (1993) has given the following equation for f, which is valid for laminar flow, turbulent flow and the transition in between:

$$f = \left[\left(\frac{50.27\upsilon D}{Q}\right)^{8} + 9.5\left\{\ln\left(\frac{\varepsilon}{3.7D} + 4.818\left\langle\frac{\upsilon D}{Q}\right\rangle^{0.9}\right) - \left(\frac{1963\upsilon D}{Q}\right)^{6}\right\}^{-16}\right]^{0.125}\qquad(9)$$

in which υ = kinematic viscosity of fluid; and ε = average roughness height of the pipe wall. Eliminating h_{o} between Equations 6 and 8; one gets:

$$F_{\text{p}} = KLD^{\text{m}} + \frac{8K_{\text{T}}\rho f L Q^3}{\pi^2 D^5} + \rho g K_{\text{T}} Q(H + z_{\text{L}} - z_{\text{o}}).\qquad(10)$$

Differentiating F_{p} with respect to D, equating it to zero and simplifying, one gets the optimal diameter D^*_{p} as:

$$D^*_{\text{p}} = \left[\frac{40\rho K_{\text{T}} f Q^3}{\pi^2 K\text{m}}\right]^{\frac{1}{\text{m}+5}}\qquad(11)$$

Using Equations 10 and 11 the optimal pumping main cost F^*_{p} is obtained as:

$$F^*_{\text{p}} = KL(1+0.2\text{m})\left[\frac{40\rho K_{\text{T}} f Q^3}{\pi^2 K\text{m}}\right]^{\frac{\text{m}}{\text{m}+5}} + \rho g K_{\text{T}} Q(H + z_{\text{L}} - z_{\text{o}}).\qquad(12)$$

The second term on the right hand side of Equation 12 is the cost of pumping against gravity. For the case under consideration, this term is negative. Since the negative term is not going to reduce the cost of pumping, it is taken as zero. Thus:

$$F^*_{\text{p}} = KL(1+0.2\text{m})\left[\frac{40\rho K_{\text{T}} f Q^3}{\pi^2 K\text{m}}\right]^{\frac{\text{m}}{\text{m}+5}}.\qquad(13)$$

The cost of gravity main F_g comprises the pipe cost only, i.e.

$$F_g = KLD^m: \tag{14}$$

and the head loss is expressed as:

$$h_f = z_o - z_L - H = \frac{8fLQ^2}{\pi^2 g D^5}. \tag{15}$$

Equation 15 gives the diameter of the gravity main D_g as:

$$D_g = \left[\frac{8fLQ^2}{\pi^2 g(z_o - z_L - H)} \right]^{0.2}. \tag{16}$$

Using Equations 14 and 16 the cost of the gravity main F_g is obtained as:

$$F_g = KL \left[\frac{8fLQ^2}{\pi^2 g(z_o - z_L - H)} \right]^{0.2m}. \tag{17}$$

The gravity main is economical when $F_g < F_p^*$. Using Equations 13 and 17, the optimality criterion is obtained as:

$$z_o > z_L + H + \frac{L}{g} \left[\frac{5}{m+5} \right]^{\frac{5}{m}} \left[\frac{8fQ^2}{\pi^2} \right]^{\frac{m}{m+5}} \left[\frac{mK}{5\rho K_T Q} \right]^{\frac{5}{m+5}}. \tag{18}$$

Inequality 18 is a criterion for a water transmission main; however, generally, Equation 18 can be applied to get an idea about the adoption of a gravity distribution network of an arbitrary geometry.

NETWORK DESCRIPTION

In order to describe the synthesis algorithm properly, a typical water distribution network as shown in Figure 1 is considered. The geometry of the network is described by the following data structure.

Pipe link data

The pipe link i has two end points with the nodes $J_1(i)$ and $J_2(i)$; and a length L_i for $i = 1,2,3 \ldots i_L$; i_L being the total number of pipe links in the network. The elevations of the end points are $z(J_1)$ and $z(J_2)$.

Input point data

The nodal number of the input point is designated as $S(l)$ for $l = 1$ to n_L (total number of input points).

Loop data

The description of loops is not an independent information. The loop data can be generated by any of the loop generation algorithms such as that of Swamee and Sharma (1991).

Node-pipe connectivity

There are $N_p(j)$ number of pipe links meeting at the node j. These pipe links are numbered as $I_p(j,l)$ with l varying from 1 to $N_p(j)$. Scanning Table 2, the node pipe connectivity data can be formed. These data for Figure 1 are given in Table 3.

PIPE—MATERIAL AND CLASS SELECTION

The selection of the material and class of pipes largely depends on its per metre cost, working pressure and availability in that commercial size range. Figure 2 shows economically available commercial pipe sizes with the working pressure head as per design practice of the Uttar Pradesh Jal Nigam (Water Corporation), India. The effective pressure head h_{oi} of the pipe link i is the maximum of its two nodal point pressure heads under no flow conditions, i.e.

$$h_{oi} = z_L + h_o - \min\{z[J_1(i)], z[J_2(i)]\}. \tag{19}$$

Knowing h_{oi} and assuming $D_i = 0.20$ m, the economic pipe type for the entire network can be obtained as an initial design.

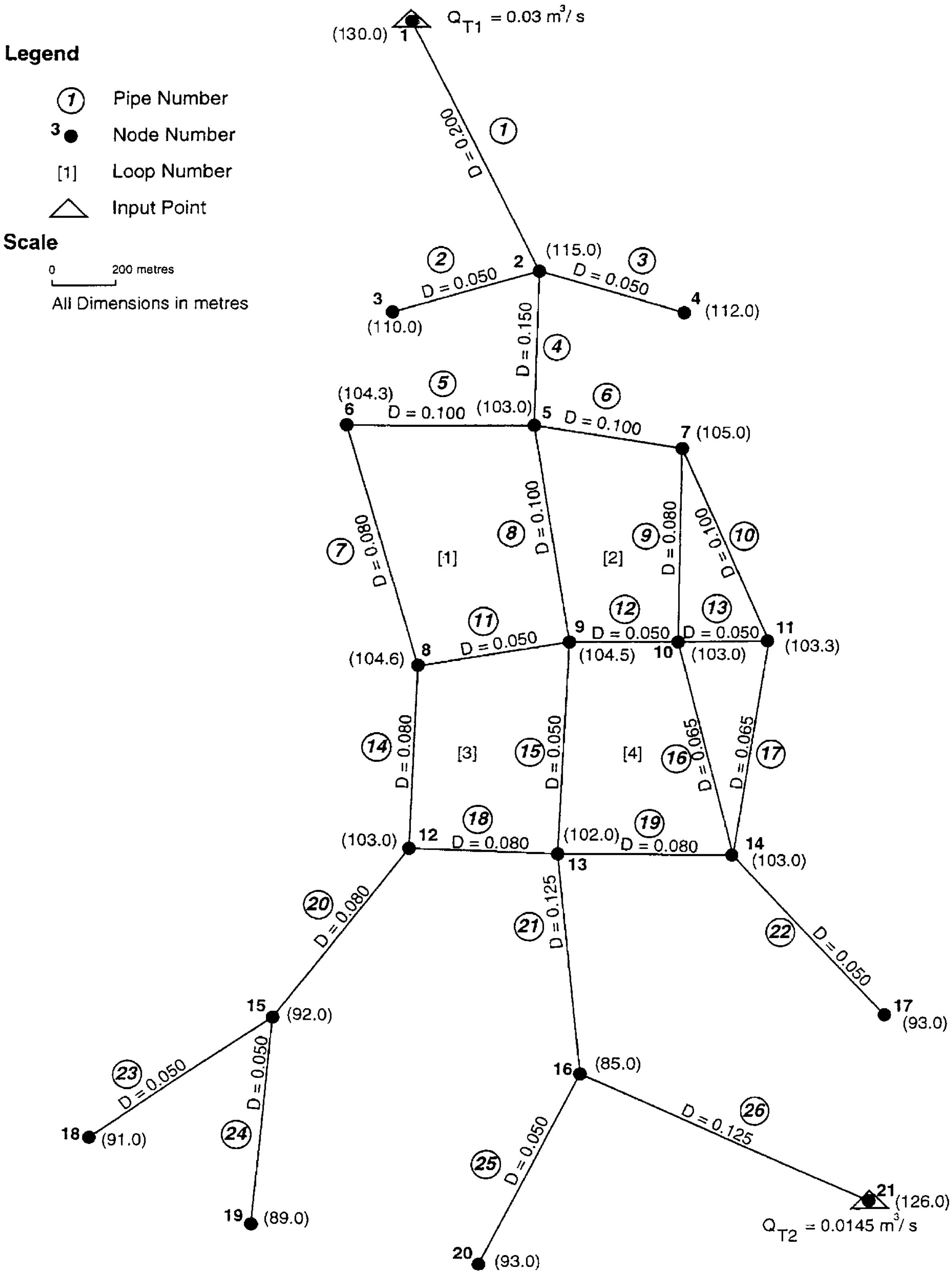

Figure 1 | Typical looped gravity flow distribution system.

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Table 2 | Pipe link data

i	J_1	J_2	$Z(J_1)$ (m)	$Z(J_2)$ (m)	*L* (m)	*Q* ($m^3 s^{-1}$)
1	1	2	130.0	115.0	800.0	0.0295
2	2	3	115.0	110.0	400.0	0.0009
3	2	4	115.0	112.0	400.0	0.0015
4	2	5	115.0	103.0	420.0	0.0240
5	5	6	103.0	104.3	500.0	0.0058
6	5	7	103.0	105.0	420.0	0.0080
7	6	8	104.3	104.6	700.0	0.0038
8	5	9	103.0	104.5	720.0	0.0062
9	7	10	105.0	103.0	600.0	0.0026
10	7	11	105.0	103.3	580.0	0.0028
11	8	9	104.6	104.5	420.0	0.0010
12	9	10	104.6	103.0	280.0	0.0026
13	10	11	103.0	103.3	240.0	0.0009
14	8	12	104.6	103.0	520.0	0.0026
15	9	13	104.5	102.0	620.0	– 0.0006
16	10	14	103.0	103.0	600.0	0.0013
17	11	14	103.3	103.0	560.0	0.0012
18	12	13	103.0	102.0	200.0	– 0.0050
19	13	14	102.0	103.0	460.0	0.0027
20	12	15	103.0	92.0	600.0	0.0044
21	13	16	102.0	85.0	600.0	– 0.0124
22	14	17	103.0	93.0	600.0	0.0017
23	15	18	92.0	91.0	620.0	0.0010
24	15	19	92.0	89.0	700.0	0.0007
25	16	20	85.0	93.0	630.0	0.0090
26	16	21	85.0	126.0	800.0	– 0.0150

Table 3 | Node-pipe connectivity

	$I_P(j,\ell)$				
j	ℓ=1	ℓ=2	ℓ=3	ℓ=4	$N_P(j)$
1	1				1
2	1	2	3	4	4
3	2				1
4	3				1
5	4	5	6	8	4
6	5	7			2
7	6	9	10		3
8	7	11	14		3
9	8	11	12	15	4
10	9	12	13	16	4
11	10	13	17		3
12	14	18	20		3
13	15	18	19	21	4
14	16	17	19	22	4
15	20	23	24		3
16	21	25	26		3
17	22				1
18	23				1
19	24				1
20	25				1
21	26				1

NETWORK ANALYSIS

To start the algorithm, a preliminary analysis is carried out using the Hard-Cross method. The pipe discharges (a

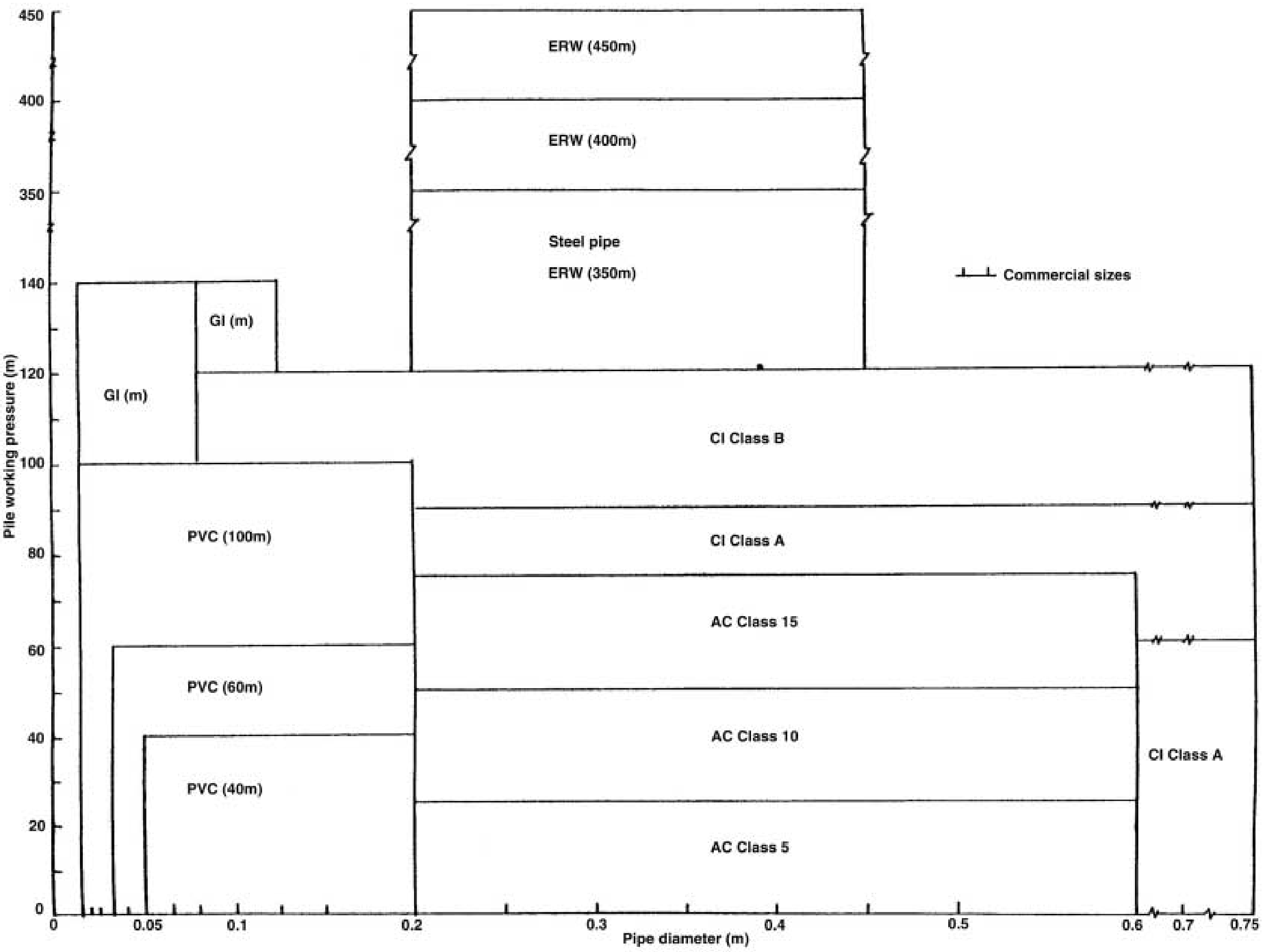

Figure 2 | Pipe material and class selection.

positive discharge flows from a lower node number to a higher node number) as obtained by the preliminary analysis are listed in Table 2.

NETWORK DESIGN

Assuming that the pipe link L_i consists of two commercially available discrete sizes of diameter D_{i1} and D_{i2} having lengths x_{i1} and x_{i2}, respectively, then the pipe network system cost can be written as:

$$F = \sum_{i=1}^{i_L} (c_{i1}x_{i1} + c_{i2}x_{i2}) \; ; \quad (20)$$

in which c_{i1} and c_{i2} are the costs of 1 m length of the pipes of diameters D_{i1} and D_{i2} respectively.

The cost function F has to be minimised subject to the following constraints:

1. the sum of lengths x_{i1} and x_{i2} is equal to the pipe link length L_i; and
2. the pressure head at each node is greater than or equal to the prescribed minimum head H.

The first constraint can be written as:

$$x_{i1}+x_{i2}=L_i;\ i=1,2,3\ \ldots\ i_L. \tag{21}$$

Flow path description

In order to formulate head loss constraint for a node, the paths (set of pipes) through which the node receives water from various input points must be known. The flow path can be obtained by starting from a node and proceeding in a direction opposite to the flow. In order to involve all the pipe links in the constraints, it is necessary to determine the flow paths for each pipe link; i.e. find the flow path through which the pipe receives the discharge. Considering the pipe link $i=9$ (see Figure 1), a set of pipe links should be found through which pipe link 9 is connected to an input point. The node $J_t(i)$ to which pipe link i is supplying the discharge is the terminal node of the flow path. Following Table 2, it can be seen that pipe link 9 is connected to nodes 7 and 10. Also from Table 2, the discharge in pipe link 9 is positive meaning that water flows from node 7 to node 10. Thus if one moves from node 10 to 7, the movement will be opposite to the flow direction in pipe link 9. So node 10 is the terminal node, i.e. $J_t(9)=10$.

Starting from node 10 one reaches node 7. Scanning Table 3 for node 7, one finds that it connects three pipe links, namely $i=6$, 9 and 10. One has already travelled along pipe link 9, therefore, considering pipe links 6 and 10 one finds from Table 2 that the discharge in pipe link 6 is positive and the other node of pipe link 6 is 5. Thus a positive discharge flows from node 5 to node 7 in pipe link 6. A similar argument for pipe link 10 yields that the discharge flows from node 7 to node 11. Thus by moving against the flow from node 7 one can move along pipe link 6 and not along pipe link 10 (in which the movement will be in the direction of flow). Repeating this procedure one moves along pipe links 4 and 1 to reach node 1, which is an input point.

Thus starting from pipe link 9, one encounters four pipe links before reaching an input point. The total number of pipes in the path, $N_t(9)=4$, and the pipes encountered along the path are designated by $I_t(i,l)$ for $l=1,2,3\ \ldots\ N_t(i)$. In the present case the following values were obtained:

$$I_t(9.1)=1,\ I_t(9,2)=4,\ I_t(9,3)=6 \text{ and } I_t(9,4)=9. \tag{22a}$$

Pipe link 9 thus receives water from node 1 and discharges to node 10. This can be written as:

$$J_s(9)=1, \text{ and } J_t(9)=10 \tag{22b}$$

in which J_s is the starting node of the flow path.

The head loss constraint for the node J_t(i) is written as:

$$\sum_{i=I_t(i,\ell)}\left[\frac{8f_{i1}Q_i^2}{\pi^2 gD_{i1}^5}x_{i1}+\frac{8f_{i2}Q_i^2}{\pi^2 gD_{i2}^5}x_{i2}\right]\le z_{J_s(i)}+h_o-z_{J_t(i)}-H;$$
$$\ell=1,2,3\ N_t(i) \text{ For } i=1,2,3......i_L \tag{23}$$

in which h_o = head at the input point. Inequality 23 represents i_L constraints. If the formulation was made node-wise, this number could have been reduced to J_L (total number of nodal points in network). In such a case many pipe links would not have appeared in any of the constraints. These links would finally be eliminated in an optimal solution thus resulting in forced conversion of the looped network into a branched system. Equations 20, 21 and 23 form a LP problem in the decision variable x_{i1} and x_{i2} for i varying from 1 to i_L. Thus the LP problem involves $2i_L$ decision variables, i_L equality constraints and i_L inequality constraints. An LP solution will indicate the optimal choice between the two diameters D_{i1} and D_{i2} for all i values.

Approximate pipe link diameter

Although the optimal design algorithm can be started by assuming a set of arbitrary diameters for all pipe links, a good starting solution will reduce the computational effort considerably. It is difficult to develop a criterion for determining an approximate diameter of a pipe link for a network of an arbitrary geometry, however a study can be conducted for a gravity flow distribution main. Figure 3 shows a gravity flow distribution main with i_L number of withdrawals separated at intervals L_1, L_2, $L_3\ \ldots\ L_{i_L}$; with z_o and z_{i_L} being the elevations of the input point and the terminal node of the main. The cost of the main comprises the pipe cost only, i.e.

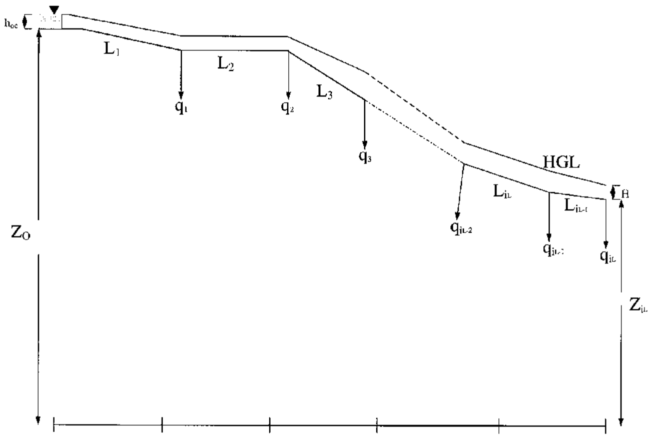

Figure 3 | A single gravity distribution system.

$$F=\sum_{i=1}^{i_L} KL_iD_i^m . \quad (24)$$

Considering the constraint at the terminal node to be tight, it can be written as:

$$\sum_{i=1}^{i_L} \frac{8f_iL_iQ_i^2}{\pi^2gD_i^5} - z_o - h_{oc} + z_{i_L} + H = 0; \quad (25)$$

where h_{oc} = effective head at input point.

Minimising Equation 24 using the Lagrange multiplier method with Equation 25 as the constraint, the optimal diameter D_i^* is obtained as:

$$D_i^*=[f_iQ_i^2]^{\frac{1}{m+5}}\left[\frac{8}{\pi^2g(z_o+h_{oc}+z_{i_L}-H)}\sum_{i=1}^{i_L}L_i(f_iQ_i^2)^{\frac{m}{m+5}}\right]^{0.2} \quad (26)$$

Using Equation 26 the diameters can be obtained for all the pipe links. For some of the pipe links more than one optimal diameter will be obtained. In such a case the larger diameter should be selected as the pipe diameter to start the algorithm. The two consecutive commercially available sizes close to D_i^* should be selected as the starting solution, i.e.

$$D_{i1} \geqslant D_i^* \leqslant D_{i2} \quad (27)$$

Corresponding to these diameters the constants C_{i1} and C_{i2} are obtained and the objective function (20) is obtained. Similarly, knowing D_{i1} and D_{i2}, the constraints in Equation 23 are formed. Solving the LP problem, the lengths x_{i1} and x_{i2} are obtained. In most cases either x_{i1} or x_{i2} is zero, indicating the choice between the diameters D_{i1} and D_{i2}. In the case where neither x_{i1} or x_{i2} is zero, the larger diameter is retained. If $x_{i1}=0$, D_{i1} and D_{i2} are changed to the next larger commercially available pipe sizes; whereas if $x_{i2}=0$, D_{i1} and D_{i2} are changed to the next smaller commercial sizes.

Knowing the pipe diameters, the pipe material is revised using Equation 19 and Figure 2. The network is again analysed using the Hardy-Cross method. This results in the pipe discharges which are used to obtain the new flow paths and the new constraints are formed using Equation 23. The LP algorithm is repeated to indicate the choice of the pipe sizes. The process is repeated until the two consecutive iterations yield identical results.

The proposed algorithm can be summarised in the following steps:

1. Assume the pipe link diameters
2. Compute the maximum pipe link pressure heads using Equation 19
3. Decide the pipe material and class using Figure 2
4. Initially assume f, otherwise compute from Equation 9
5. Analyse the network using the Hardy-Cross method to yield the pipe discharges
6. Find the flow path and the input point for each pipe link
7. Find the approximate diameter of the all the pipe links using Equation 26
8. Formulate the cost function and the constraints using Equations 20 and 23 respectively
9. Obtain the pipe link diameters using LP
10. Repeat steps 3 to 10 until the two successive solutions are identical.

A looped gravity flow water distribution system with a design population of 10,275 (see Figure 1) has been designed for a per capita water demand of 150 litres per day and the terminal pressure of 10 m. The resulting pipe link diameters are depicted in Figure 1. The CPU time taken in this case is 50 s on a DEC 2050 system. The variation of system cost with LP iteration is shown plotted in Figure 4.

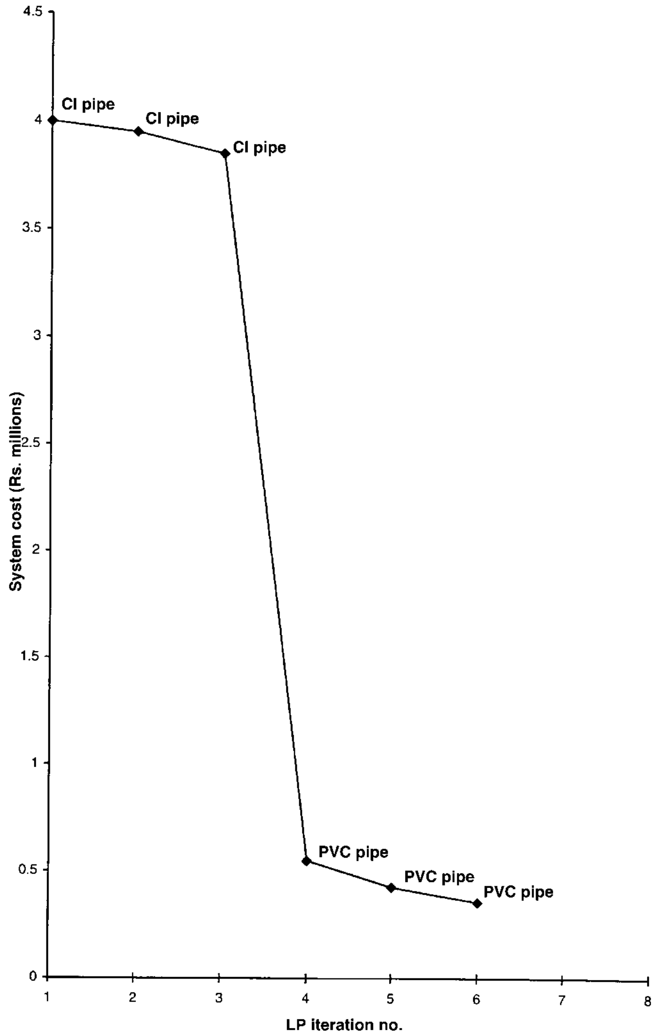

Figure 4 | Variation of system cost with LP iteration.

CONCLUSIONS

A sequential linear programming algorithm capable of preserving the looped configuration has been evolved. The algorithm selects the pipe material and class depending upon the available commercial sizes and the working pressure head. In order to reduce the computation time, an approximate pipe link diameter equation has been developed for the initiation of the algorithm. A criterion for the choice between a gravity and a pumping system, where the elevation difference between supply and demand points is marginal, has also been developed.

NOTATION

The following symbols are used in this paper:

C = sectional cost coefficient
D = pipe link diameter
F = cost function
F_A = annual averaging factor
F_D = daily averaging factor
f = coefficient of surface resistance
g = gravitational acceleration
H = minimum prescribed terminal head
h_f = head loss
h_o = pumping head
h_{oc} = effective head at input point
I_p = pipe links meeting at a node
I_t = pipe links in a track
i = pipe index
J_1, J_2 = pipe link node
J_s = starting node of a track
J_t = terminal node of a track
j = node index
K = pipe cost coefficient
K_p = pump cost coefficient
K_T = pump and pumping cost coefficient
L = pipe link length
ℓ = index
m = pipe cost exponent
N_p = number of pipe links meeting at a node
n = input point index
Q = pipe link discharge
R_E = cost of electricity per kilowatt hour
r = rate of interest
s = standby fraction
T = life of component
x = sectional pipe link length
z = elevation
α = scrap value fraction
β = maintenance fraction
ρ = mass density of water
η = efficiency

Superscript

$*$ = optimal value

Subscript

g = gravity system

i = pipe index

$i1$ = first section of pipe link

$i2$ = second section of pipe link

o = entry point

p = pumping system

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