On the Elastic Scattering of Charged Particles by Nuclei in the Intermediate Energy Region

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(Received January 4, 1957)

An expression has been obtained for the differential cross-section for the elastic scattering of intermediate energy charged particles by heavy nuclei, using a complex potential and employing the variational method due to Montroll and Greenberg. The potential considered is of the type \( V = -(V_0 + iW_0) \) inside the nucleus and \( \frac{Z^2e^2}{r} \) outside.

The expression obtained for the differential elastic scattering cross-section has been used in the case of scattering of 32Mev protons from gold. These have been compared with the experimentally determined cross-sections. It has also been shown that the present method of calculation gives results which are in good agreement with those obtained by using the partial wave method.

Theoretical calculations of the angular distribution of the elastic scattering of protons using a complex square-well potential have been carried out by a number of workers,\(^1\) up to an energy of 18.5 Mev. In all these, partial wave method of calculation has been employed which becomes somewhat tedious for higher proton energies and heavier scatterers.

For the calculation of the scattering cross-sections of protons of higher energy from heavier scatterers we have employed the Montroll-Greenberg\(^2\) variational method with complex square-well potential. This method has recently been used by Izumo\(^3\) for investigating alpha-scattering and by Mohr and Robson\(^4\) in the case of neutron-scattering.

The potential assumed in the present calculations is

\[
V(r) = \begin{cases} 
-(V_0 + iW_0) & \text{for } r < a \\
\frac{Z^2e^2}{r} & \text{for } r > a 
\end{cases}
\]

(1)

The scattered wave function for a uniform isotopic spherical scatterer can be written as an integral in the following manner.\(^2\)

\[
\psi_s(R) = \frac{e^{ik_0R}}{R}A_i\int_0^\infty \left\{ k^2(r) - k_0^2 \right\} r^2 \left\{ \frac{\sin \omega r}{\omega r} - Ke^{i\omega k_0 \frac{\sin \varphi r}{\varphi r}} \right\} dr 
\]

(2)

where

\[
A_i = \frac{2k_e^{i\omega(k_1-k_0)}}{(k_1+k_0)(1-K^2e^{i\omega k_0})}
\]

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\[ K = \frac{k_1 - k_0}{k_1 + k_0}, \quad k^2 (r) = \frac{2M}{b^2} [E_p - V (r)], \quad k_0^2 = \frac{2M}{b^2} E_p, \]

\[ \omega^2 = k_1^2 + k_0^2 - 2k_1k_0 \cos \theta \quad \text{and} \quad \nu^2 = k_1^2 + k_0^2 + 2k_1k_0 \cos \theta. \]

\( k_i \) is the internal wave number of the scattered particle. For a potential of arbitrary shape \( k_i \) and \( a \) have to be determined by applying the variational method.

The Montroll and Greenberg method gives the following expression for the scattered wave function for the type of potential given by equation (1).

\[ \psi_s (r) = \frac{e^{i\theta_0 r}}{R} \left\{ A_i (V_0 + iW_0) \frac{2M}{b^2} d^2 \sqrt{\pi/2} \times \right. \]

\[ \left. \times \left[ J_{\nu/2} \left( \omega a \right) - k_0^2 J_{\nu/2} \left( \omega a \right) \right] - \frac{2M}{b^2} \frac{Ze^2}{\omega^2} \cos (\omega a) \right\} \]

where \( \omega' = 4k_0^2 \sin^2 \theta/2. \)

Fig. 1. Comparison of differential elastic scattering cross-sections calculated by two methods for 18.3 MeV protons scattered from copper. The results of partial wave calculations are due to Chase and Rehrlich. The square-well type of nuclear potential is used in both the methods.
The differential scattering cross-section is given by
\[ \sigma_s(\theta) = R^4 |\psi_s(\vec{R})|^2 d\Omega. \]

In equation (3), \( \mathcal{A}_s, K, k_l, \omega a \) and \( \nu a \) are complex quantities. In order to obtain the scattering cross-section, it is necessary to separate real and imaginary parts in equation (3).

To do this, we first note that both \( |\omega a| \) and \( |\nu a| \) are large compared to \( 3/2 \) for the values of the parameters employed here. Hence it is possible to write the Bessel functions in the asymptotic forms:
\[ J_{n+1/2}(x) = \sqrt{2/\pi x} \sin(x - n\pi/2). \]
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The separation of real and imaginary parts of equation (3) is now fairly simple.

We also make the assumption that the absorptive part of the complex potential \( W_0 \) is small compared to \((E_p + V_0)\). Then if we write \( k_1 = c_1 + id_1 \), the following approximate expressions for \( c_1 \) and \( d_1 \) can be easily obtained:

\[
c_1 = \frac{\sqrt{2M(E_p + V_0)}}{\hbar} \quad \text{and} \quad d_1 = \frac{W_0}{\hbar \sqrt{2(M(E_p + V_0))}}.\]

The evaluation of real and imaginary parts of \( K \) then follows simply.

Using similar approximations the real and imaginary parts of \( \omega \) and \( \nu \) can be evaluated.

The following values were used in carrying out the calculations: \( V_0 = 45 \) Mev, \( W_0 = 20 \) Mev and \( a = 1.42 \times 10^{13} \times 10^{-13} \) cm. The above method has been used to calculate the differential elastic scattering cross-section for the two elements, gold and copper for the proton energies of 32.1 Mev and 18.3 Mev respectively. The latter have been compared with the cross-sections calculated by Chase and Rohrlich employing the partial wave method. These are plotted in Figure 1. The qualitative agreement between the two curves justifies the use of this approximation method in calculating the differential elastic scattering cross-section of intermediate energy protons. From Figure 1, it is found that the cross-sections calculated by the M-G method are in general higher than those calculated by the exact method. This has also been observed by Mohr and Robson who point out that the M-G cross-sections in the case of a complex potential are expected to be higher than the exact ones because of the fact that the former include a factor equal to the square of the modulus of the complex potential. For scattering angles below about 40°, however, M-G cross-sections in the present case turn out to be lower than the exact ones.

In Figure 2, the cross-sections calculated from the above considerations have been compared with the experimentally determined cross-sections for the scattering of 32 Mev protons from gold*. The poor agreement is probably due to the use of the square-well type of potential in the present calculations. It is known that the square-well type complex potential gives results which are in agreement with experimental observations only for scatterers of low nuclear charge, e.g. aluminium. For heavier nuclei where back scattering is relatively low this model does not give satisfactory agreement. It has been shown by Wood and Saxon that better agreement with experimental results can be obtained by using a diffused boundary model of the nucleus. Calculations are in progress using this type of potential.

* A report of this experimental work has been communicated elsewhere.
References

1) Aaron, W., McIntosh, J. S.; Schrank, G., and Bigelow, J. H., Phys. Rev. 99 (1955), 629A.