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A MODEL FOR ESTIMATING ACTUAL EVAPOTRANSPIRATION FROM POTENTIAL EVAPOTRANSPIRATION

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A model for calculating the daily actual evapotranspiration based on the potential one is presented. The potential evapotranspiration is reduced according to vegetation density, water content in the root zone, and the rainfall distribution. The model is tested by comparing measured (EA_m) and calculated (EA_c) evapotranspirations from barley, fodder sugar beets, and grass over a four year period. The measured and calculated values agree within 10 %. The model also yields information on soil water content and runoff from the root zone.

The potential evapotranspiration, defined as the climatic demand, can be measured with reasonable accuracy by evapotranspirometers of different kinds (W.M.O. 1966, Heldal 1969, Kristensen 1971), or it can be calculated from climatic parameters (Penman 1948, Aslyng 1965, W.M.O. 1966). The evapotranspiration that actually takes place from a given land surface is more difficult to ascertain, however, as, in addition to the climatic demand it also depends on the water supply to the evaporating surfaces, soil water content and rainfall distribution. An estimation of actual evapotranspiration may be made directly, from climatic measurements (Fritschen 1966, Tanner 1968), or indirectly, by measuring precipitation and change in the water content of the root zone. Another indirect method is that of relating the actual evapotranspira-

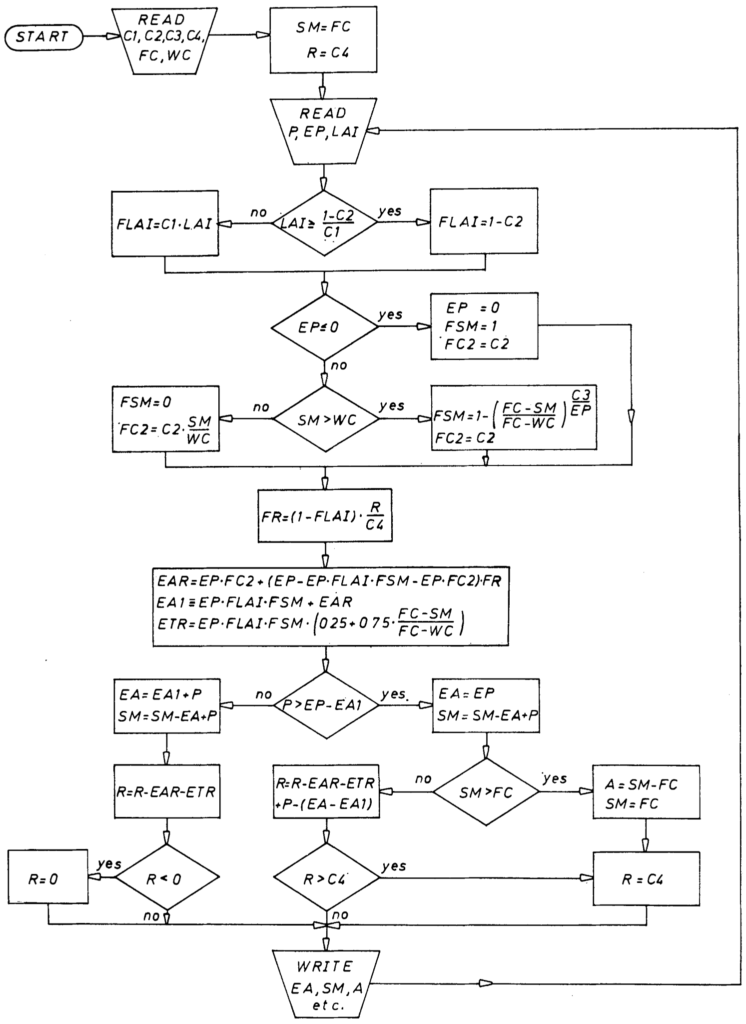

Fig. 1.

Flow chart for calculation of actual evapotranspiration based on potential evaporation.

tion to the potential, taking into account the density of the vegetation (leaf area index or other suitable parameter), the soil water content, and the rainfall distribution.

In the following presentation, a model for an indirect estimation of the actual evapotranspiration, based on the measured values of potential evapotranspiration is given. The calculated results are compared with measured ones for different agricultural crops over a period of from 3 to 5 years. In addition to the actual evapotranspiration, the model yields information on runoff, plus storage change and soil water content.

THE MODEL

It is an *a priori* assumption that the actual evapotranspiration can reach, but not exceed, the potential, regardless of the type of vegetation considered. The actual evapotranspiration will be below the potential if the evaporating surface is not supplied with water at a rate sufficiently high to meet the potential demand. Insufficient supply rate may be caused by incomplete vegetation density or none at all and/or by drying out of the soil. A model describing the actual evapotranspiration, therefore, must include functions involving the density of active (green) leaves, the root zone soil water, the intercepted water (vegetation and upper soil layer), and the rainfall distribution.

The model is presented in Fig. 1 as a flow diagram. In order to operate, certain constants have to be entered. These are: field capacity (FC), and wilting capacity (WC) of the whole root zone for the soil and vegetation in question. Furthermore, the constants (C1 – C4) required in the different functions need to be evaluated. These constants depend partly on soil conditions and partly on the vegetation.

Before starting, the initial conditions must be defined. These are: the actual soil moisture (SM) at starting time and the evaporative reserve in the upper soil layer (R). The nature of the latter is explained in a later section. If starting with the soil at field capacity, then SM = FC and R = C4.

Finally, the variables are entered. These are daily values of potential evapotranspiration (EP) and precipitation (P) (both in mm), and the density of the crop green material, i.e., leaf area index (LAI).

DEFINITION OF FUNCTIONS AND CONSTANTS

The values of FC and WC are found by ordinary methods used for soil characterization (Black 1965). Both are expressed in mm for the whole soil layer constituting the root zone. In evaluating FC and WC, both soil type and vegetation type should be considered with respect to root depth. The difference FC–SM is the amount of soil water removed from the soil by evapotranspiration. The difference FC – WC is the maximum amount of soil water that the vegetation can extract from the soil. When initiated, the model will supply itself with information on SM.

The constants C1 and C2 in the transpiration function F(LAI) are defined in Fig. 2 as the slope and intercept (for LAI = 0), respectively, of the straight line relation of EA/EP vs. LAI in the interval LAI = 0 to LAI = (1 - C2)/C1. For LAI in excess thereof, F(LAI) is assumed constant and equal to 1 - C2. Consequently, LAI needs only to be evaluated if it is below (1 - C2)/C1.

C2 is defined as a basic evaporation taking place, regardless of vegetation density and soil dryness, when SM of the root zone is not below WC. For SM below WC, F(C2) is assumed to decrease linearly with decreasing SM. C2 results from the process of diffusion between the moist soil atmosphere and the generally drier atmosphere above the soil.

For vegetated soils, it is difficult to separate evaporation and transpiration. However, as EA it not allowed to exceed EP, it is not necessary to distinguish between these two processes in order to estimate EA.

In the model, it is suggested that LAI be used as a vegetation characteristic. Other suitable characteristics (height, development stage, age of vegetation etc.) may possibly be used. The constant C1 should then be changed accordingly.

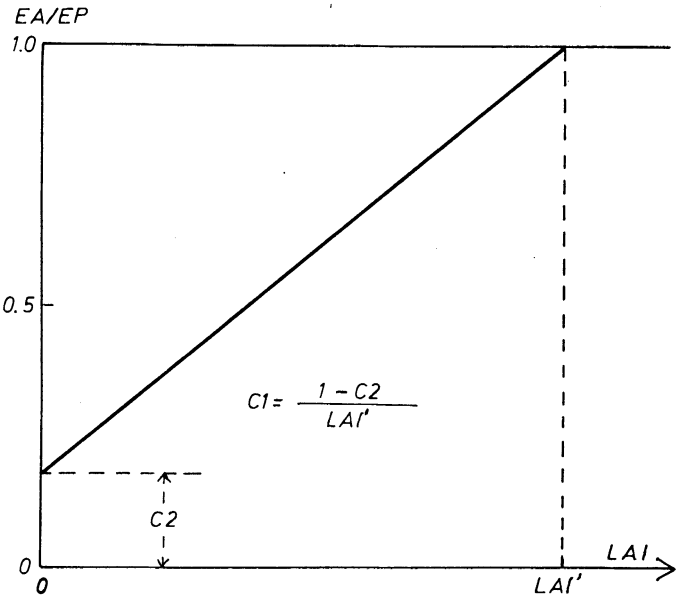

Fig. 2.

Definition of the constants C1 and C2. EA/EP is assumed to be a rectilinear function of leaf area index (LAI) in the interval from LAI = 0 to LAI = (1 - C2)/C1. For LAI greater than LAI' the transpiration function is a constant, 1 -C2.

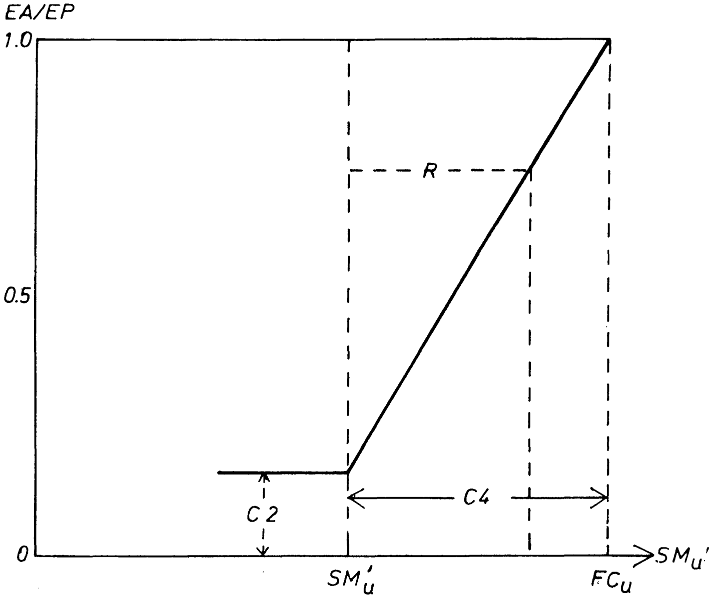

Fig. 3.
Definition of the constants C2 and C4. Subscript refers to the upper soil layer.

C4 and R are defined in Fig. 3. The constant C4 is the amount of soil water, in mm, that should be removed from the upper soil layer in order to reduce the evaporation from bare soil to the basic one defined above as EP · C2. The evaporative reserve (R) is the amount of soil water in the upper soil layer in excess of SM'_u ($= FC_u - C4$), where the subscript *u* refers to an undefined upper soil layer. R consequently can vary between zero and C4. Oversaturation is excluded by definition. Although it is realized that SM_u occasionally may exceed FC_u, this is assumed to be of relatively short duration, with only minor influence on the evaporation. For a bare soil, it is assumed that EA = EP when the upper soil layer is at field capacity, and that EA/EP decreases linearly as R is reduced to zero. From Fig. 3,

$$EA/EP = C2 + (1 - C2)\ (SM_u - FC_u + C4)/C4$$

is obtained.

The expression ($SM_u - FC_u + C4$) is equal to R, and the evaporation from bare soil (EAR) consequently is described by:

$$EAR = EP \cdot C2 + (EP - EP \cdot C2)R/C4$$

If the soil is covered by vegetation, the amount of energy penetrating to the soil surface is restricted and the evaporation from R is thereby also restricted. In order to adjust for that, the general function F(R) for R is written:

$$F(R) = (R/C4)\,(1 - FLAI)$$

As the vegetation will extract water from the upper soil layer, R must be reduced by a fraction of the transpired soil water (EP · FLAI · FSM). In the model this fraction (ETR) is arbitrarily chosen as:

$$ETR \equiv EP \cdot FLAI \cdot FSM\,[0.25 + 0.75(FC - SM/FC - WC)]$$

This implies that when SM ≡ FC, 25 % of the transpired soil water is taken from the upper soil layer. When SM approaches WC, approximately all soil water transpired is taken from the possible reservoir (R). The function has only a minor influence on the calculated EA and is merely introduced in order to bring R down to acceptable level when vegetation is present.

R is continuously adjusted by the model. The unevaporated part of a rainfall (P) is added to R, even if the upper soil layer might be dried out in excess of C4. The reason is that any surplus rainfall will moisten the soil to approximately field capacity from above to a depth determined by the amount of rainfall, soil type, and dryness of the soil.

The ratio EA/EP is also influenced by the dryness of the deeper root zone soil, as the rate at which plants can extract water is assumed to decline as the root zone soil dries out. The degree to which the ratio is influenced may depend on the evaporative demand (EP) and the relative dryness of the soil. Experimental results reported (Makkink & van Heemst 1956, Kristensen 1961, Denmead & Shaw 1962, Shaw & Laing 1965, Cowan 1965) show that the actual evapotranspiration and the crop yield decrease gradually as the root zone dries out, and that this decrease is initiated at a higher soil water content and is more pronounced the higher the evaporative demand. A function describing the effect, therefore, must involve both the root zone dryness and potential evapotranspiration. The exact relationship is not at hand, and, even if it was, it might be a rather complicated one, as different soil layers and actual root depth (varies in time for most agricultural crops) should most likely be involved. It is therefore suggested that an empiric soil moisture function be used, of the type:

$$F(SM) \equiv 1 - \left(\frac{FC - SM}{FC - WC}\right)^{\frac{C3}{EP}}$$

A family of curves, showing the relationship EA/EP versus relative dryness

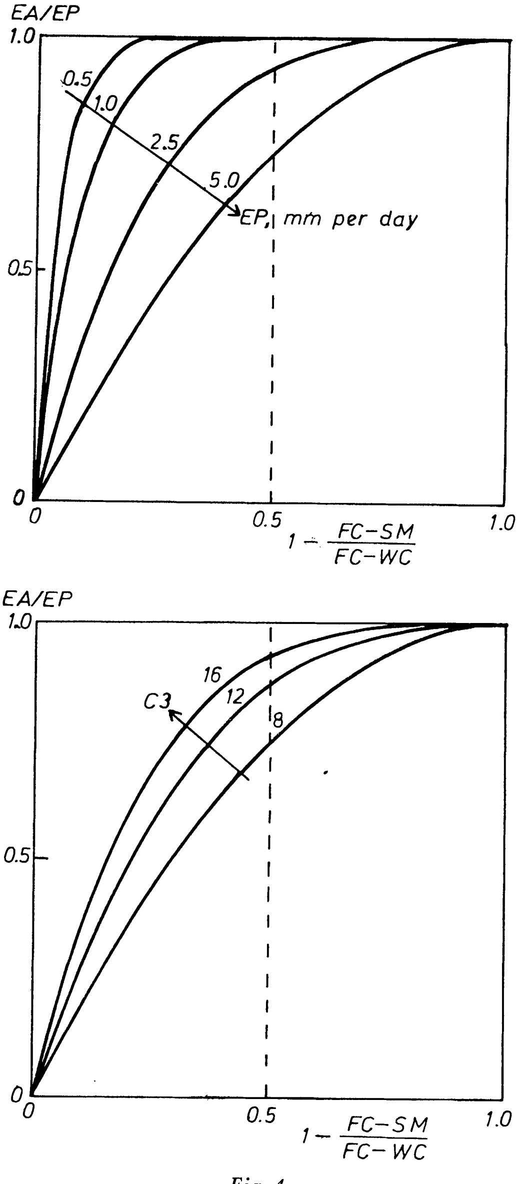

Fig. 4.

The empirical soil moisture function at constant C3 (10 mm/day) and varying EP (upper), and at constant EP (4 mm/day) and varying C3 (lower).

of the soil, are depicted in Fig. 4. In the upper part of this Figure, C3 is constant (10 mm/day) and EP is varied as indicated. The influence of C3 is illustrated in the lower part of figure 4, where EP is kept constant (4 mm/day), and C3 varied.

The influence of soil dryness is reduced when C3 is increased. The selection of an appropriate C3 value is a matter of guesswork and experience. The value to be chosen may depend on soil type (e.g. soil moisture retention curve) and vegetation (root density). The more water released at low matrix potential, and the denser the root system, the greater the C3 which should be chosen, and vice versa (Fig. 4, lower part). Generally, a higher value of C3 should be chosen for light soils and shallow rooted vegetations, than for heavier soils and vegetations with deeper root systems.

OPERATION OF THE MODEL

When the appropriate constants and variables are entered (Fig. 1) the function for vegetation cover, F(LAI), is calculated according to the value of LAI. Negative values of EP are not allowed. If $EP \leqq 0$, F(SM) is set to 1 and F(C2) to C2. With $EP > 0$ F(SM) is adjusted by EP and SM and the function for basic evaporation is defined as $F(C2) = C2$ for $SM > WC$ and $F(C2) = C2(SM/WC)$ for $SM \leqq WC$. The latter situation may be rare, but might occur for shallow soils overlaying a bedrock and in very dry climates. Finally, the function for the reserve evaporation is defined by $F(R) = (1 - F(LAI))R/C4$.

From the function for leaf area index and soil moisture, a transpiration is calculated as $EP \cdot F(LAI) \cdot F(SM)$. The evaporation (EAR) consists of the basic evaporation $(EP \cdot F(C2))$, plus evaporation from a possible topsoil reserve.

$$EAR = EP \cdot F(C2) + [EP - (EP \cdot F(LAI) \cdot F(SM) + EP \cdot F(C2)]F(R)$$

The sum of calculated transpiration and evaporation $(EP \cdot F(LAI) \cdot F(SM) + EAR)$ gives a temporary evapotranspiration (EA1), which is identical to the actual one, if no rainfall has occurred during the day in question.

If rainfall (P) has occurred, it is necessary to know if P is greater than $EP - EA1$ or not. If P is smaller than or equal to this difference, it is assumed to have evaporated, and consequently, $EA = EA1 + P$. SM is adjusted by substracting EA and adding P. If R is greater than zero, it is adjusted by substracting EAR and ETR. R is not allowed to be a negative value.

If P is greater than $EP - EA1$, EA is assumed equal to EP. SM is adjusted as mentioned above. If SM thereby exceeds FC, a runoff (A) is calculated as

A = SM – FC. The new SM is set equal to FC. R is further adjusted by adding the surplus precipitation (P – (EA – EA1)). R is not allowed to exceed C4.

The calculation procedure is repeated by reading the variables for the following day, using the calculated values of SM and R.

TESTING THE MODEL

The ability of the model to simulate the actual evapotranspiration is tested by comparing measured (EA_m) and calculated (EA_c) values of EA. EA_m was obtained by periodical measurement of the soil water content with a neutron scattering instrument (Basc D/M). For periods with SM below FC, EA_m is equal to $P + \triangle SM$, where $\triangle SM$ is the initial SM less the final SM of the period. A possible drainage is disregarded, as experiments from plastic-covered areas have shown that for the clayey loam soil used, FC remained fairly constant during the whole growing season, when rainfall and evaporation were prevented.

Soil water measurements were taken at 20 cm increments, starting at 10 cm

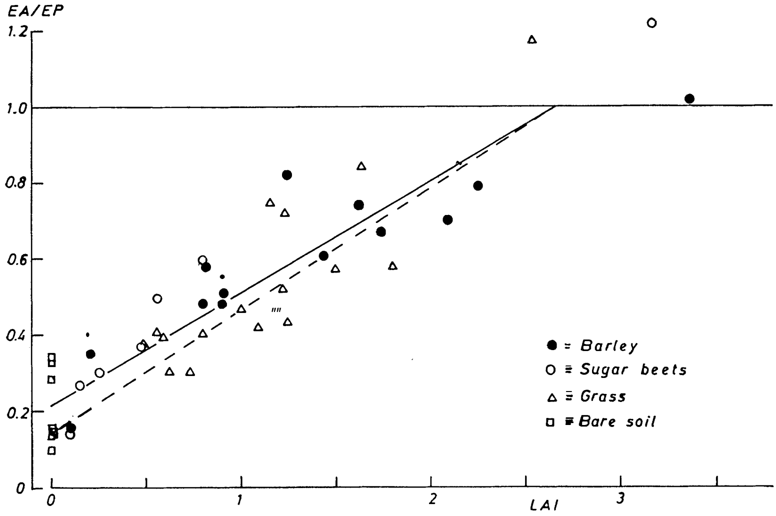

Fig. 5.

Relative evapotranspiration as a function of leaf area index (points). The line of best fit (solid line) and the one resembling transpiration dependence (broken line) are shown, the latter yielding C1 = 0.31 and C2 = 0.15.

depth and going to a depth of 150 cm. Four crops or crop treatments were involved (Barley, Fodder sugar beets, long grass (Grass L), and short grass (Grass S)). The grasses (lolium perenne) resembled hay grass and grazing grass, respectively. The measurements were carried out only during the growing season (April–May to October–November). Measurements of soil water in that period were carried out at 1–2 weeks intervals.

Concurrent with the soil water measurements, the leaf area index was estimated whenever crops were present. LAI was calculated by measuring the length and average width of a number of leaves as described by Kristensen (1974a). Daily values of LAI were estimated by assuming linear change in the period between actual measurements. For the winter period, an LAI of 1.0 was assumed for grass.

In order to calculate EA_c, the constants and functions mentioned earlier have to be evaluated. FC and WC are about 30 and 10 vol. % respectively for the clayey loam soil, corresponding to 3 and 1 mm per 1 cm soil depth, respectively

The value of C1 is calculated from results reported by Kristensen (1974b). The selected results, being for periods with no rainfall, are presented in Fig. 5. As periods with R = 0 are very rare, an evaporation in excess of the basic one has occurred in several of the periods, and EA/EP consequently is higher than

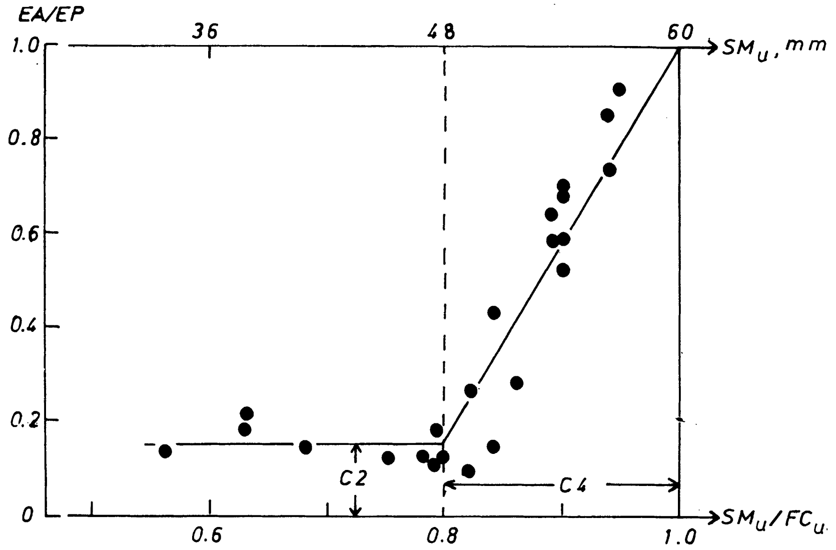

Fig. 6.

Relation of EA/EP to relative soil water content (SM_u/FC_u) and actual soil water content (SM_u) in mm for the upper (0–20 cm) layer of a clayey loam soil.

it would be if only transpiration and basic evaporation took place. The calculated straight line for the points presented in Fig. 5 is

$$EA/EP = 0.30(LAI) + 0.20$$

From other observations in bare soil (Fig. 6), however, it was found that the basic evaporation is about 0.15 EP. This value is assumed to be more correct than the 0.20 produced by the equation. The broken line given in Fig. 5 intercepts at EA/EP = 0.15, which is then identical to C2, and it is assumed to cross the value of EA/EP = 1.0 at the same value of LAI as the calculated function. The equation of the broken line is

$$EA/EP = 0.31(LAI) + 0.15$$

and C1 = 0.31.

C2 and C4 are found by measuring evaporation loss from bare soil during rainless periods. The relative actual evaporation related to the relative and actual soil water content of the upper (0–20 cm) soil layer is presented in Fig. 6. The ratio EA/EP declines linearly as the water content of the soil decreases from field capacity. At a relative soil water content below about 0.8, the relative evaporation remains fairly constant at about 0.15 (C2). C4 is then $FC_{(0-20)}-0.8\ FC_{(0-20)}$ or, for the soil in question, $0.2 \cdot 60 = 12$ mm.

The constant C3 has not been experimentally evaluated. For the present calculation C3 is assumed to be 10 (mm/day). For this value of C3, the relative actual transpiration governed by the soil water content is reduced to about 0.95 of medium (2.5 mm/day) evaporation intensity, when 50 % of the plant-available soil water is removed (cf. Fig. 4). When the soil water content is reduced further, EA/EP decreases rapidly. The function proposed is in rather

Table 1.
Constants and functions used for calculation of actual evapotranspiration (EA_C) from crops grown on clayey loam soil.

Crop	Dimensionless		mm/day	mm		
	C1	C2	C3	C4	FC	WC
Barley	0.31	0.15	10	12	450	150
Fodder sugar beets	0.31	0.15	10	12	450	150
Grass L	0.31	0.15	10	12	360	120
Grass S	0.31	0.15	10	12	240	80

good agreement with the assumption that about 50 % of the available soil water in the root zone can be extracted without serious consequences for the vegetation.

The constants and functions used for calculation of EA_c are summarized in Table 1. The root depth, estimated as the maximum depth from which water has been extracted, for barley and fodder sugar beets is 150 cm; for grass L, 120 cm; and for grass S, 80 cm.

Monthly values of calculated (EA_c) and measured (EA_m) actual evapotranspiration are presented in Fig. 7. The 1:1 line is drawn for comparison. This line obviously fits the points satisfactorily. The scattering of the points around the 1:1 line may have several explanations:

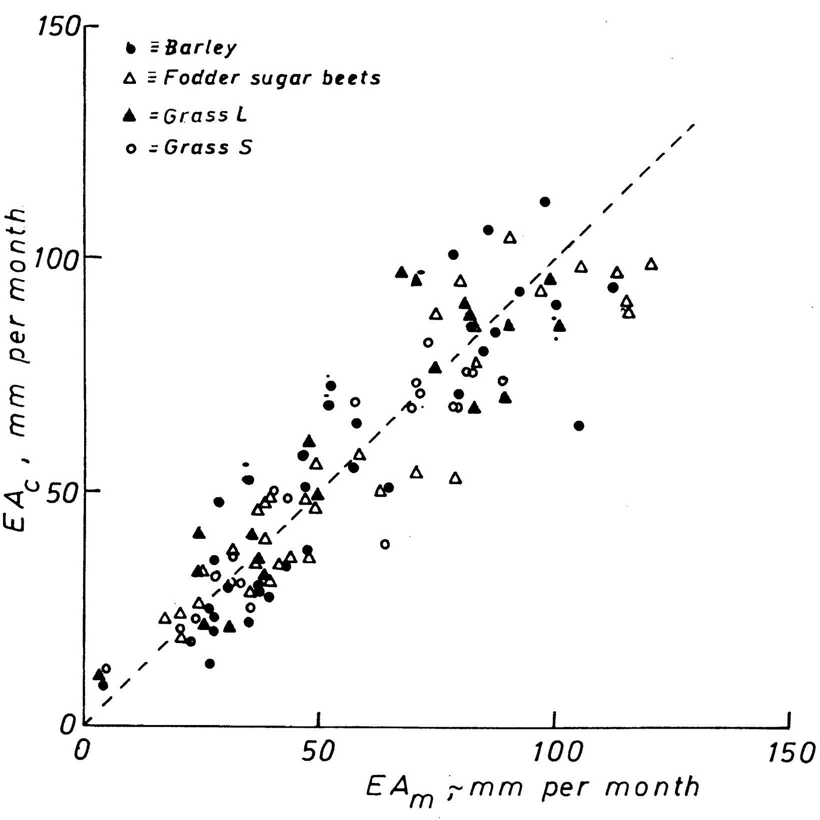

Fig. 7.
Calculated (EA_c) vs. measured (EA_m) actual monthly evapotranspiration from different agricultural crops.

1. Uncertainty of the neutron scattering method, estimated to be about ± 5 mm for a 0–150 cm soil layer.

2. Insufficient correctness of the instrument's calibration function, caused by, for example, incorrect soil densities assumed.

3. The constants and functions used for calculation of EA_c are not precise.

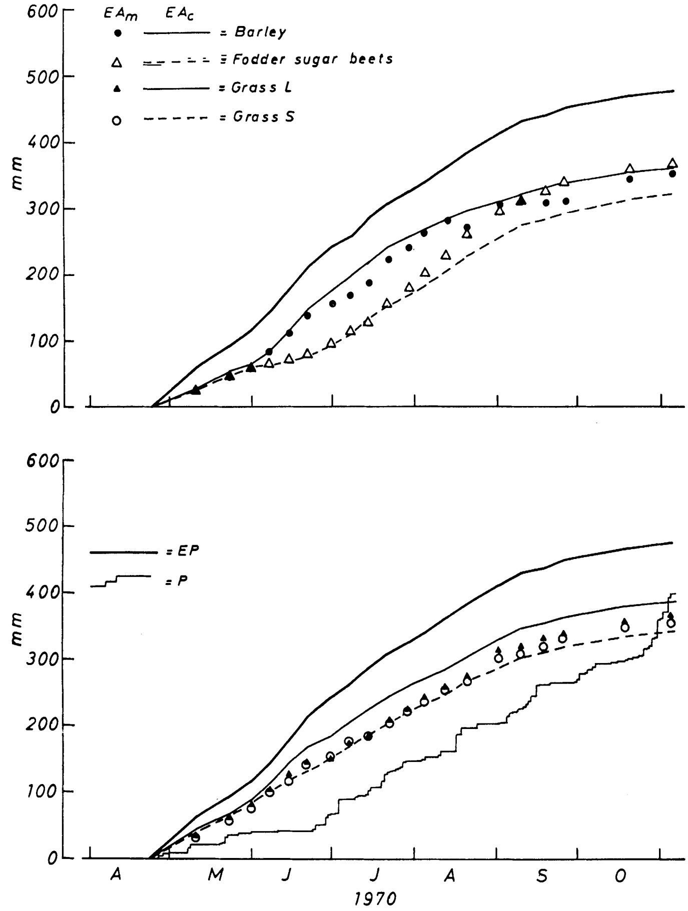

Fig. 8.

1970. Potential (EP) actual measured (EA_m), and calculated (EA_c) evapotranspiration summarized for the growing season. Upper part: Barley and Fodder sugar beets. Lower part: Grass L and grass S. Summarized rainfall is only shown in the lower part of the figure.

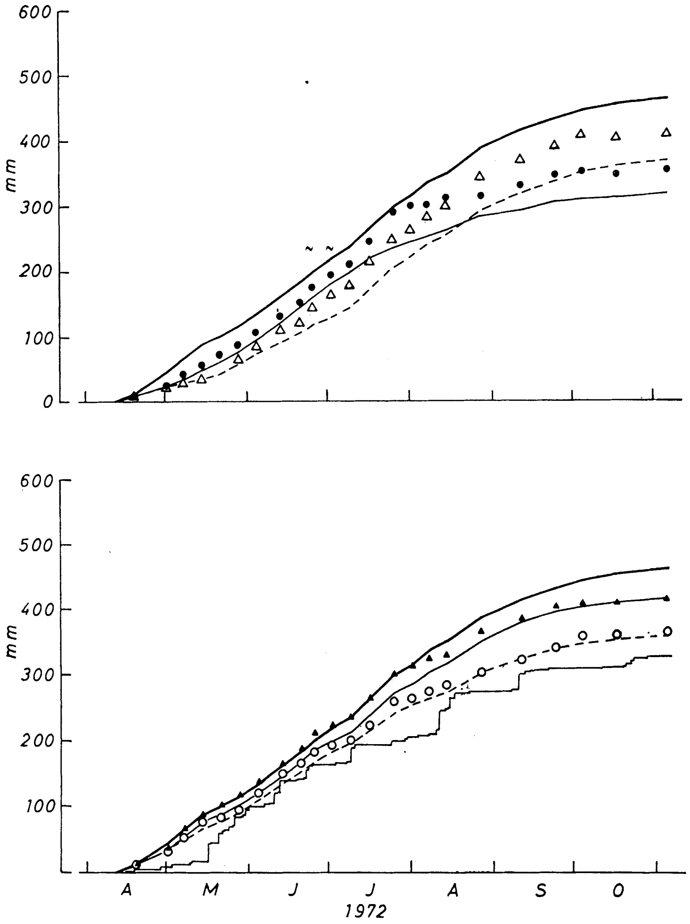

Fig. 9.
1972. Explanation same as Fig. 8.

4. Failure of the model to describe all situations, e.g. evaporation from an area covered with inactive vegetation such as ripening or ripe grain crops.

5. The EP used (Penman's function) may not be the upper limit of EA for all crops or development stages.

Summarized values of EP, EA_m, EA_c', and precipitation are presented in Figs. 8–10 for the years 1970, 1972, and 1973, respectively. In order to reduce confusion, each of these figures is divided in two parts, showing barley and fodder sugar beets in the upper part and the two grass crops in the lower part. The potential evaporation is shown in both parts of the figures. Precipitation is shown only in the lower part, but it is valid for the upper part as well.

The summarized precipitation characterizes the humidity conditions of the growing season. 1970 (Fig. 8) had a rather long, dry period in the first part of

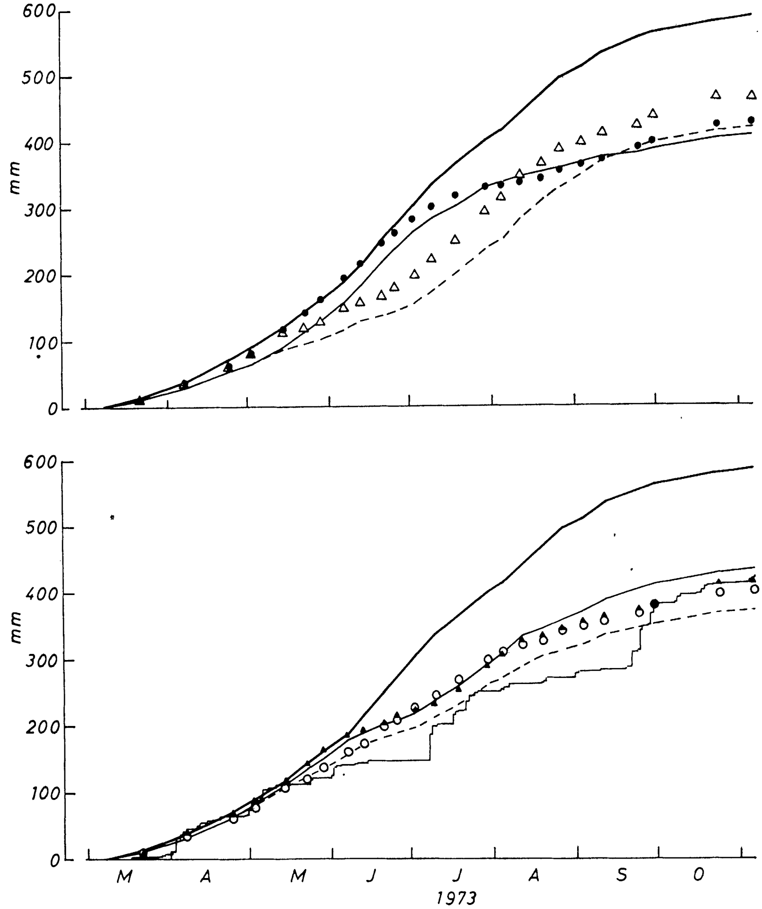

Fig. 10.
1973. Explanation same as Fig. 8.

Table 2:
Precipitation, potential evaporation, and actual (measured and calculated) evapotranspiration (mm) from different types of vegetation 1969–1973.

Year	Period	P	EP	Barley		Sugar beets		Grass L		Grass S	
				EA_m	EA_c	EA_m	EA_c	EA_m	EA_c	EA_m	EA_c
1969	Apr. 1–Nov. 5	310	522	389	378	409	391				
1970	Apr. 24–Nov. 5	397	476	352	359	366	319	366	387	355	342
1971	Apr. 8–Nov. 5	316	534	367	392	359	379				
1972	Apr. 12–Nov. 6	326	462	353	318	408	367	415	417	363	359
1973	Mar. 8–Nov. 7	418	586	427	409	465	420	412	433	400	371

the season, causing a relatively low evapotranspiration from uncropped soil (sugar beets and barley) while the evaporation rate from grass was rather high. The calculated evapotranspiration for Grass L is considerably above the measured one in the dry period. A possible explanation for this may be an overestimation of the available water in the soil at that rather early time of the year. The years 1972 and 1973 (Figs 9 & 10) were rather rainy in the first part of the season. Consequently, the lagging behind of incompletely vegetated soils is not so pronounced in these years. The dry period in the middle of the season causes a reduction of evapotranspiration from the sugar beets. The depicted values of EA_m for barley and sugar beets may be too high in 1972, as the rather high precipitation intensity in May–June caused drainage from the soil under these crops. For such periods, EA_m was assumed equal to EP, which may be in excess of the true EA_m.

The summarized values of P, EP, EA_m, and EA_c are given in Table 2 for the years 1969–1973. LAI was not measured for grass in 1969, nor for any of the crops in 1971. In 1971 LAI values for barley and sugar beets were estimated from observations of emergence and development stages.

The measured and calculated values of actual evapotranspiration agree reasonably well on a full-season basis. The differences between them are at most of the order of 10 %. As the exact values of EA are not known, it may be concluded that the calculated values are acceptable.

DISCUSSION AND CONCLUSIONS

The model presented for calculation of actual evapotranspiration yields acceptable values when used for common agricultural crops grown on well-drained clayey loam soil. The ability of the model to work for any soil and vegetation type depends, however, on the exactness of the constants and functions used. Thus far these constants and functions have been evaluated only for barley, fodder sugar beets, and grass grown on clayey loam soil with a great reservoir of plant-available water. For soil with less available water, the importance of, for example, the soil moisture function (C3) will be greater, while the purely plant-dependent function (C1) may be independent of soil type.

In the present calculation, the crop density function is assumed to be linear. Judged from experimental results, this assumption seems reasonable (Fig. 5), and is a very convenient one, as observation of the crop is only required in periods with crop densities below a certain value.

The constants C2 and C4 found for clay loam soil are not necessarily valid for other soil types. The physical significance of C2 is the gaseous diffusion taking place between the moist soil atmosphere and the generally more dry atmosphere above the soil.

As the amount of material transported is nearly proportional to the area through which diffusion takes place, a slightly higher value for C2 may be expected for soil with greater air porosity, e.g. sandy soil, and a slightly lower value for soil with lesser air porosity. The constant C4 is experimentally found to be FC – 0.8 FC of the upper (0–20 cm) soil layer. The 20 cm layer was selected because the neutron scattering measurements were referred to 20 cm soil layers. The value FC – 0.8 FC for C4 may be an acceptable one for other soil types as well.

The field capacity and wilting capacity of the soil is readily measured by ordinary laboratory methods. The soil layer constituting the root zone is more difficult to ascertain, as it depends upon soil structure, soil texture, drainage conditions, vegetation type, and cultivation practice. A vegetation root depth of 100–150 cm can be assumed in good sandy soils and light clay soils if no extreme structural conditions are present, whereas a more shallow root depth may be expected in heavier clay soils and in light sandy soils.

When the plant- and soil-dependent information is at hand, the operation of the model only requires day-to-day information on P, EP, and vegetation density. P is measured by ordinary rain gauges mounted in such a way that the influence of wind is as small as possible. EP can be either measured (evaporimeters) or calculated (climatic observations). The crop density can be measured either by sampling or by measuring light transmission through the canopy, if inactive plant material can be disregarded or corrected for, or it can be estimated from plant development stages. Due to the linear relationship used in the model, the plant observation period is limited to the periods in which the density is below a certain value. The daily calculation is needed in order to use the functions at the correct level.

In the model it is assumed that runoff cannot occur before the soil is resaturated to field capacity after drying out. If this requirement is not fulfilled (e.g. due to steep slopes, impermeable soils, some cracking soils) the model fails to work. For most normal soils, the model can be used, and will yield information on the day-by-day actual evaporation, change in soil water content and runoff.

REFERENCES

Aslyng, H. C. (1965) Evaporation, evapotranspiration and water balance investigations at Copenhagen, *Acta Agr. Scand.* XV, 284–300.

Black, C. A. (1965) Methods of soil analysis, *Agronomy* 9, Vol. 1. Madison, Wisc. U.S.A.

Cowan, I. R. (1965) Transport of water in the soil-plant-atmosphere system, *J. Appl. Ecol.* 2, 221–239.

Denmead, O. T. and R. H. Shaw (1962) Availability of soil water to plants as affected by soil moisture content and meteorological conditions, *Agron. J.* 54, 385–390.

Fritschen, L. J. (1966) Evapotranspiration rates of field crops determined by the Bowen ratio method, *Agron. J.* 58, 339–342.

Heldal, B. (1969) Evaporation from different pans in relation to meteorological conditions, Sci. Rep. Agr. Coll. Norw., Vol. 48, 1–42.

Kristensen, K. J. (1961) Crop yield as a function of soil moisture supply, Roy. Vet. Agr. Univ. Yearb. 1961, 31–53.

Kristensen, K. J. (1971) Potentiel vandfordampning bestemt ved forskellige metoder (Summary), *Vannet i Norden* 1971:3, 11–28.

Kristensen, K. J. (1974a) Production and solar energy utilization by two different strains of rye grass, Roy. Vet. Agr. Univ. Yearb. 1975, 1–16.

Kristensen, K. J. (1974b) Actual evapotranspiration in relation to leaf area, *Nordic Hydrology 5,* 173–182.

Makkink, G. F. and H. D. J. van Heemst (1956) The actual evapotranspiration as a function of the potential evaporation, *Neth. J. Agr. Sci.* 4, 67–72.

Penman, H. L. (1948) Natural evaporation from open water, bare soil and grass, *Proc. Roy. Soc.* A 193, 120–146.

Shaw, R. H. and D. R. Laing (1965) Moisture stress and plant response, In: Plant environment and Efficient Water Use (Ed. W. R. Pierre et al.). A.S.A.E. Madison, Wisc., U.S.A

Tanner, C. B. (1968) Evaporation of water from plants and soil, In: Water Deficit and Plant Growth (Ed. T. T. Kozlowski). Acad. Press, London.

W.M.O. (1966) Measurement and estimation of evaporation and evapotranspiration, W.M.O. Tech. Note No. 83.

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