Determination of the critical pressure drop for filtration of super-compactible cakes

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Abstract
In accord with Darcy’s law, the flow rate through a porous bed depends upon the pressure drop \( \Delta p_c \). In general, increasing \( \Delta p_c \) leads to increased values of flow rate and average percentage solids in filtration operations. When cakes become super-compactible, their behavior undergoes an unexpected change in which both the flow rate and the percentage solids reach maximum values and thereafter are unaffected by increasing \( \Delta p_c \). The critical pressure drop \( \Delta p_{cR} \) is defined as that value at which the flow rate reaches 90% of its ultimate value. When \( \Delta p_c \) is greater than \( \Delta p_{cR} \) and is doubled or tripled, the cake resistance approximately doubles or triples leaving the rate virtually unchanged. The super-compactibility problem is analyzed theoretically, and is verified by stepped pressure filtration experiments on different materials from Houston and Korea.

Keywords
Filter cake, super-compactible materials, critical pressure drop, flow rate, solidity \( e_s \) (volume fraction of solids), specific cake resistance, stepped pressure filtration

Introduction
Solid/Liquid Separation (SLS) involves the relative movement of solids and liquid in settling of suspensions and liquid flow through a matrix of particles, called “cake” or “sediment”, and formed by successive layers of particles on a supporting medium. The particulate matrix at the surface has a null-stress solidity (volume fraction of solids) \( e_{so} \). At any given time, the solidity \( e_s \) increases to a maximum at the point where the cake rests on the supporting medium. Cakes and sediments may be incompressible (sand), moderately compactible (kaolin clay), or highly or super-compactible (floculated biosolids, activated sludge) according to the fragility of the particulate structure and its response to load. A cake with a low \( e_{so} \) tends to be fragile and compactible (Tiller and Kwon, 1999; Tiller and Li, 2000). Highly floculated suspensions of biosolids, or activated sludge in wastewater treatment plants with very low values of \( e_{so} \) and very fragile particulate structures fall in the super-compactible category.

For incompressible cakes, there is a proportional relation between liquid flow rate and the pressure drop across the cake, as determined by Darcy’s law. On the other hand, for super-compactible cakes, as pressure increases, a critical region is reached where the flow rate does not respond to the increases of the overall pressure drop, and it appears as though Darcy’s law has been repealed. Determination of cake compactibility and the critical pressure drop are important to SLS process control and optimization. In this paper, the fundamental laws of flow through porous cake are used to explain the strange behavior of super-compactible materials, and a simple stepped pressure filtration experiment for measuring compactibility parameters is presented.

Fundamentals of flow through porous compactible cakes
Theory of the flow through a porous medium is based on Darcy’s law, stress analysis, and empirical equations for a particulate cake structure. A typical cake structure for linear flow cake filtration is shown in Figure 1. There are two pressures \( p_L \) and \( p_s \) in the cake. The
pressure $p_L$ is the internal liquid pressure decreasing from the pump pressure $p$ at the cake surface to $p_m$ at the medium in Fig. 1. The “effective pressure” $p_s=F_s/A$ resulting from the accumulated frictional drag forces $F_s$ as shown in Figure 2 increases from zero at the cake surface to a maximum value at the medium. A relationship involving $p_s$ and $p_L$ was obtained by stress analysis (Tiller and Kwon, 1999; Tiller and Li, 2000) in the form

$$p_L + p_s = p \quad \text{or} \quad dp_L + dp_s = 0 \quad (1)$$

Darcy’s differential equations involving both spatial and material coordinates illustrate the relationship of local values of permeability $K$ and specific resistance $\alpha$ to the gradients of $p_L$ with respect to $x$ and $\omega$ as

$$q = \frac{K}{\mu} \frac{dp_L}{dx} = \frac{1}{\mu \alpha} \frac{dp_s}{d\omega} \quad (2)$$

in which $q$ is the flow rate; the spatial coordinate $x$ is the distance from the cake surface, and the material coordinate $\omega$ is the volume of solids/unit area in distance $x$ as shown in Figure 1. As $K$ and $\alpha$ in Eq. 2 are functions of the effective or compressive pressure $p_s$ rather than $p_L$, Eq. 1 cannot be integrated unless $p_L$ is substituted by $p_s$. Making the substitution $dp_L = -dp_s$ (Eq. 1) yields the “Particulate Structure Equation”

$$q = \frac{K}{\mu} \frac{dp_s}{dx} = -\frac{1}{\mu \alpha} \frac{dp_s}{d\omega} \quad (3)$$

A relationship of $K$ and $\alpha$ to $p_s$ is provided by the empirical constitutive eqs. (Tiller and Leu, 1980)

$$
\left( \frac{\varepsilon_s}{\varepsilon_m} \right)^{\beta/\delta} = \left( \frac{K}{K_c} \right)^{1/\delta} = \left( \frac{\alpha/\alpha_c}{1} \right)^{1/\delta} = 1 + p_s/p_a \quad (4)
$$

in which $p_a$ is an empirical parameter; $\beta$, $\delta$, $n$ are cake compressibility coefficients. They provide a measure of the rate of change of $\varepsilon_s$, $\alpha$, $K$ with $p_s$ subject to $\varepsilon_s$, $\alpha K=1$ and $\delta=n+\beta$.

Combining the Eqs. (3) and (4), and integrating on $x$ and $\omega$ subject to the boundary conditions at the cake surface ($x=L$ or $\omega=\omega_c$, $p_s=0$) and at the medium ($x=0$ or $\omega=0$, $p_s=\Delta p_c=p_m$) yields the flow rate and average solidosity as functions of pressure drop across the cake $\Delta p_c$ (Tiller, Kwon, 1999)

$$q = \frac{\Delta p_c}{\mu \alpha_m \omega_c} = \frac{p_a}{\mu \omega_c \alpha_c (1-n)} \left( \left( 1 + \frac{\Delta p_c}{p_a} \right)^{1-n} - 1 \right) = \frac{K_m \Delta p_c}{\mu L} = \frac{K_m p_a}{\mu L (1-\delta)} \left( \left( 1 + \frac{\Delta p_c}{p_a} \right)^{1-\delta} - 1 \right) \quad (5)
$$

$$\varepsilon_{av} = \varepsilon_{av} \left( \frac{1-n}{1-n} \left( 1 + \frac{\Delta p_c}{p_a} \right)^{1-n} - 1 \right)$$

$$= \varepsilon_{av} \left( \frac{1-\delta}{1-\delta} \left( 1 + \frac{\Delta p_c}{p_a} \right)^{1-\delta} - 1 \right) \quad (6)$$

**Figure 1** Diagram of a cake with spatial $x$ and material $\omega$ coordinates

**Figure 2** Frictional drag in a particulate bed
It is clear from Eqs. 5 and 6 that the flow rate and the volume fraction of cake solids are dependent on the null-stress cake structure parameters $\varepsilon_{so}, K_o, \alpha_o$, and cake compressibility coefficients $n, \delta, \beta$.

**Behavior of super-compactible materials**

The relative compactibility of cakes can be crudely classified in accord with the magnitudes of $n$ and $\delta$ of the empirical constitutive equations (4) as shown in Table 1.

The cake compactibility parameters for an incompressible Carbonyl Iron, a moderately compactible Kaolin Flat D, and a super-compactible Mierlo Biosolid are given in Table 2.

Plots of calculated values of flow rate and average solidosity as functions of $\Delta p_c$ from Eqs. 5 and 6 with $\omega_0=0.002m^3/m^2$ are shown in Figure 3 and Figure 4 for the three materials in Table 2. Whereas the flow rate $q$ increases linearly with $\Delta p_c$ for incompressible materials like the Carbonyl Iron, it increases with a power function of $\Delta p_c$ at a rate less than unity for the compactible kaolin. Unfortunately, for the super-compactible Mierlo Biosolid with $n>1$, and $\delta>1$, the flow rate and the average solidosity increase with pressure only in a relatively low pressure range, and undergo little change as pressure increases beyond some critical value. Compared with the incompressible or moderately compactible material, the behavior of super-compactible materials is strange and unusual. For super-compactible materials when $n>1$ and $\delta>1$, Eqs. 5 and 6 are best rearranged as follows:

$$q = P_c \mu \omega_{o, \alpha_o}(n-1) \left[ 1 - \frac{1}{\left(1 + \frac{\Delta p_c}{p_o}\right)^{n-1}} \right]$$

(7)

**Table 1** Classification of cake compactibility

<table>
<thead>
<tr>
<th>Incompressible</th>
<th>$n = 0$</th>
<th>$\delta = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderately Compactible</td>
<td>$n = 0.4-0.7$</td>
<td>$\delta = 0.5-0.9$</td>
</tr>
<tr>
<td>Highly Compactible</td>
<td>$n = 0.7-0.8$</td>
<td>$\delta = 0.9-1.0$</td>
</tr>
<tr>
<td>Super-compactible</td>
<td>$n &gt; 1$</td>
<td>$\delta &gt; 1.0$</td>
</tr>
</tbody>
</table>

**Table 2** Cake Compactibility Parameters for three materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>$\varepsilon_{so}$</th>
<th>$\alpha_o$, m$^{-2}$</th>
<th>$\beta$</th>
<th>$n$</th>
<th>$\delta$</th>
<th>$p_o$, Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbonyl Iron, Grade E (Grace, 1953)</td>
<td>0.575</td>
<td>$2.34 \times 10^{13}$</td>
<td>0.001</td>
<td>0.005</td>
<td>0.006</td>
<td>–</td>
</tr>
<tr>
<td>Kaolin Flat D (Chen, 1986)</td>
<td>0.14</td>
<td>$2.98 \times 10^{13}$</td>
<td>0.12</td>
<td>0.4</td>
<td>0.52</td>
<td>11</td>
</tr>
<tr>
<td>Mierlo Biosolid (LaHeij, 1994)</td>
<td>0.03</td>
<td>$4.02 \times 10^{12}$</td>
<td>0.47</td>
<td>1.83</td>
<td>2.3</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Figure 3** Variation of $q$ against $\Delta p_c$

**Figure 4** Variation of $\varepsilon_{sav}$ against $\Delta p_c$
As $\Delta p_c$ increases indefinitely in Eq. 7 and 8, the last term approaches zero. The limiting values of the flow rate and average solidosity become

$$q_{\text{max}} = \frac{p_a}{\alpha_s(n-1)\mu\omega_c}$$  

(9)

$$\varepsilon_{\text{av, max}} = \varepsilon_{\infty} \left( \frac{n-1}{n} \right)$$  

(10)

The critical pressure drop $\Delta p_{cR}$ is defined as the pressure at which $q$ reaches 90% of its maximum value given by Eq. (9). With $q/q_{\text{max}} = \gamma = 0.9$, $\Delta p_{cR}$ becomes (Tiller and Li, 2000)

$$\Delta p_{cR} = p_a \left[ \left( \frac{1}{1 - \gamma} \right)^{1/(n-1)} - 1 \right] = p_a \left[ \left( \frac{1}{1 - 0.9} \right)^{1/(n-1)} - 1 \right] = p_a \left[ 10^{1/(n-1)} - 1 \right]$$  

(11)

The critical pressure drop provides a guide for specifying practical operating pressures for filtration of super-compactible materials. As pressure is increased beyond the critical pressure drop, which is only 15kPa for the Mierlo Biosolid, there will be negligible effect of pressure on either the flow rate or the average percentage of cake solids.

The underlying theory of the strange behavior of super-compactible cakes involves the transmission of stress through the particulate bed (Tiller, Li, 2000) and the formation of a resistant skin near the medium. After the skin is formed when the operating pressure is beyond the critical pressure drop, increasing pressure will simply proportionally increase the resistance of skin leading to little effect of pressure on the flow rate. Furthermore, since the skin absorbs most of the pressure drop across cake, increase pressure will have little change on solidosity over most of the Mierlo Biosolid cake in contrast to the kaolin.

**Stepped-pressure filtration experiment**

**Volume versus time and flow rate**

A stepped-pressure filtration experiment of flow through already formed cakes was designed to study the behavior of super-compactible materials. The apparatus includes a filter with an area of 0.00456m$^2$, an air compressor, a digital balance and a computer data acquisition system. The experimental materials includes: 2% (volume) kaolin with a total volume of cake solids per unit area $\omega_c = 0.00228$m$^3$/m$^2$, 2%(volume) flocculated kaolin with $\omega_c = 0.0022$m$^3$/m$^2$, and 0.34%(volume) raw activated sludge from South Korea. The experimental results of volume of filtrate versus time for different materials under different pressures are shown in Figure 5.

![Figure 5 Volume of filtrate versus time](image-url)

![Figure 6 $q$ versus pressure drop](image-url)
Average specific cake resistance

Assuming the medium resistance can be neglected, the pressure drop across cake $\Delta p_c$ equals the total pressure $p$. The average specific cake resistance can be obtained from Eq. 5 by substituting pressure, $q$, and the known total volume of solids per unit area $\omega_c$ into the equation. Calculated average specific resistance are plotted against pressure in Figure 7, where there is a large increase with pressure of $\alpha_{av}$ for the 2% flocculated kaolin and a moderate increase for the 2% kaolin.

Average solidosity

During the experiments, the variations of cake thickness were recorded. With constant known $\omega_c$ values, the average solidosity could be calculated (Tiller, 1990). The calculated average solidosity as function of pressure is shown in Figure 8.

The average solidosity of flocculated 2% kaolin range from 0.04 to 0.09 compared to the higher values of 0.15 to 0.228 for the 2% kaolin without flocculation. The large difference in the average solidosity results from the increased fragility of the particle structure after flocculation with the resultant substantial increase of cake compactibility.

Cake Compactibility Parameters

Cake compactibility parameters $\alpha_o$, $\varepsilon_{so}$, $n$, $\delta$, $\beta$, and the empirical value $p_a$ in Eq. 10 are important cake characteristics for determining $q$ and $\varepsilon_{sav}$ and the critical pressure drop for super-compactible materials. The null-stress state parameters $\alpha_o$, $\varepsilon_{so}$ can be interpolated from Figure 7 and Figure 8 when $p=0$. The exponents $n$, $\delta$, $\beta$, and the empirical $p_a$ can be obtained by data fitting based on the Eqs. 5, 6 and the experimental $\alpha_{av}$ versus $p$ and $\varepsilon_{sav}$ versus $p$ data. The calculated cake compactibility parameters are shown in Table 3.

Comparison of experimental and theoretical results

Calculated and experimental $v$-$t$, $q$-$v$, $\alpha_{av}$-$p$, and $\varepsilon_{sav}$ versus $p$

The comparison of $v$-$t$, $q$-$p$, $\alpha_{av}$-$p$, and $\varepsilon_{sav}$ versus $p$ of calculated and experimental values are shown in Figures 5, 6, 7 and 8. From the figures, the calculated $v$-$t$, $q$-$p$ and $\alpha_{av}$-$p$ match the experimental values fairly well for both materials in the experimental pressure ranges. Unfortunately, a difference of about 50% between the experimental and theoretical $\varepsilon_{sav}$

Table 3 Cake Compactibility Parameters

<table>
<thead>
<tr>
<th>Materials</th>
<th>$\alpha_o$, 1/m²</th>
<th>$\varepsilon_{so}$</th>
<th>$n$</th>
<th>$\delta$</th>
<th>$\beta$</th>
<th>$p_a$, Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>2% kaolin</td>
<td>$5.00 \times 10^{14}$</td>
<td>0.15</td>
<td>0.62</td>
<td>0.86</td>
<td>0.24</td>
<td>36000</td>
</tr>
<tr>
<td>Flocculated 2% kaolin</td>
<td>$1.44 \times 10^{14}$</td>
<td>0.034</td>
<td>1.9</td>
<td>2.3</td>
<td>0.4</td>
<td>3100</td>
</tr>
</tbody>
</table>

Figure 7 $\alpha_{av}$ versus pressure drop

Figure 8 Average solidosity against pressure drop
occurred for flocculated kaolin, and 20% for kaolin in Figure 8. Some underlying mechanism such as the collapse of the cake structure, which has not been taken into the theoretical equation, might account for the large difference.

**Critical pressure drop**
With known cake compactibility parameters, the calculated critical pressure drop $\Delta p_{cR}$ from Eq. 12 for super-compactible flocculated kaolin equals 5.38psi(36.9kPa), which is very close to the estimated critical pressure drop 5psi from Figure 6. Any pressure beyond the critical pressure drop will have little effect on the flow rate. The explanation of the strange behavior of super-compactible cake lies in the existence of a concentrated skin near the medium as discussed before. The values of $\Delta p_{cR}$ at which the filtration rate is 90% of its maximum value is of critical importance to design and operation of filtration systems involving super-compactible cakes. For those materials with low critical pressure drop and low percentage of cake solids below 10%, the use of gravity filtration followed by expression with belts or pistons to increase the average cake solidosity is recommended for the dewatering of this type of materials.

**Conclusion**
Compared with moderately compactible materials, the super-compactible material is characterized by maximum values of flow rate and percentage of cake solids. The critical pressure drop is defined as the pressure at which flow rate reaches 90% of its maximum value. The critical pressure drops are very low values for super-compactible materials. A gravity filtration followed by expression operation is recommended for dewatering of those materials.

To verify the theoretical analysis, stepped pressure filtrations of flow through already formed cake were carried out on kaolin and flocculated kaolin, and activated sludge. Based on the original volume versus time data at different pressures, the variations with pressure of average flow rate, average specific resistance, and average solidosity were calculated. The cake compactibility parameters $\alpha^c$, $\varepsilon_{so}$, $n$, $\delta$, $\beta$, and the empirical value $p_a$ were obtained by analysis of the experimental data. The experimental behavior of the flocculated 2% kaolin reflected the predicted performance of super-compactible materials. The critical pressure drop was determined as 5.38psi (36.9kPa) for the flocculated kaolin.

**References**