GEOPHYSICS, VOL. 59, NO. 11 (NOVEMBER 1994); P. 1729–1734, 4 FIGS.

Understanding air-gun bubble behavior

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ABSTRACT

An air-gun bubble behaves approximately as a spherical bubble of an ideal gas in an infinite volume of practically incompressible water. With this simplification, the equation of bubble motion and its far-field signature is more understandable than with the more exact theory commonly cited in the literature. The terms of the equation of bubble motion are explained using elementary physics and mathematics, computation of numerical results is outlined, and an example signature is shown. An air-gun bubble is analogous to a simple harmonic oscillator consisting of a mass on a spring, with an equivalent mass equal three times that of the water displaced by the bubble, and air pressure following an ideal gas law corresponding to a spring. With this understanding, one is prepared to deal with the effects of interactions among air guns and with the higher-order terms and other features that must be included to model the air-gun signature of actual seismic source arrays.

INTRODUCTION

The air gun is the most common seismic source for marine exploration surveys. A typical marine air-gun array signature, such as shown in Figure 1, is the sum of many individual interacting air guns and their ghosts caused by reflection at the sea surface. Analysis of array signatures into individual "notional" sources is treated by Ziolkowski et al. (1982). The notional sources are the point source signatures for each air gun, modified by interactions with the others and their ghosts, so that the array signature is the superposition of the notional pressure fields. An individual air bubble oscillates with a period related to air-gun volume (and other parameters), and an array is tuned to achieve destructive interference among the bubble oscillations and to concentrate signature amplitude in the initial peak (and its ghost).

Understanding the oscillation of a single bubble is the basis for treating interaction among air guns and modeling array signatures. Qualitatively this oscillation is rather intuitive. The air bubble released by the gun is relatively soft and springy compared to the surrounding water which is much more massive, and these conditions provide the components of a harmonic oscillator, a mass on a spring as shown in Figure 2. However, rigorous and quantitative analysis is quite complex, even in the ideal case of a spherical bubble at the center of an infinite volume of water where spherical symmetry allows the physics to be formulated in terms of the single dimension of radius from center (Ziolkowski, 1970). Because of this complexity, most geophysicists abandon any wish to understand air-bubble oscillation beyond the qualitative level, since the problem is peripheral to usual exploration concerns. In this paper I present no new physics, but develop an explanation of air-gun bubble behavior using only elementary physics and mathematics. Hopefully, this will enhance the understanding of this phenomenon among a wider group of geophysicists.

Essentially, the air in the bubble behaves like an ideal gas, with pressure being a simple exponential function of volume. This results in a force on the bubble wall that acts like a spring on a coupled mass of water three times that displaced by the bubble. Effectively, the air spring is coupled only to a shell of water with outside radius $4^{1/3}$ times the inside (bubble) radius. This equivalent harmonic oscillator radiates an acoustic pressure wave with a $1/R$ divergence of the pressure at the bubble wall, where the air pressure combines with dynamic and hydrostatic pressure terms. Oscillations of the bubble are damped by this resistive radiation, as well as by turbulence and other dissipative mechanisms.

APPROXIMATE THEORETICAL MODEL

For the ideal case of a spherical bubble in an infinite volume of water at hydrostatic pressure p_0, the bubble motion approximately obeys

$$\frac{d^2r}{dt^2} = \frac{p}{\rho r} - \frac{3}{2r}\left(\frac{dr}{dt}\right)^2 + \frac{1}{\rho \nu}\frac{dp}{dt}, \tag{1}$$

Manuscript received by the Editor August 5, 1993; revised manuscript received April 29, 1994.
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where r is bubble radius as a function of time t, p is the acoustic pressure at the bubble wall [see equation (3) below], ρ is the density of the water, and ν is the acoustic velocity of the water. This approximation is equivalent to Safar's (1976) equation (8). Density ρ and acoustic velocity ν are assumed constant in this equation. The approximate notional signature corresponding to this, referenced to unit distance, is

$$\text{signature} = r\left(p + \frac{\rho u^2}{2}\right), \tag{2}$$

where $u = (dr/dt)$ is the particle velocity at the bubble wall. This signature is the source wavelet that propagates into the far-field with a $1/R$ attenuation of amplitude. That is, the

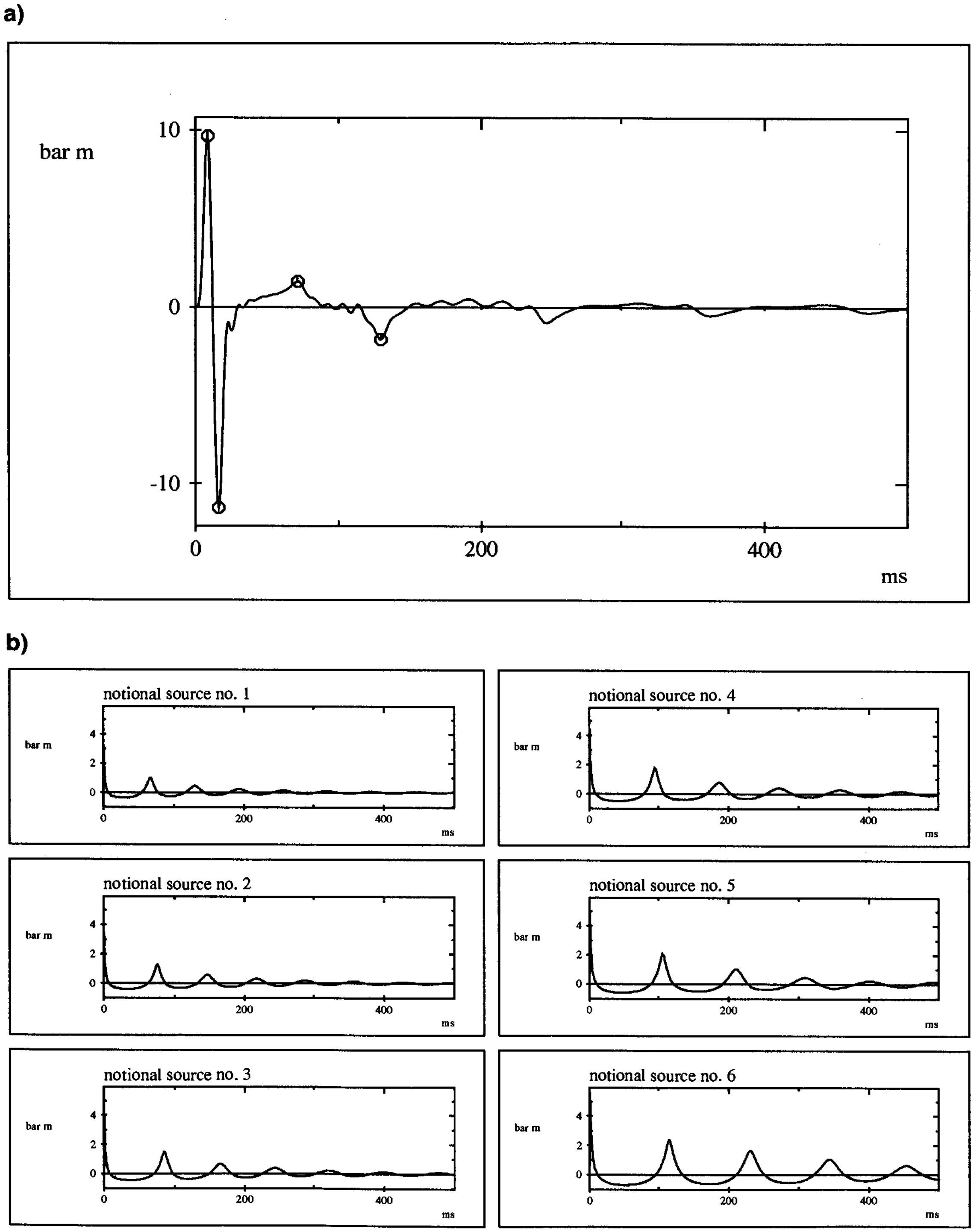

FIG. 1. An array of six air guns modeled using the Modgun software package from SeRes. The vertical far-field signature is shown in (a), with a DFS-V instrument response filter. The notional signatures of the six individual air guns are shown in (b) with no filtering. In number order, they have volumes of 1.00, 1.41, 2.00, 2.83, 4.00, and 5.66 dm^3, respectively, and are separated by 5 m to limit interaction effects. All guns were at 6 m depth and operated at a pressure of 13 MPa.

pressure field at a distance R from the origin is $(p + \rho u^2/2)r/R$, provided $R \gg r$. This expression for the signature corresponds to applying the first-order approximation of Ziolkowski (1970) directly at the bubble wall.

The acoustic pressure p at the bubble wall is the absolute air pressure p_a inside the bubble minus the hydrostatic pressure p_0 in the water:

$$p = p_a - p_0. \tag{3}$$

The absolute air pressure in the bubble is given by

$$p_a = p_0 \left(\frac{r_0}{r} \right)^{3\gamma}, \tag{4}$$

where r_0 is the equilibrium bubble radius at hydrostatic pressure, and γ is a thermodynamic parameter having a value 1.0 for isothermal, 1.4 for adiabatic, and about 1.13 based on fitting empirical air-gun data, according to Ziolkowski (1970).

The computer pseudocode shown in Figure 3 models these equations by a simple modified Euler method, with the substitution

$$\frac{dp}{dt} = \frac{-3\gamma p_a}{r}\frac{dr}{dt} = \frac{-3\gamma p_a u}{r} \tag{5}$$

derived by differentiating equation (4). Results for a specific case are shown in Figure 4; details are given in the Appendix. For comparison, Figure 4a also shows a result calculated using Ziolkowski's (1970) more exact theory. The agreement is good, and much closer than the match of either theoretical result with a signature derived from observed data as shown in Figure 4b. The simple modeled signature has a much larger amplitude than observed, taking about six cycles to decay down to the observed initial amplitude. Safar (1976) explains this as caused by the energy loss when releasing the air through the ports of an actual gun, although his methods have been disputed (Ziolkowski, 1977; Buchanan, 1977; Safar, 1978). The observed signature also decays much

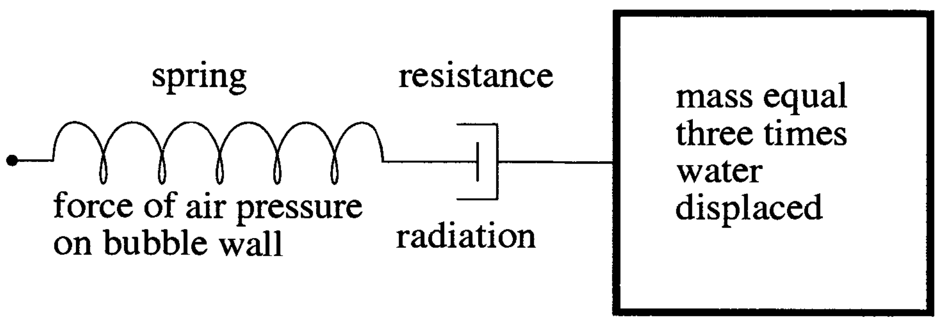

FIG. 2. Harmonic oscillator equivalent to an air-gun bubble. Parameters change with bubble size.

```
define constants v, ρ, γ
set parameters r0, p0
initialize a, u, r, pa for t = 0
iterate while (t < 1):
      output t, r(pa − p0 + ρu²/2)        signature from equations (2) and (3)
      ap ← a                              save previous values
      up ← u
      rp ← r
      dt ← 0.004r²                        variable time step
      t ← t + dt
      do 2 times:                         2 iterations per time step
            pa ← p0 (r0/r)^3γ             equation (4)
            a ← ((pa − p0)/ρ − 1.5u² − 3γpa u/(ρv))/r
                                          equations (1), (3), and (5)
            u ← up + dt (ap + a)/2        trapezoid integration
            r ← rp + dt (up + u)/2
      end do
end iterate                               loop for next time step
```

FIG. 3. Algorithm to model bubble behavior in computer pseudocode. With suitable initialization, this code produces the simple model result shown in Figure 4a.

faster than the model, presumably because of turbulence and other energy losses.

DISCUSSION

The approximate model described above can be derived by rigorous fluid dynamics as outlined by Ziolkowski (1970), and then eliminating less important higher-order terms as discussed by Schulze-Gattermann (1972) and Safar (1976). However, an understanding of the essential terms can be gained directly from elementary physics, with some sacrifice of rigor and elegance.

Consider a fluid element just outside the bubble wall, with its inner face at radius r having cross-sectional area A. If there is a steady acoustic pressure at r of p, the pressure on

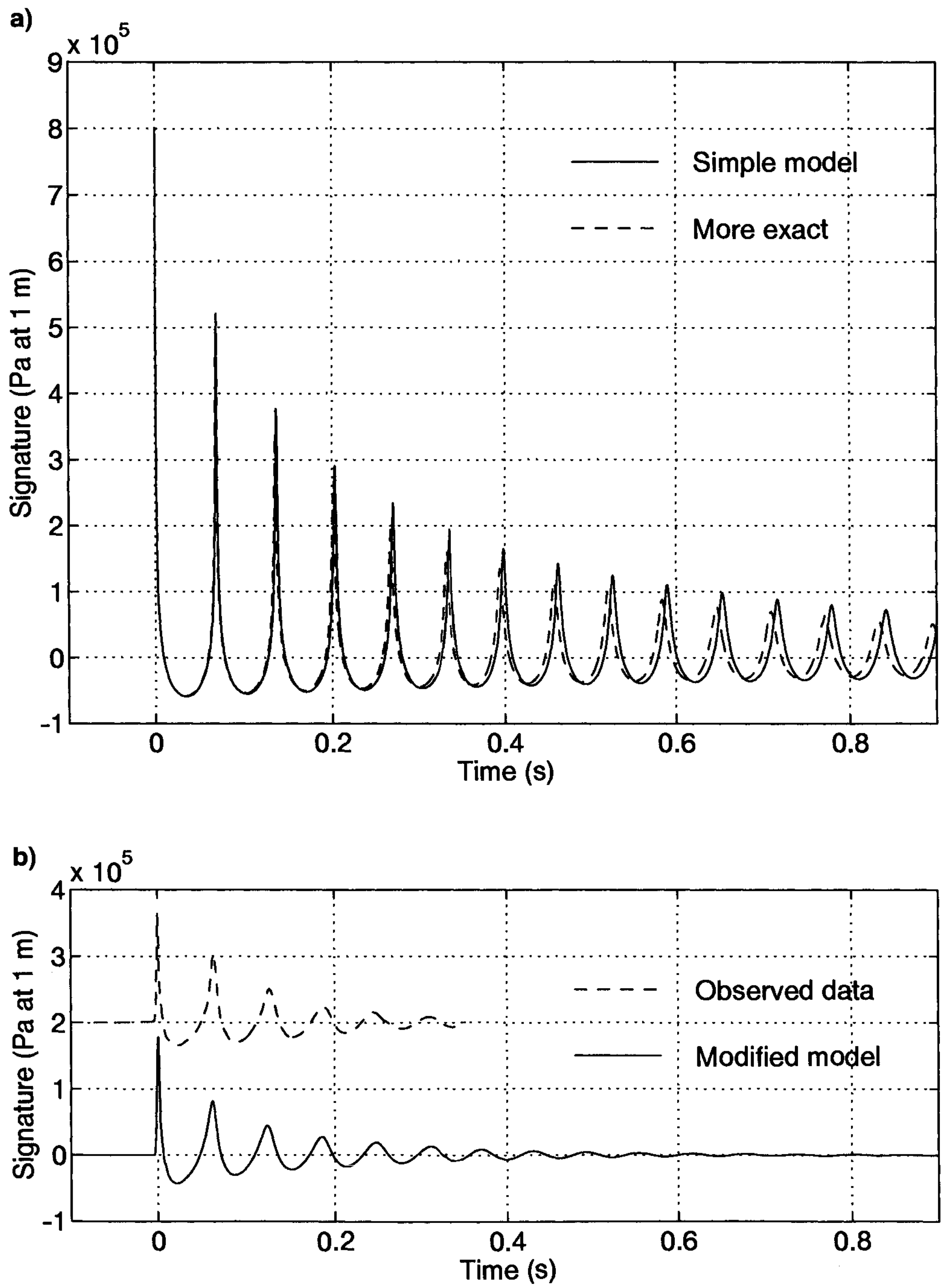

FIG. 4. Signature of a 1.0 dm^3 air gun with 13 MPa air pressure and hydrostatic pressure corresponding to 6 m depth in seawater. Results of the simple model presented here are shown in (a) with a solid curve and compared to the more exact model of Ziolkowski (1970), dashed curve. For comparison, a signature derived from observed data is shown in (b) with a dashed curve; results from a modification to the simple model are shown with a solid curve. A DC offset of 2×10^5 Pa at 1 m has been added to the observed signature.

the outer face of the element at $(r + \Delta r)$ is reduced by $1/R$ spherical divergence to

$$p_2 = p \frac{r}{r + \Delta r}, \tag{6}$$

and the force F_p acting on the fluid element caused by the pressure difference is

$$F_p = (p - p_2)A = p\left(1 - \frac{r}{r + \Delta r}\right)A \cong p\left(\frac{\Delta r}{r}\right)A. \tag{7}$$

Although there is spherical symmetry, it is best to consider a rectangular fluid element that has the same cross-sectional area on its outer and inner faces, because the forces on its other faces that are parallel to the radius cancel on opposite sides. The mass of the fluid element is $\rho A \Delta r$, so by Newton's second law, $F = ma$, there is an acceleration of

$$a = \frac{d^2 r}{dt^2} = \frac{F_p}{m} = \frac{pA\Delta r}{r\rho A \Delta r} = \frac{p}{\rho r}, \tag{8}$$

which gives the first term on the right of equation (1). This informal derivation is based on $1/R$ spherical divergence rather than the more fundamental differential equations of fluid dynamics, since acoustic wave propagation is so familiar to geophysicists and simply assuming it enables an intuitive short cut to understanding bubble behavior.

The surface area of a sphere is $4\pi r^2$, so the whole air bubble applies a force to the water of

$$F_s = 4\pi r^2 p. \tag{9}$$

With the acceleration of equation (8), we can use $F = ma$ to derive an equivalent coupled mass of the water,

$$m_e = \frac{F_s}{a} = \frac{4\pi r^2 p}{(p/\rho r)} = 4\pi\rho r^3. \tag{10}$$

Since the volume of a sphere is $(4/3)\pi r^3$, this is three times the mass of the water displaced by the bubble. The coupled mass corresponds to the phenomenon of near-field flow. This mass can be combined with a linear approximation of the air pressure as an equivalent spring to treat low amplitude bubble oscillations as a simple harmonic oscillator (see the Appendix).

The variation of coupled mass with bubble size cannot be neglected in modeling larger amplitude oscillations. The second term on the right of equation (1) can be understood by applying a conservation of energy constraint to the bubble and coupled mass system, where m_e varies as the bubble changes size. The kinetic energy E of the coupled mass is $m_e u^2/2$, and changes in this energy must equal the work done by F_s, so

$$\begin{aligned} F_s u &= \frac{dE}{dt} = \frac{1}{2} m_e 2u \frac{du}{dt} + \frac{1}{2} u^2 \frac{dm_e}{dt} \\ &= m_e u a + \frac{1}{2} u^2 \frac{d}{dt}(4\pi\rho r^3) \\ &= m_e u a = m_e u a + 6\pi\rho r^2 u^3. \end{aligned} \tag{11}$$

Rearranging to solve for the acceleration a and substituting F_s from equation (9) gives the first two terms on the right of equation (1):

$$a = \frac{F_s u - 6\pi\rho r^2 u^3}{m_e u} = \frac{4\pi r^2 p - 6\pi\rho r^2 u^2}{4\pi\rho r^3} = \frac{p}{\rho r} - \frac{3u^2}{2r}. \tag{12}$$

This is known as the Rayleigh equation for bubble oscillation in an incompressible fluid, and applies approximately to air guns in water, since even at typical air-gun pressures of about 13 MPa, water is compressed by only about 0.5 percent. The addition of the second term of equation (1) modifies the sinusoidal low amplitude bubble motion to the sharp peak and broad trough character of air-gun signatures.

Because energy is conserved by the first two terms, bubble oscillation would continue forever without the third term on the right of equation (1), which describes the radiated acoustic energy and damps the oscillations of the bubble. This radiation term can be understood as the well-known formula for plane-wave particle velocity, $u = p/\rho v$, differentiated to give acceleration. In the equivalent harmonic oscillator of Figure 2, it is a dashpot resistance between the air spring and the water mass. An electrical analog can also be made with capacitance, resistance, and inductance corresponding to the spring, dashpot, and mass, respectively. This approximate theoretical model ignores the compressibility and therefore finite acoustic velocity of water. Thus, equation (2) for the signature does not account for the acoustic traveltime over the bubble radius. In addition to the direct bubble pressure p in equation (2), the second term is caused by the dynamic pressure of the water flow, the familiar Bernoulli effect.

The basic simple model was modified as described in the Appendix and shown in Figure 4b to give a better fit to the observed data by decreasing the model's initial amplitude and increasing the damping. Ziolkowski (1970), Schulze-Gattermann (1972), and Safar (1976) discuss further complicating factors like noninstantaneous release of the air, energy losses in air release, and the presence of the air-gun body in the bubble. Interaction with ghosts and other bubbles in an air-gun array can be modeled by considering their pressure fields as additional, time-varying, hydrostatic pressure. A thorough application of the complete theory to the case of the GI (Generator Injector) gun is given by Landro (1992) and to the water gun by Landro et al. (1993).

CONCLUSIONS

The simple approximate theoretical model presented here is more understandable than the conventional, more exact theory and fits observed air-gun signatures almost as well. Improved modeling of signatures is achieved primarily by empirical modifications for mechanical factors and energy losses, rather than by the higher-order terms included in the more exact theory. These more sophisticated models and the literature on air-gun modeling can be understood more intuitively with the simple model of air-gun bubble behavior presented here.

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APPENDIX

EXAMPLE PARAMETERS AND LINEAR CASE

Figure 4 shows results for an air gun with a volume of 1.0 dm^3 (61.02 cubic inch) operating at a pressure of 13 MPa (1885 psi) at a depth of 6 m. Seawater parameters of $\rho = 1030$ kg/m^3 and $\nu = 1500$ m/s were used. A depth of 6 m at this density with standard gravitation and atmospheric pressure gives a hydrostatic pressure p_0 of 0.162 MPa. The initial bubble radius for 1.0 dm^3 is 0.062 m (by inverting $(4/3)\pi r^3$), and solving equation (4) for r_0 from the initial pressure of 13 MPa gives $r_0 = 0.226$ m. The initial bubble wall velocity is the plane-wave particle velocity,

$$\frac{dr}{dt} = u = \frac{p}{\rho\nu} = \frac{(13 - 0.162) \times 10^6}{1030 \times 1500} = 8.4 \text{ m/s}. \quad \text{(A-1)}$$

It is interesting to note that the empirical value of $\gamma = 1.13$ implies that the initial temperature of the air in the gun must be quite hot if the ideal gas law ($PV = nRT$) holds and the final temperature is that of the water.

To better fit the observed data, lower amplitude initial conditions were used with the same mass of air satisfying equation (4), a radius of 0.102 m, and a pressure of 2.43 MPa. Stronger damping was introduced by multiplying the third term on the right of equation (1) by seven. The modified signature was convolved with the instrument response of a DFS-V recording system with a conventional 1 ms antialias filter to produce the result in Figure 4b. The skew of the observed signature, with the troughs not centered between the peaks, is caused by the recording instrument filter. The instrument filter also smooths the discontinuity at $t = 0$ and attenuates the initial amplitude.

For low amplitude bubble oscillations, a linear approximation can be made of equation (1) that can be solved explicitly. This is best understood in terms of the equivalent harmonic oscillator. The equivalent mass has already been derived in equation (10) and for the 1.0 dm^3 example with $r \cong r_0 = 0.226$ m, $m_e = 148$ kg. We can now treat the air as an equivalent spring by considering how the force of the air on the bubble wall varies with radius. The force, from equations (3), (4), and (9), is

$$F_s = 4\pi r^2(p_a - p_0) = 4\pi r^2 p_0\left(\left(\frac{r_0}{r}\right)^{3\gamma} - 1\right)$$
$$= 4\pi p_0(r_0^{3\gamma}r^{2-3\gamma} - r^2). \quad \text{(A-2)}$$

Differentiation gives the spring "constant" (which is not constant in this case, but a function of r):

$$k = -\left(\frac{dF_s}{dr}\right) = -4\pi p_0(r_0^{3\gamma}(2 - 3\gamma)r^{1-3\gamma} - 2r). \quad \text{(A-3)}$$

At the equilibrium radius r_0,

$$k_0 = -4\pi p_0((2 - 3\gamma)r_0 - 2r_0) = 12\pi\gamma p_0 r_0. \quad \text{(A-4)}$$

We now have both parts of the equivalent harmonic oscillator, with the mass m_e from equation (10) and the spring from equation (A-4). The period of such an oscillator is given by

$$\tau = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\rho r_0^3}{12\pi\gamma p_0 r_0}} = 2\pi r_0\sqrt{\frac{\rho}{3\gamma p_0}}. \quad \text{(A-5)}$$

Schulze-Gattermann (1972) attributes this frequency relationship to Minnaert (1933). In our example, $k_0 = 1.56 \times 10^6$ N/m. Note that this is a very stiff spring; hanging the 148 kg equivalent mass from it in standard gravitation would stretch it by less than 1 mm. From equation (A-5), the period of oscillation is 61.5 ms, the frequency of 16.3 Hz, is consistent with Figure 4.