Waveform inversion in the frequency domain for the simultaneous determination of earthquake source mechanism and moment function

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SUMMARY
We propose a method of waveform inversion to rapidly and routinely estimate both the moment function and the centroid moment tensor (CMT) of an earthquake. In this method, waveform inversion is carried out in the frequency domain to obtain the moment function more rapidly than when solved in the time domain. We assume a pure double-couple source mechanism in order to stabilize the solution when using data from a small number of seismic stations. The fault and slip orientations are estimated by a grid search with respect to the strike, dip and rake angles. The moment function in the time domain is obtained from the inverse Fourier transform of the frequency components determined by the inversion. Since observed waveforms used for the inversion are limited in a particular frequency band, the estimated moment function is a bandpassed form. We develop a practical approach to estimate the deconvolved form of the moment function, from which we can reconstruct detailed rupture history and the seismic moment. The source location is determined by a spatial grid search using adaptive grid spacings, which are gradually decreased in each step of the search. We apply this method to two events that occurred in Indonesia by using data from a broad-band seismic network in Indonesia (JISNET): one northeast of Sulawesi (\(M_w = 7.5\)) on 2007 January 21, and the other south of Java (\(M_w = 7.5\)) on 2006 July 17. The source centroid locations and mechanisms we estimated for both events are consistent with those determined by the Global CMT Project and the National Earthquake Information Center of the U.S. Geological Survey. The estimated rupture duration of the Sulawesi event is 16 s, which is comparable to a typical duration for earthquakes of this magnitude, while that of the Java event is anomalously long (176 s), suggesting that this event was a tsunami earthquake. Our application demonstrates that this inversion method has great potential for rapid and routine estimations of both the CMT and the moment function, and may be useful for identification of tsunami earthquakes.

Key words: Inverse theory; Tsunamis; Earthquake dynamics; Earthquake source observations.

1 INTRODUCTION
The source location, origin time, magnitude, focal mechanism and moment function are the fundamental parameters that characterize an earthquake. Rapid estimations of the source parameters are critically important for the evaluation of possible seismic hazards and prompt emergency response. Advances in the deployment of global and regional seismic networks, the development of various waveform inversion techniques, and recent increases in computer capacity enable us to perform routine and near real-time estimations of quantitative source information for earthquakes.

Centroid moment tensor (CMT) solutions for moderate to large earthquakes are routinely estimated by the Global CMT Project (http://www.globalcmt.org) and the National Earthquake Information Center (NEIC) of the U.S. Geological Survey (Sipkin 1994) by using waveform data from global broad-band seismic networks. For more rapid estimations of CMT solutions, near real-time CMT inversions are performed using regional broad-band seismic networks in Japan (Kawakatsu 1995, 1998; Fukuyama et al. 1998; Ito et al. 2006) and in central California (Dreger & Helmberger 1993; Pasyanos et al. 1996; Dreger et al. 1998; Tajima et al. 2002). Assuming a point source and using the hypocentre and origin time information independently determined from high-frequency onset readings, these inversions determine six independent moment tensor components by solving inverse problems with or without the non-volumetric source constraint. The detailed source location is estimated by an iterative method or a grid search in space. The centroid time is estimated by cross-correlations of observed seismograms with Green’s functions or by a grid search in time. These methods assume that the moment function can be represented by a simple
Waveform inversion in the frequency domain

2 Method

2.1 Waveform inversion in the frequency domain

The displacement field excited by a point seismic source may be written in a discrete form (e.g. Aki & Richards 2002)

\[ u_n(j \Delta t) = \sum_{i=1}^{N_m} \sum_{l=0}^{N_s-1} G_m(j \Delta t - l \Delta t) m_i(l \Delta t) \Delta t, \]

where \( \Delta t \) is the sampling interval, \( u_n(j \Delta t) \) is the \( n \)th trace of a displacement seismogram at time \( j \Delta t \), \( m_i(j \Delta t) \) is the \( i \)th base of the moment function tensor, and \( G_m(j \Delta t) \) represents the spatial derivative of the corresponding Green’s function. \( N_m, N_t, \) and \( N_s \) are the number of independent bases of moment tensor components, number of seismic traces, and number of samples in each trace, respectively. Eq. (1) is written in matrix form as

\[ \mathbf{d} = \mathbf{Gm}, \]

where \( \mathbf{d} \) is the data vector consisting of \( u_n(j \Delta t) \), \( \mathbf{G} \) is the matrix of Green’s functions \( G_m(j \Delta t) \), and \( \mathbf{m} \) is the model parameter vector consisting of \( m_i(j \Delta t) \). The dimension of the matrix \( \mathbf{G} \) is \( N_t N_m \times N_s N_i \). The convolution relation in eq. (1) means that the displacement \( u_n(j \Delta t) \) includes the entire time history of the moment tensor functions and Green’s functions before time \( j \Delta t \). This results in solving a large matrix in eq. (2), which requires much computer memory and long computation time. A more efficient approach is to solve the problem in the frequency domain (Stump & Johnson...
The Fourier transform of eq. (1) is given by
\[ \tilde{u}_k(\omega_k) = \sum_{i=1}^{N_m} G_{ii}(\omega_k) \tilde{m}_i(\omega_k), \quad k = 1, \ldots, N_f, \] (3)
where \( \omega_k \) is the angular frequency; \( \tilde{u}_k(\omega_k), G_{ii}(\omega_k) \) and \( \tilde{m}_i(\omega_k) \) are the Fourier transforms of \( u_k(\Delta t) \), \( G_{ii}(\Delta t) \) and \( m_i(\Delta t) \), respectively; and \( N_f \) is the number of frequency components used for the waveform inversion. In this form, the equations for all frequencies are independent of each other and can be solved separately (Stump & Johnson 1977). Eq. (3) is written as \( N_f \) sets of matrix equations as
\[ \tilde{d}(\omega_k) = \tilde{G}(\omega_k) \tilde{m}(\omega_k), \quad k = 1, \ldots, N_f, \] (4)
where \( \tilde{d}(\omega_k) \) is the data vector consisting of \( \tilde{u}_k(\omega_k) \), \( \tilde{G}(\omega_k) \) is the data kernel matrix with its element of \( G_{ii}(\omega_k) \), and \( \tilde{m}(\omega_k) \) is the model parameter vector consisting of \( \tilde{m}_i(\omega_k) \).

The dimension of the data kernel matrix in eq. (4) is \( N_f \times N_m \); which is much smaller than that of eq. (2) in the time domain. Solving eq. (4) \( N_f \) times is much faster than solving the single large matrix represented by eq. (2). Therefore, the computation time required for inversion in the frequency domain is much shorter than in the time domain (e.g. Auger et al. 2006; Nakano et al. 2007).

In our waveform inversion, we assume a pure double-couple source mechanism, which does not change throughout an event. This assumption reduces the number of free parameters in the waveform inversion. Accordingly, the source mechanism can be stably determined by using waveform data from a small number of stations (Nakano et al. 2006). Assuming a pure double-couple mechanism, \( \tilde{m}(\omega_k) \) can be decomposed into the moment function and moment tensor as follows:
\[ \tilde{m}(\omega_k) = \tilde{m}^{DC}(\phi, \delta, \lambda). \] (5)
Here, \( \tilde{m}^{DC}(\omega_k) \) is a frequency component of the scalar moment function for an assumed double-couple mechanism; \( \phi, \delta, \lambda \) are the strike, dip, and rake angles, respectively; and \( m^{DC}(\phi, \delta, \lambda) \) represents the moment tensor corresponding to a double-couple mechanism whose components are (e.g. Aki & Richards 2002)
\[ \begin{align*}
m_{sx}^{DC} & = -(\sin \delta \cos \lambda \sin 2 \phi + \sin 2 \delta \sin \lambda \sin^2 \phi), \\
m_{sy}^{DC} & = \sin \delta \cos \lambda \sin 2 \phi - \sin 2 \delta \sin \lambda \cos^2 \phi, \\
m_{sz}^{DC} & = \sin 2 \delta \sin \lambda, \\
m_{xy}^{DC} & = \sin \delta \cos \lambda \cos 2 \phi + \frac{1}{2} \sin 2 \delta \sin \lambda \sin 2 \phi, \\
m_{xz}^{DC} & = -\cos \delta \cos \lambda \cos \phi + \cos 2 \delta \sin \lambda \sin \phi, \\
m_{yz}^{DC} & = -\cos \delta \cos \lambda \sin \phi - \cos 2 \delta \sin \lambda \cos \phi. \end{align*} \] (6)
The normalized residual \( R \) is given by
\[ R = \frac{\sum_{k=1}^{N_f} |\tilde{d}(\omega_k) - \tilde{G}(\omega_k) \tilde{m}^{DC}(\omega_k)|^2}{\sum_{i=1}^{N_m} |	ilde{d}(\omega_k)|^2}. \] (7)
where \( \tilde{m}^{DC}(\omega_k) = \tilde{m}^{est}(\omega_k) \tilde{m}^{DC}(\phi^{est}, \delta^{est}, \lambda^{est}) \) is the estimated model parameter vector \( \tilde{m}^{est}(\omega_k) \); \( \tilde{m}^{est}(\omega_k) \) is a frequency component of the estimated scalar moment function; \( \phi^{est}, \delta^{est}, \lambda^{est} \) are the estimated fault and slip orientation angles; and |·| represents the length of a vector. Note that the residual calculated in the frequency domain by eq. (7) yields the same value as the residual calculated in the time domain after the inverse Fourier transform. We solve the inverse problem by minimizing \( R \) in the frequency domain to determine the moment function for each set of the assumed three angles.

A grid search with respect to the angles and space is conducted to find the best-fitting double-couple mechanism, moment function, and source location. The moment function in the time domain is then obtained by the inverse Fourier transform of \( \tilde{m}^{est}(\omega_k) \). This approach is an extension of the method of Nakano & Kumagai (2005) proposed for the analysis of source mechanisms of volcano-seismic signals. In the spatial grid search, we use an adaptive grid spacing similar to that proposed by Dreger et al. (1998). In this approach, if the location yielding a minimum residual lies at the edge of a search area, the grid is extended to surround the location of the minimum residual. When the location of a minimum residual lies within a search area, a new search is performed around the location using a reduced grid spacing to find a detailed source location.

### 2.2 Method to reconstruct the moment function

It is necessary to apply a bandpass filter to observed waveforms before the inversion because of the limited frequency response of seismometers and limited knowledge of fine structural heterogeneities in the earth. Therefore, the moment function determined by the inversion represents a bandpassed form. The actual moment function may be recovered by deconvolution of the filter that was applied to the observed waveforms. However, an ordinary method of deconvolution, in which data are divided by the filter characteristics in the frequency domain, is not stable because of small values in the denominator. Some methods have been proposed to stabilize the deconvolution by introducing the ‘water level’ (Clayton & Wiggins 1976), for which a suitable selection of the water level depends on data quality. The frequency components beyond the response of a seismometer, such as the DC component, cannot be reconstructed by any methods based on deconvolution.

We propose a method to estimate the deconvolved form of the moment function in the time domain from its bandpassed form determined by the inversion. Let us consider a moment function \( m^j(\Delta t) \), which is obtained from the inverse Fourier transform of the frequency components \( \tilde{m}^{est}(\omega_k) \) determined by inversion using observed waveforms bandpassed between \( \omega_1 \) and \( \omega_{N_f} \). We assume that the deconvolved form of the moment function \( m_j(\Delta t) \) is represented by a series of elementary functions \( s(j \Delta t) \) with differing weights (e.g. Langston 1981)
\[ m_j(\Delta t) = \sum_{i=1}^{N_f} a_i s(j \Delta t - i \Delta r). \] (8)
where \( \Delta r \) is the time interval of \( s(j \Delta t) \), the coefficient \( a_i \) represents the amplitude of \( s(j \Delta t) \) at time \( i \Delta r \), and \( N_f \) and \( N_{f2} \) are the indices for the initial and final times of \( s(j \Delta t) \). We assume that the actual moment function is represented by a monotonically increasing function. Therefore, a step-like function may be appropriate for the elementary function and the coefficients \( a_i \) are non-negative.

We define a function \( m^j_{\Delta}(j \Delta t) \), which is the bandpass filtered \( m_j(\Delta t) \) between \( \omega_1 \) and \( \omega_{N_f} \), as follows:
\[ m^j_{\Delta}(j \Delta t) = \sum_{i=1}^{N_f} a_i s^j(j \Delta t - i \Delta r). \] (9)
where \( s^j(j \Delta t) \) is the bandpass filtered \( s(j \Delta t) \) between \( \omega_1 \) and \( \omega_{N_f} \). The coefficients \( a_i \) are estimated by minimizing the residual \( E \) defined as
\[ E = \sum_{j=0}^{N_f-1} \frac{[m^j(j \Delta t) - m^j_{\Delta}(j \Delta t)]^2}{\sum_{j=0}^{N_f-1} [m^j(j \Delta t)]^2} \] (10)
under a non-negative constraint on \( a_i \). We use the non-negative least squares (NNLS) algorithm (Lawson & Hanson 1974) to estimate \( a_i \),
which solves the least-squares problem

$$\mathbf{m}' = \mathbf{S}' \mathbf{a}$$

subject to

$$\mathbf{a} \geq 0,$$

where $\mathbf{m}'$ and $\mathbf{a}$ are the vectors consisting of $m'(j \Delta t)$ and $a_i$, respectively, and $\mathbf{S}'$ is the matrix whose elements are given by the function $s'(j \Delta t)$. The two indices $N_{12}$ and $N_{11}$ are determined by minimizing the Akaike Information Criterion (AIC, Akaike 1974) defined as

$$\text{AIC} = N_t \ln E + 2(N_{12} - N_{11} + 1),$$

where the constant term is omitted. The two indices are estimated by the following procedure. First, $N_{12}$ is increased with a fixed small value of $N_{11}$. The coefficients $a_i$ are estimated for individual values of $N_{12}$ to find the value of $N_{11}$ that yields a minimum value of AIC. Next, we use the estimated value of $N_{12}$ and increase $N_{11}$ until it reaches $N_{12}$. Finally, we adopt the values of $N_{11}$ and $N_{12}$ and corresponding coefficients $a_i$ that yield the minimum AIC in this search. Then, $m_i(j \Delta t)$, our estimate of the moment function, is obtained by substituting the coefficients $a_i$ into eq. (8) with the estimated values of $N_{11}$ and $N_{12}$. The rupture duration can be determined from the time interval $(N_{12} - N_{11}) \Delta t$ plus the rise time of the elementary function. Since this procedure automatically estimates the origin time, there is no need to assume it prior to the inversion.

3 NUMERICAL TESTS

To investigate the capability of our inversion method, we carried out a series of numerical tests using synthesized seismograms. In the first series of tests, we investigated the ability of our inversion method to reconstruct the moment function. In the second series of tests, we investigated the capability of our method to estimate both the source mechanism and location.

In our numerical tests, we used a station distribution that mimics JISNET (Fig. 1) and assumed input source parameters corresponding to the earthquake that occurred south of Java on 2006 July 17 (e.g. Ammon et al. 2006). A point source is located at 10.0° S, 107.5° E at a depth of 10 km (Fig. 2a). The source mechanism is represented by an EW-striking reverse fault, which corresponds to the fault parameters (strike, dip, rake) = (90, 45, 90)/(270, 45, 90) (Fig. 3a). We synthesized three-component seismograms at three stations: LEM, BJI and KSI. Since the source is off the coast of Java, the stations are distributed only to the north of the source with limited azimuthal coverage. The seismograms at these stations were synthesized by using the discrete wavenumber method (e.g. Bouchon 1979) and standard earth model ak135 (Kennet et al. 1995). Green’s functions used for the inversion tests were also calculated with the same method and structure.

3.1 Reconstruction of the moment function

In the first series of tests, we investigated the capability of our method to recover the input moment function. We assume the following function for the input moment function:

$$m(t) = \begin{cases} t/t_p - (1/2\pi) \sin(2\pi t/t_p), & 0 \leq t \leq t_p \\ 1, & t > t_p \end{cases}$$

where $t_p$ is the rise time of the input moment function. We assume $t_p = 10, 20, 30$ and 40 s with a seismic moment of 1 Nm. Seismograms at each station are synthesized at a sampling frequency of 0.5 Hz. The synthesized seismograms are padded with zeros over the first 30 s, and the total data length of 512 s (256 data points for each trace) is used. The seismograms are then bandpass filtered (two-pole Butterworth) with corner periods of 50 and 100 s. The moment function is estimated by the inversion in the frequency domain, in which eq. (4) is solved for the entire frequency components except for the DC and Nyquist frequency components. The source location and focal mechanism are fixed to the input values.

Fig. 4(a) shows an example of the moment function $m'$ in the time domain obtained from the inverse Fourier transform of the frequency components determined by the inversion, in which $t_p = 10$ s is used in the input moment function. The estimated moment function $m'$ displays a pulsive signature of much smaller amplitude than the
input value (1 Nm), since the estimated function is a bandpassed form of the input function.

To reconstruct the input moment function, we use a step-like function as the elementary function:

\[
s(t) = \begin{cases} 
  t/t_r, & 0 \leq t \leq t_r \\
  1, & t > t_r
\end{cases}
\]  

(15)

where \( t_r \) specifies the rise time of the step-like function. Assuming \( t_r = 2 \) s in the elementary function (eq. 15), we reconstruct the input moment function from the estimated function \( m'\) using the approach described in Section 2.2. The reconstructed moment function \( m_f \) and its bandpassed form \( m'_f \) are shown in Figs 4(a) and (b), respectively, in which the time interval of the elementary function \( \Delta t = 2 \) s is used. The function \( m'_f \) fits well the function \( m' \) (Fig. 4a), and the function \( m_d \) shows a good recovery of the input moment function (Fig. 4b).

We also reconstruct the function \( m_d \) assuming \( t_r = 4 \), 6 and 8 s from the function \( m'/d \) shown in Fig. 4(a), in which \( \Delta t \) is assumed to be identical to \( t_r \). The reconstructed moment functions assuming \( t_r = 4 \), 6 and 8 s are shown in Figs 4(c)–(e), respectively, and the corresponding fits to the function \( m'/d \) are shown in Fig. 4(a). As shown by these results, as well as by the result for \( t_r = 2 \) s in Fig. 4(b), the recovery of the input moment function depends on the value of \( t_r \). While the functions \( m_d \) assuming \( t_r = 2 \) and 4 s show good recoveries of the input moment functions (Figs 4b and c), the recovery deteriorates as \( t_r \) gets longer (Figs 4d and e).

We next use the input moment functions with other values of \( t_r \) and reconstruct the moment functions \( m_d \) by using the same procedure as above. The results, summarized in Fig. 5, also show that the recoveries of the input functions depend on the value of \( t_r \). For example, for \( t_r = 30 \) s (Fig. 5c) the input function is well recovered by the elementary function with \( t_r = 4 \) or 6 s, but the recoveries are worse for other values of \( t_r \).

As shown above, an appropriate value of \( t_r \) in the elementary function may depend on the value of \( t_p \) in the input moment function. Therefore, an objective measure is required for the selection of \( t_r \). We again use AIC calculated by eq. (13), and adopt a value of \( t_r \) that yields a minimum value of AIC. Fig. 6 shows the values of AIC.
plotted against the trial values of $t_r$. Minimum values of AIC for the input moment functions with $t_p = 10$, 20, 30 and 40 s are found at $t_r = 2$, 4, 6 and 6 s, respectively. Fig. 5 shows that these values of $t_r$ provide good recoveries of the input moment functions among the trial values. Therefore, we can justify the use of AIC for the selection of $t_r$ in the elementary function. The rupture durations can be estimated as 8, 16, 24 and 30 s from the reconstructed functions $m_f$ for the input values of $t_p = 10$, 20, 30 and 40 s, respectively, indicating about 80 per cent recoveries of the input values. The slight reductions of rupture durations may occur because the emergent onset and terminal portions of the input moment functions are not fully reconstructed. The seismic moments estimated from the functions $m_f$ are about 97–99 per cent of the input values, showing almost complete recoveries.

3.2 Estimations of source location and focal mechanism

In the second series of tests, we investigated the capability of our method to recover the source location and focal mechanism. We used the input moment function with $t_p = 10$ s and a seismic moment of $2.50 \times 10^{20}$ Nm ($M_o = 7.5$) to synthesize input seismograms at individual stations. For a grid search in space, we start with grid spacings of 0.5° horizontally and 10 km vertically. At each node, the strike, dip, and rake angles are searched in 5° steps. The spatial grid spacing is then reduced to 0.2° horizontal and 5 km vertical around the source location that yields a minimum residual in the first search. Finally, the horizontal grid spacing is reduced to 0.1°, and we obtain the best-fitting source location, focal mechanism and the moment function.

The horizontal and vertical residual distributions are shown in Figs 2(b) and (c), respectively. The estimated best-fitting source location is identical to the input location. The contour lines of the horizontal residual distribution show an elongation in the NS direction, which may be attributable to the one-sided station distribution with limited azimuthal coverage. Fig. 2(b) shows the focal mechanisms obtained at individual node points by the spatial grid search. The result obtained at the best-fitting source is identical to the input focal mechanism (see Fig. 3b). The mechanisms around the best-fitting source are very similar to the input mechanism in both horizontal and vertical directions (Figs 2b and c, respectively), indicating that

![Figure 6](https://example.com/image6.png)

**Figure 6.** Values of AIC plotted against the rise time ($t_r$) of an elementary function for the reconstructions of the input moment functions with the rise times ($t_p$) of (a) 10 s, (b) 20 s, (c) 30 s and (d) 40 s.

![Figure 7](https://example.com/image7.png)

**Figure 7.** (a) Waveform match obtained from our inversion test. Black solid lines indicate input (synthetic observed) seismograms, in which Gaussian white noise is added to each seismogram. Grey solid lines indicate synthesized seismograms calculated for the obtained source model by our inversion method. The station code and component of motion are indicated at the upper left of each seismogram. (b) Black and grey solid lines indicate the obtained moment function ($m_f^0$) and the function $m_f^0$ fitted to $m_f$, respectively. (c) Black and grey solid lines represent the input moment function and the reconstructed moment function ($m_f$), respectively.

The focal mechanism is not much affected by source mislocation in our method. The reconstruction of the moment function is shown in Figs 4(a) and (b).

To evaluate the effect of noise in our inversion, Gaussian white noise is added to the input seismograms. Resultant signal-to-noise (S/N) ratios are orders of magnitude smaller than those of actual observed seismograms. The waveform inversion is performed by using these input seismograms in the same manner as mentioned above. We find that the best-fitting location is identical to the input one. The focal mechanism estimated at the best-fitting source location is slightly different from the input one: two nodal planes are estimated as (strike, dip, rake) = (99, 46, 100)/(265, 45, 80) (Fig. 3c). Fig. 7(a) shows the waveform match between the input and synthesized seismograms at the best-fitting location ($R = 0.13$). The bandpassed and reconstructed forms of the moment function are shown in Figs 7(b) and (c), respectively. The rupture duration and seismic moment are estimated as 12 s and $2.61 \times 10^{20}$ Nm, respectively, which are slightly overestimated by the effect of noise.

We investigated the effect of incorrect seismic structure on the source parameter estimations. We assume a seismic structure with...
Table 1. Seismic structure with a thin crust.

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>P-wave velocity (km s⁻¹)</th>
<th>S-wave velocity (km s⁻¹)</th>
<th>Density (kg m⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.80</td>
<td>2.19</td>
<td>2300</td>
</tr>
<tr>
<td>5</td>
<td>5.90</td>
<td>3.41</td>
<td>2700</td>
</tr>
<tr>
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<td>6.55</td>
<td>3.78</td>
<td>2700</td>
</tr>
<tr>
<td>15</td>
<td>8.04</td>
<td>4.48</td>
<td>3320</td>
</tr>
<tr>
<td>35</td>
<td>8.04</td>
<td>4.48</td>
<td>3320</td>
</tr>
</tbody>
</table>

a thin crust (Table 1) to synthesize the seismograms. We use the 
P-wave velocity \((v_p)\) and density model of Kopp et al. (2002), who 
studied velocity and density structures across the subduction zone 
off the south coast of Java using reflection seismic and gravity data. 
We assume the S-wave velocity \(v_s = v_p/\sqrt{3}\), and the structure 
below 35 km depth is set to be identical to the structural model 
ak135 of Kennet et al. (1995). The seismograms are synthesized 
with this structure by using the input moment function with \(t_p = 
10\) s. No noise is added to the input seismograms. The inversion is 
carried out by using Green’s functions calculated with the seismic 
structure of the ak135 model. Our grid search with respect to the 
horizontal source location is the same as the input location, but the 
depth is 5 km shallower than the input value. The best-fitting focal 
mechanism is slightly different from the input mechanism: two nodal 
planes are estimated as (strike, dip, rake) = (85, 30, 80)/(277, 60, 
96) (Fig. 3d). The rupture duration estimated from the reconstructed 
moment function \(m_T\) is 16 s, which is slightly longer than the input 
duration. The seismic moment is estimated as 92 per cent of the 
input value. These results suggest that structural uncertainty slightly 
affects the estimates of all the source parameters.

3.3 Inversion without the assumption of a pure 
double-couple mechanism

We perform the inversion without the assumption of a pure double-
couple mechanism to demonstrate how the assumption stabilizes 
the inversion solution. Using only the non-volumetric source con-
straint, the five independent bases of moment tensor components 
\((M_1, \ldots, M_5)\) in Kikuchi & Kanamori (1991) are estimated by the 
waveform inversion in the frequency domain. We use the ak135 
model for both the input seismograms and Green’s functions. The 
input seismograms are synthesized with and without noise as in 
Section 3.2. In the first test, we perform the inversion to determine 
the five bases of moment tensor components using the noise-free 
input seismograms. The horizontal residual distribution and the fo-
cal mechanisms obtained at individual node points are shown in 
Fig. 8(a). The best-fitting source location is identical to the input 
location. However, the broadly distributed residuals indicate that 
the source location is not well constrained. This is clear contrast to 
the result of the inversion assuming a pure double-couple source 
mechanism (Fig. 2b). Non-double-couple components appear in the 
focal mechanisms at most of node points (Fig. 8a). These spurious 
components are attributable to a trade-off between the source lo-
cation and non-double-couple components. When we use the input 
seismograms with noise, the best-fitting source location does not 
coincide with the input location (Fig. 8b) and the best-fitting focal 
mechanism largely deviates from the input one (Figs 3e and 8b). 
These results indicate that the assumption of a double-couple focal 
mechanism is crucial for stable estimations of the source location 
and focal mechanism, especially for the inversion using data from 
a small number of stations.

4 APPLICATION TO OBSERVED DATA

We applied our inversion method to earthquakes in Indonesia ob-
erved by JISNET (Fig. 1). This network is operated by the Na-
tional Research Institute for Earth Science and Disaster Prevention 
(NIED) and the Meteorological and Geophysical Agency of Indone-
sia (BMG) (Nakano et al. 2006). Each JISNET station is equipped 
with a CMG-3T EBB three-component broad-band seismograph, 
which has a bandwidth of 0.02–360 s. Data from the seismograph at 
each station are sampled at 20 Hz for each channel and transmitted 
to BMG and NIED in nearly real time. We selected two events for 
our application: the earthquake that occurred northeast of the island 
of Sulawesi on 2007 January 21 \(M_w = 7.5\) based on our estimate) 
and another off the southern coast of Java on 2006 July 17 \(M_w = 
7.8\) based on the estimate of Ammon et al. 2006). We assumed the 
ak135 model structure (Kennet et al. 1995) for the calculation of 
Green’s functions.

To estimate the source location, we start with horizontal grid spac-
ing of 0.5° and vertical grid spacing of 10 km. The grid spacings 
are then reduced to 0.2° and 5 km for the horizontal and vertical 
directions, respectively. Finally, the horizontal grid spacing is re-
duced to 0.1°. The fault parameters of strike, dip, and rake angles 
are searched in 5° steps. We use initial hypocentre locations deter-
mined by the automatic GEOFON global seismic monitor system 
maintained by GeoForschungsZentrum Potsdam, Germany (GFZ) 
(http://www.gfz-potsdam.de/geofon/seismon/globmon.html).

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4.1 January 21, 2007, event northeast of Sulawesi

We first investigated the event northeast of Sulawesi (Fig. 9a). We used waveforms observed at five stations (KDI, PCI, TARA, TLE and BAKI; Fig. 1), which are located at epicentral distances ranging from approximately 600 to 1100 km. Data from station MNI were not used because the waveforms are clipped. The azimuthal coverage of the stations is much better than that of the numerical tests discussed in the previous section. The observed velocity waveforms were corrected for instrument response, and then integrated in time to obtain the displacement seismograms. These waveforms were bandpass filtered between 50 and 100 s, and decimated to a sampling frequency of 0.5 Hz. We used the total data length of 512 s (256 data points in each channel) for the inversion.

Figs 9(a) and (b) show the horizontal and vertical residual distributions, respectively. We found a well-defined global minimum residual, which is at $1.1^\circ$N, 126.3$^\circ$E at a depth of 25 km. The estimated focal mechanism is shown in Fig. 9(c): two nodal planes correspond to (strike, dip, rake) = (215, 50, 105)/(12, 42, 73). The moment function $m_f$ obtained by the inverse Fourier transform of the frequency components determined by the waveform inversion shows a pulsive signature as displayed in Fig. 10(a). The fits between observed and synthesized seismograms are shown in Fig. 11. The small residual ($R = 0.21$) indicates that the observed

Figure 9. (a) Map around the source area of the event northeast of Sulawesi on 2007 January 21. The black solid star indicates the centroid source location of this event estimated by our inversion method and the grey solid diamonds are those determined by NEIC and the Global CMT Project. Crosses denote node points for the spatial grid search, and contour lines represent the spatial distribution of the residuals obtained from our inversion. (b) Plots of the residuals at the best-fitting epicentre as a function of a depth. (c) Comparison of source parameters obtained from our inversion with those determined by NEIC and the Global CMT Project.

Figure 10. (a) Comparison of the moment function $m_f$ (black solid line) obtained from the waveform inversion of the Sulawesi event and the function $m_d$ (grey solid line). (b) The reconstructed moment function ($m_d$). (c) The moment-rate function corresponding to the moment function shown in (b).

Figure 11. Waveform match obtained from the waveform inversion of the Sulawesi event. Black and grey solid lines represent the observed and synthesized seismograms, respectively. The station code and component of motion are indicated at the upper left-hand side of each seismogram.
seismograms are well explained by the source parameters. Figs 9(a) and (c) compare our estimated source location and mechanism with those determined by NEIC and the Global CMT Project. Note that the results of NEIC and the Global CMT Project are from their preliminary analyses. Our source location and focal mechanism are similar to those determined by the Global CMT Project. The focal mechanism of NEIC shows relatively large non-double-couple components, and its source depth is shallower than the other two estimates.

To reconstruct the deconvolved form of the moment function \( m_f \), we used trial values of \( t_r = 2, 4, 6, \cdots, 20 \) s, and \( \Delta \tau = t_r \) for the elementary function given by eq. (15). We found that \( t_r = 4 \) s yields the minimum AIC (\( E = 0.03 \)). The reconstructed moment function \( m_f \) and corresponding moment-rate function are shown in Figs 10(b) and (c), respectively. As shown in Fig. 10(a), the function \( m_f \) fits well the bandpassed form of the moment function \( m_f^{\text{tr}} \). The seismic moment is estimated as \( M_0 = 2.10 \times 10^{20} \text{Nm} \), and the corresponding moment magnitude \( M_w \) is 7.5. These estimates are almost identical to those determined by the Global CMT Project. The rupture duration is estimated as 16 s from the reconstructed moment function \( m_f \). This is slightly shorter but comparable to a typical duration for earthquakes of this magnitude (e.g. Ekström & Engdahl 1989).

### 4.2 2006 July 17, event south of Java

We next investigated the earthquake that occurred off the southern coast of Java on 2006 July 17 (Fig. 12a). We used waveforms observed at stations LEM, BJI and KSI. Since the EW component at BJI was not available because of seismograph problems, our waveform inversion relies on eight seismograms. These stations are located at epicentral distances ranging from approximately 300 to 900 km. The observed velocity seismograms corrected for instrument response were integrated in time to obtain the displacement seismograms. These seismograms were bandpass filtered between 50 and 200 s and decimated with a sampling frequency of 0.5 Hz. The total data length of 512 s for each trace was used for the inversion.

Figs 12(a) and (b) show the horizontal and vertical residual distributions, respectively. The best-fitting source location is at 9.8°S, 107.5°E at a depth of 10 km. The estimated focal mechanism (Fig. 12c) is characterized by two nodal planes (strike, dip, rake) = (100, 55, 75)/(305, 38, 110). The moment function \( m_f^{\text{tr}} \) obtained from the waveform inversion is shown in Fig. 13(a). Waveform fits between observed and synthesized seismograms are shown in Fig. 14 and show a residual \( R = 0.34 \). The source locations and mechanisms determined by NEIC and the Global CMT Project are also shown in Figs 12(a) and (c), respectively. These locations are about 100 km apart, and our source location lies between them. The focal mechanism we estimated is similar to those determined by NEIC and the Global CMT Project, although the dip angle is slightly different.

We estimated the deconvolved form of the moment function \( m_{d} \) using the same trial values of \( t_r \) and \( \Delta \tau \) as those used in the analysis of the Sulawesi event. The minimum AIC was obtained at \( t_r = 16 \) s (\( E = 0.09 \)). The fit between \( m_f^{\text{tr}} \) and \( m_{d}^{\text{tr}} \) and the reconstructed moment function \( m_{d} \) are shown in Figs 13(a) and (b), respectively. The corresponding moment-rate function is shown in Fig. 13(c). The seismic moment and rupture duration for the entire event are estimated as \( M_0 = 2.63 \times 10^{20} \text{Nm} \) (\( M_w = 7.5 \)) and 176 s, respectively. Our estimate of the seismic moment lies between those determined by NEIC and the Global CMT Project (Fig. 13c). The duration of 176 s is anomalously longer than the typical duration for earthquakes of this magnitude.

We also performed the inversion using Green’s functions calculated with a thin crust model (Table 1), which placed the source about 30 km to the north of the location obtained in our previous inversion. This is close to the location estimated by NEIC, although the resolution of the source location is weak in this direction as seen in the contour plot of the residual obtained from our previous inversion (Fig. 12a). We obtained a slightly shorter rupture duration (130 s), but the source mechanism and seismic moment are almost identical to our previous estimations. The residual (\( R = 0.34 \)) from this inversion is almost the same as that obtained in our previous inversion.

### 5 DISCUSSION

The application of our method to the observed data shows that the deconvolved forms of the moment functions are recovered for both the Sulawesi and Java events, and that they indicate normal and abnormal rupture durations, respectively, for those two events. Ammon
et al. (2006) estimated detailed spatio-temporal slip distributions for the Java event by using waveform data from global seismic networks. It is useful to compare our result with that determined by Ammon et al. (2006) for this event. Ammon et al. (2006) used the elementary function

\[
s(t) = \begin{cases} 
\frac{t}{t_r} - (1/2\pi) \sin \left(2\pi t/t_r\right), & 0 \leq t \leq t_r \\
1, & t > t_r 
\end{cases}
\]

with \(t_r = 16\) s and \(\Delta \tau = 8\) s. Using this elementary function, we reconstructed the deconvolved form of the moment function \(m_d\) from the moment function \(m_f\) in Fig. 13(a) following the procedure explained in Section 2.2. The reconstructed moment and moment-rate functions are shown in Figs 15(a) and (b), respectively. We obtained a seismic moment of 2.62 \(\times 10^{20}\) Nm and a rupture duration of 168 s: these values are almost identical to those obtained in our previous analysis (Fig. 13). The moment-rate function of Ammon et al. (2006) is also shown in Fig. 15(b). The common features of these two sets of results confirm that this event consisted of three subevents: an initial rupture of roughly 20 s duration (marked by ‘I’ in Fig. 15b), a second main subevent of longer duration (marked by ‘II’ in Fig. 15b), and a third small subevent (marked by ‘III’ in Fig. 15b). The amplitude of the second subevent in our function is less than a half that derived by Ammon et al. (2006). Accordingly, our estimates of the seismic moment and magnitude are smaller than those of Ammon et al. (2006) \((M_0 = 7.0 \times 10^{20}\) Nm and \(M_w = 7.8)\). The difference in these results may be attributed to the different frequency bands used in these two studies. Our inversion used the band between 50 and 200 s, while Ammon et al. (2006) used a wider band including short-period teleseismic body waves (\(\sim 10\) s) and long-period Rayleigh waves (up to 800 s). Despite the differences, the comparison clearly supports that our method retrieves the essential feature of abnormally long rupture, implying that this event was a tsunami earthquake as concluded by Ammon et al. (2006).

Our method, therefore, has great potential to identify tsunami earthquakes from the routinely determined source parameters including the moment function.

In our waveform inversion, we assume a pure double-couple source mechanism, which does not change during an event. This assumption reduces the number of free parameters in the waveform inversion, and the source mechanism can be stably estimated by using waveform data from a small number of stations (Nakano et al. 2006). Although three-component seismograms from three stations
are sufficient to estimate the five independent bases of moment tensor components for a fixed source location (Dreger et al. 1998), waveform data from more stations are required to accurately estimate both the moment tensor and source location (e.g. Tajima et al. 2002). If the estimated source location is not accurate, spurious non-double-couple components appear in the source mechanism owing to a trade-off between the source location and non-double-couple components as shown by our numerical test (Fig. 8). Therefore, the simultaneous determination of both the source location and five independent moment-tensor components becomes unstable, especially using data from a small number of stations. Our assumption stabilizes the solution by minimizing the trade-off, and avoids estimation of a non-realistic source mechanism. This approach is especially useful for seismic monitoring based on a sparse regional network, as was the case in this study.

A suitable choice of the passband in the inversion may depend on the knowledge of fine crustal structures and noise level of observed seismic records. We found, through trial and error, that the passband between 50 and 100 s is most suitable for the inversions of earthquakes with magnitudes between 5 and 8 by using waveform data from JISNET. For events having long rupture durations, such as earthquakes with $M_w > 8$ and tsunami earthquakes, the passband should be extended to a longer period to improve the accuracy of the moment function estimations. Since S/N ratios of long-period components in seismic records generally decrease with decreasing magnitude, the use of long-period components may not always improve the inversion results. Practically, our inversion is first carried out by using the passband between 50 and 100 s. If the estimated magnitude is larger than 8 or the moment function shows anomalously long rupture duration, the inversion is carried out again by using the passband extended to 200 s.

Langston (1981) developed a method to estimate the moment function of an earthquake, assuming a point source, based on waveform inversion in the time domain. Similar approaches have been used by Ohminato et al. (1998) and Nakano et al. (2003) to estimate the moment functions of volcano-seismic signals. As pointed out by Auger et al. (2006), the computation time required for the time-domain approach is much longer than that for the frequency-domain approach. We compare the computation times required for both the approaches. First, the assumption of a pure double-couple mechanism and the non-negative constraint for the moment function estimation are incorporated into the time-domain inversion method of Ohminato et al. (1998). Then, the time-domain and frequency-domain inversions are performed at the source location fixed to the input one by using the input seismograms and Green’s functions utilized in the first test in Section 3.2. The parameters $N_{11}$ and $N_{12}$ in eq. (8) and $t_1$ in eq. (15) are estimated by searching the minimum AIC in both the approaches. Our comparison indicates that the computation time for the time-domain inversion is several thousands times longer than that for the frequency-domain inversion. Therefore, the time-domain approach may not be practical for rapid estimations of the source parameters.

Waveform inversions in the frequency domain have previously been used to estimate complicated moment functions of seismic events caused by mine collapses (Yang et al. 1998), mining cast blasts (Yang et al. 1999), and volcanic processes (Auger et al. 2006). In these studies, the moment functions were obtained solely by inverse Fourier transform of the frequency components determined by inversions. The moment function obtained in this way is a bandpassed form, which shows a pulsive signature in the case of a tectonic earthquake as seen in our application. It is almost impossible to estimate the rupture duration and seismic moment directly from the bandpassed form. We have developed a practical approach to reconstruct the moment function for a tectonic earthquake from the bandpassed form, and successfully recovered the moment functions for earthquakes in Indonesia. Our method, however, does not always guarantee an accurate reconstruction of the moment function, especially when S/N ratios in observed seismograms are small. In such a case, the residual $E$ (eq. 10) is large, even if the residual $R$ is small. Therefore, not only $R$ but also $E$ must be used to measure the validity of the inversion result.

Our method of reconstructing the deconvolved form of the moment function is especially valuable for two reasons: our technique provides a practical approach to reconstruct the deconvolved form, and it is a technique that can be easily automated. The waveform inversion including the grid search is effectively a series of automatic processes and, therefore, our method can determine the source parameters including the moment function in a systematic and automated way. We are currently using the method proposed in this paper in an automated system to determine the source parameters of earthquakes with $M_w > 5$ in Indonesia (see http://www.isn.bosai.go.jp/en/index.html). The computation time required for finding the best-fitting solution (including the focal mechanism, moment function, and source location) depends on computer capacity. In our automated system, we use a personal computer (Dual core Intel Xeon 3 GHz) with a Linux operating system. Our inversion using a grid search in 15 steps for fault and slip orientation angles provides the final solution within 2–5 min after obtaining observed waveforms with a total length of 512 s (30 s before and 482 s after the origin time). The program system is triggered by receiving initial hypocentre information from the GE-OFON system, which usually arrives within 10 min of the origin time. Therefore, we can obtain source parameters within 15 min of the occurrence of an earthquake. The calculation time is clearly shorter than that required to derive a global CMT solution and may be rapid enough to allow timely issue of tsunami warnings.

6 CONCLUSIONS

We have developed a method of waveform inversion to rapidly and routinely estimate both the moment function and a CMT solution for an earthquake. In this method, we perform the inversion in the frequency domain assuming a pure double-couple source mechanism to rapidly and stably estimate a solution by using data from a small number of seismic stations. A grid search with respect to the fault and slip orientations and space is performed to find the best-fitting source mechanism and location. The deconvolved form of the moment function is estimated by fitting a function represented by a superposition of elementary functions to a bandpassed form of the moment function determined by the inversion. The application of this method to two Indonesian earthquakes shows successful recovery of the moment functions and CMT solutions for both. The moment function obtained for the earthquake northeast of Sulawesi ($M_w = 7.5$) shows a rupture duration of 16 s, which is comparable to a typical value for earthquakes of this magnitude. The moment function for the earthquake south of Java ($M_w = 7.5$), on the other hand, shows an anomalously long rupture duration of 176 s, indicating this event was a tsunami earthquake. The inversion procedure can be easily automated, and we can estimate source parameters within 15 minutes of the occurrence of an earthquake with $M_w > 5$ in Indonesia. Seismic monitoring based on our inversion method may provide early notification of detailed characterizations of earthquakes including the moment function, and may provide information that will allow timely issue of tsunami warnings.

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