Bimodal and multimodal descriptions of soil-water characteristic curves for structural soils
Shiyu Liu, Noriyuki Yasufuku, Qiang Liu, Kiyoshi Omine and Hazarika Hemanta

ABSTRACT
In the last decades several approaches have been developed to describe bimodal or multimodal soil-water characteristic curves (SWCCs). Unfortunately, most of these models were derived empirically. In the presented study, physically based bimodal and multimodal SWCC functions have been developed for structural soils. The model involved two or more continual pore series; the probability density functions for each pore series were assumed to be lognormal distribution and can be superposed to obtain the overall probability density function of the structural soils. The proposed functions were capable of simulating bimodal or multimodal SWCCs using parameters which can be related to physical properties of the structural soils. The experimental SWCC data were used to verify the proposed method. The fitting results showed that the proposed approaches resulted in good agreement between measurement and simulation. These functions can potentially be used as effective tools for indentifying hydraulic porosities in the structural mediums.

Key words | pore-size distribution, soil-water characteristic curves, structural soils

INTRODUCTION
Modeling water flow and solute transport through the vadose zone requires knowledge of the relationship between moisture and suction. The soil-water characteristic curves (SWCCs), also known as the water retention curves (WRCs), constitute a basic relationship between moisture and suction used for prediction of the hydraulic behavior of unsaturated porous materials.

Numerous unimodal functions (e.g. Brooks & Corey 1964; Van Genuchten 1980; Fredlund & Xing 1994; Kosugi 1994) have been developed to describe SWCCs. These functions were generally derived from idealized pore-space models that assume a unimodal pore-size distribution. Nevertheless, evidence has shown that the pore-size distributions of some soils are often bimodal or multimodal (Othmer et al. 1991; Durner 1994; Spohrer et al. 2006; Zhang & Li 2010). For example, the structural soils consist of interconnected networks of matrix (inter-aggregate) and structural (intra-aggregate) pores forming two (or more) distinct pore spaces (Kutílek 2004). Typically, spaces of sizes in the order of $10^{-4}$ to $10^{-2}$ m are associated with structural pores, while the porous matrix contains smaller pore sizes in the range of $10^{-2}$ to $10^{-3}$ m (Tuller & Or 2002). The existence of two vastly different pore domains results in pore-size distributions of structural soils that are often bimodal. In such soils the independent draining of the structural and matrix pores frequently results in two distinct air-entry values, which any single unimodal function does not reproduce adequately (Othmer et al. 1991; Durner 1994). To counter these problems, in the last decades several approaches have been developed to describe bimodal or multimodal SWCCs. Peters & Klavetter (1998) first proposed the superposition of two unimodal pore systems to represent the bimodal pore-size distributions. This approach was generalized by Othmer et al. (1991), Wilson et al. (1992), and Durner (1994) to consider multimodal pore-size distributions, each of which is characterized by its own SWCC function. In these approaches the SWCC function of the whole porous medium has been described by linearly overlapping functions of the same form or of different forms (Coppola 2000). Durner (1994) extended the unimodal van Genuchten functions (Van Genuchten 1980) to fit bimodal and multimodal SWCCs by introducing weighting factors for combining individual functions. Burger & Shackelford (2001a, b) proposed piecewise-continuous forms of the

Shiyu Liu (corresponding author)
Noriyuki Yasufuku
Qiang Liu
Kiyoshi Omine
Hazarika Hemanta
Geotechnical engineering research group, Faculty of Engineering, Kyushu University, Fukuoka, 819-0395, Japan
E-mail: liushiyu518@gmail.com

doi: 10.2166/wst.2013.046
Brooks & Corey (1964), Van Genuchten (1980), and Fredlund & Xing (1994) SWCC functions to fit the bimodal experimental SWCCs for diatomaceous earth (DE) and sand-DE mixtures. Although the above approaches were successfully applied to structural soils, there is a lack of a physical basis for their parameters due to the fact that the unimodal SWCC functions they extended were known as empirical equations.

The primary objective of this study was to propose a physical basis SWCC model for structural soils with two distinct pore spaces. The specific objectives were: (i) derivation of SWCC function for structural soils based on pore-size distribution; (ii) simple tests of the proposed model using available datasets. The hysteretic nature of the SWCC can significantly influence water flow and solute transport in unsaturated porous media (Šimůnek et al. 1999). In this paper the proposed model only performs on the drying curves, because if it successfully captures the bimodal shape of drying curves it also fits the bimodal wetting curves.

METHODS

Theoretical analysis of dual-porosity structural soils

Assume that ideal dual-porosity structural soils consist of uniform sphere aggregates packed in a simple cubic structure and that intra-aggregates also have the same structure (see Figure 1). There are two major pore series, i.e. structural pores (i.e. inter-aggregate pores) and matrix pores (i.e. intra-aggregate pores). Two pore series in the soil are assumed to be connected. Therefore, the suctions in the two characteristic pore series are the same.

The SWCC of ideal dual-porosity structural soil is shown in Figure 2(a). The soil remains saturated before suctions reach the air entry value (stage 1). The air entry value is controlled by the routing radius (RR1) (i.e. the radius of the largest sphere that can pass through the porous medium) of structural pores, which is related to the packing structure and the sphere diameter. As the suctions reach the air entry value (stage 2), all the bulk water in the structural pores (BW1) will drain without additional suction. After all the bulk water in the structural pores (BW1) has drained, a small amount of water (i.e. the water-pendular rings (RW1)) around aggregate-to-aggregate contact points remains and drains slowly while the matric suction increases rapidly (stage 3). Further, when the suction increases to the starting-drainage-suction of bulk water stored in the matrix pores, the bulk water stored in the matrix pores (BW2) will drain (stage 4). After the suction reaches the end-of-drainage-suction of bulk water stored in the matrix pores (BW2), the water-pendular rings in the matrix pores (RW2) starts to drain (stage 5). Hence, the volume proportion of water-pendular rings in the matrix pores (RW2) determines the residual soil water content.

In random packing geometry of aggregates, the bulk water is stored in pores with different routing radius. Then the drainage process of bulk water (stage 2) continues from the drainage suction of the largest bulk water spheres to the drainage suction of the smallest bulk water spheres. After the bulk water drains, the water-pendular rings will drain (stage 3) from the drainage suction of the largest pendular rings to the drainage suction of the smallest pendular
component with \( r_m \); and \( F_n(x) \) is the complementary normal distribution function defined as

\[
F_n(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp \left( -\frac{t^2}{2} \right) dt
\]

where \( t \) is a dummy variable.

For the dual-porosity structural soils, there are two continual pore series: structural pores and matrix pores. Probability density functions for two pore series are assumed to have a lognormal distribution and can be superposed to obtain the overall probability density function of the structural soils. Durner (1994) combined two continual pore series by introducing weighting factors to describe volumetric percentage of each pore fraction, the pore probability density function of the dual-porosity structural soil as the following form

\[
g(r) = \sum_{i=1}^{k} \phi_i g_i(r) = \sum_{i=1}^{k} \phi_i \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma_i r} \exp \left\{ -\frac{\left[ \ln \left( \frac{r}{r_m} \right) \right]^2}{2\sigma_i^2} \right\}
\]

\[0<\phi_i<1 \text{ and } \sum_{i=1}^{k} \phi_i = 1 \quad (i = 1, 2)\]

where \( k \) is the number of pore series (\( k = 1, 2 \)); \( g_i(r) \) is the pore probability density function (cm\(^{-3}\)) for the \( i \)th pore series; \( \phi_i \) is the volumetric percentage of the soil components with the \( i \)th pore series; \( r_m \) is the median pore radius (cm) for the \( i \)th pore series; \( \sigma_i \) denotes the standard deviation of \( \ln(r) \) associated with the \( i \)th pore series. SWCC function for the dual-porosity structural soil as a cumulative curve of Equation (4) can be written as

\[
S_e = \sum_{i=1}^{k} \phi_i F_n(\ln\left( \frac{h}{h_m} \right))/\sigma_i
\]

\[0<\phi_i<1 \text{ and } \sum_{i=1}^{k} \phi_i = 1 \quad (i = 1, 2)\]

where \( h_m \) is the median matric head (cm), components with the \( i \)th pore series. Figure 3(a) shows the superposition of two hypothetical pore-size distributions that define a bimodal pore series according to Equation (4). The dotted line can be interpreted as structural pores distribution, with \( \ln \left( r_m \right) \) of \(-3.64, \sigma_1 \) equal to 0.47, and maximum pore-size density around \( r = 3 \times 10^{-2} \) cm, whereas the dashed line can be seen as matrix pores distribution with \( \ln \left( r_m \right) \) \(-10.17, \sigma_2 \) equal to 0.72, and its maximum density in the range of \( r = 4 \times 10^{-5} \) cm. Figure 3(b) shows the bimodal SWCC (solid line).

**Bimodal SWCC function**

Brutsaert (1966) studied four models of pore-size distribution, among them the lognormal distribution in relation to SWCCs. A more detailed analysis was presented by Kosugi (1994), who assumed the lognormal pore probability density function for pore radii, \( g(r) \), to be

\[
g(r) = \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma r} \exp \left\{ -\frac{\left[ \ln \left( \frac{r}{r_m} \right) \right]^2}{2\sigma^2} \right\}
\]

where \( r \) the pore radius (cm), obeys the lognormal distribution; \( \theta_s \) and \( \theta_r \) are the saturated and residual volumetric water content (cm\(^3\)-cm\(^{-3}\)); \( r_m \) is the median pore radius (cm), and \( \sigma \) denotes the standard deviation of \( \ln(r) \). Based on this assumption, the SWCC function can be expressed as (Kosugi 1994)

\[
S_e = \left( \theta_e - \theta_r \right)/\left( \theta_s - \theta_r \right) = F_n\left( \ln\left( \frac{h}{h_m} \right) \right)/\sigma
\]

where \( S_e \) is the effective saturation; \( \theta \) is the volumetric water content (cm\(^3\)-cm\(^{-3}\)); \( h_m \) is the median matric head (cm)
corresponding to two hypothetical pore-size distributions; unimodal SWCC for structural pores with volumetric percentage $\phi_1$ equal to 0.54 (dotted line) and matrix pores with $\phi_2$ equal to 0.46 (dashed line).

Equation (5) is derived on the basis of there existing two combined lognormal pore radius density curves in the structural soil, hereafter referred to as bimodal lognormal (BLN) model. In the case of $k = n$ ($n > 2$), there exist multiple pore series in the soil and Equation (5) becomes the multimodal lognormal (MLN) SWCC function.

**Evaluation of the bimodal SWCC model**

We evaluated the proposed bimodal SWCC models by applying them to various soils. First, the proposed bimodal model was compared with the unimodal model. Six soil samples including loam and silt loam were selected for this purpose (see Table 1). These soil samples were obtained from the Unsaturated SOil hydraulic DAtabase (UNSODA) (Nemes et al. 2001), which consists of SWCC, hydraulic conductivity, and water diffusivity data as well as pedological information of some 790 soil samples from around the world. The unimodal lognormal (ULN) SWCC model was based on theoretical considerations and can be simplified to the currently used expressions of van Genuchten or other empirical models (Kosugi et al. 2002). For these reasons, all comparisons were performed with respect to the ULN model only. In this evaluative procedure, the parameter $\theta_s$ was set at its measured value and $\theta_r$ was assumed to be zero. For soils that did not provide porosity or $\theta_s$ value, the first point of the experimental SWCC data that corresponds to the lowest suction head was used as $\theta_s$ (Chan & Govindaraju 2004).

Further evaluations were performed for 14 artificial soils (shown in Table 2) cited from Burger & Shackelford (2001a, b), which were also used by Zhang & Chen (2005) for verification of functions they proposed. The SWCCs were measured for two commercial DE (CG1 and CG2) and 12 different mixtures of CG1 pellets with Sand 20–60, CG2 pellets with Sand 20–60, and CG2 pellets with Sand 10–20 at four different percentages of DE ranging from 4

---

**Figure 3** | Construction of a bimodal SWCC function. (a) Pore-size distribution of structural pores (dotted line) and matrix pores (dashed line). (b) Bimodal SWCC (solid line), unimodal SWCC for structural pores (dotted line) and matrix pores (dashed line).

**Table 1** | ULN and BLN models’ curve-fit parameters for the materials

<table>
<thead>
<tr>
<th>Soil Code</th>
<th>Texture</th>
<th>Fitted model</th>
<th>Structural pores</th>
<th>Matrix pores</th>
<th>RMSE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\phi_1$ (%)</td>
<td>$h_m$ (cm)</td>
<td>$\sigma_1$</td>
<td>$\phi_2$ (%)</td>
<td>$h_m$ (cm)</td>
</tr>
<tr>
<td>2530</td>
<td>Loam</td>
<td>ULN</td>
<td>–</td>
<td>2,100.49</td>
<td>3.71</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BLN</td>
<td>25.17</td>
<td>69.66</td>
<td>0.47</td>
<td>74.83</td>
</tr>
<tr>
<td>2601</td>
<td>Loam</td>
<td>ULN</td>
<td>–</td>
<td>2,943.43</td>
<td>4.37</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BLN</td>
<td>33.21</td>
<td>37.99</td>
<td>1.10</td>
<td>66.79</td>
</tr>
<tr>
<td>2603</td>
<td>Loam</td>
<td>ULN</td>
<td>–</td>
<td>1,395.24</td>
<td>4.27</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BLN</td>
<td>35.22</td>
<td>33.39</td>
<td>1.10</td>
<td>64.78</td>
</tr>
<tr>
<td>2750</td>
<td>Loam</td>
<td>ULN</td>
<td>–</td>
<td>2,214.74</td>
<td>4.51</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BLN</td>
<td>28.87</td>
<td>22.15</td>
<td>1.20</td>
<td>71.13</td>
</tr>
<tr>
<td>2751</td>
<td>Sandy loam</td>
<td>ULN</td>
<td>–</td>
<td>11,849.58</td>
<td>4.39</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BLN</td>
<td>18.22</td>
<td>23.46</td>
<td>1.09</td>
<td>81.79</td>
</tr>
<tr>
<td>2753</td>
<td>Sandy loam</td>
<td>ULN</td>
<td>–</td>
<td>4,239.51</td>
<td>3.53</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BLN</td>
<td>13.98</td>
<td>10.50</td>
<td>0.94</td>
<td>86.02</td>
</tr>
</tbody>
</table>
to 30% by dry weight. Burger & Shackelford (2000a, b) also proposed piecewise-continuous forms of the Brooks & Corey (1964), Van Genuchten (1980), and Fredlund & Xing (1994) SWCC functions to fit the bimodal experimental SWCCs. Their fitted results showed that bimodal van Genuchten function (BvG) visually fitted the data the best. Therefore our proposed BLN function was compared with BvG function. In this case, $\theta_s$ was considered as a fitting parameter to verify the effectiveness of the BLN model and $\theta_r$ was assumed to be zero.

In this study, the root mean square error (RMSE) of the measured and simulated volumetric water content $\theta$, and coefficients of determination ($R^2$) were used as measure of the goodness-of-fit.

## RESULTS AND DISCUSSION

### ULN and BLN models’ fitted results

Fitted parameters and corresponding RMSE values for ULN and BLN functions are shown in Table 1. For all soils from UNSODA, the RMSEs for the measured and simulated variables show improvement when the BLN model is used rather than the ULN model. The good fit of the proposed BLN model is indicated by the lower values of RMSE. Table 1 shows that all the materials are successfully described by the BLN model with coefficients of determination, $R^2 > 0.98$. Such high values of $R^2$ indicate the effectiveness of the BLN model in describing measured data.

In all related soils, the volumetric percentage of the structural pores ($\phi_1$) are in a range of 14–36%, in other words the matrix pores occupy the majority of total pores. Table 1 also shows that the median suction head ranges between 10.50 and 69.66 cm for the structural pores ($h_m$) and from 7,242.66 to 18,990.28 cm for the matrix pores ($h_m$). It infers that the median pore radii are from $2.14 \times 10^{-3}$ to $1.42 \times 10^{-2}$ cm for structural pores and from $7.85 \times 10^{-6}$ to $2.06 \times 10^{-5}$ cm for matrix pores ($r_m$ and $r_m$ are inversely proportional). The classification systems of soil pores by Tuller & Or (2002) is questionable when considering our results. As pointed out by Kutilek et al. (2006), our results support the belief that the classification systems of soil pores based upon fixed boundaries between pore size categories is not appropriate.

Figure 4 displays the fitted curves of the related soil samples obtained by ULN and BLN functions. All soils show a clearly structural behavior that is captured fairly well by the BLN function, while the poor fit offered by the ULN function is evident. Durner (1994) proposed that a good description of the SWCC is the basis for more accurate...
prediction of the soil’s unsaturated hydraulic conductivity function across the entire soil water content range. Our proposed approaches can potentially be used as effective tools for predicting hydraulic porosities in the medium with structures.

**BvG and BLN models’ fitted results**

The fitted results for 14 artificial soils using the BLN function are shown in Table 2 and Figure 5. The BvG function fitted results are shown in Table 7 and Figure 7 of Burger & Shackelford (2013b).

It is worth noting that the high values of $R^2$ confirm again the good ability of the proposed BLN model to describe experimental data. The low difference between the measured and simulated $\theta_s$ (RMSE = 0.010) also indicate the effectiveness of the BLN model in describing measured data.

For mixture of the three groups, the saturated volumetric water content ($\theta_s$) increases along with DE increase in the mixtures. It infers that DE can increase the total porosity of the mixtures. Furthermore, the volumetric percentage of the structural pores ($\phi_1$) decreases with the increase of DE. In contrast, the value of $\phi_2$ increases along with the DE growth. These phenomena all illustrate that DE has inner structure and increases in DE can increase the water holding capacity of the mixtures. Table 2 also shows that the median suction head ($h_{m_1}$) of the structural pores for the groups CG1 and Sand 20–60, CG2 and Sand 20–60, decreases as the amount of DE increases. It infers that the median pore radius of the structural pores ($r_{m_1}$) increases with increase of DE ($h_{m_1}$ and $r_{m_1}$ are inversely proportional). The result may be due to the particle size of CG1 and CG2 being bigger than Sand 20–60 (Figure 2 of Burger & Shackelford (2013b)). For the CG2 and Sand 10–20 group this phenomenon is not shown clearly due to their similar particle size distributions. The median suction head ($h_{m_2}$) of the matrix pores is not regular. The reason may be attributed to the fact that the measured data of the high suction portions are scatter, which can be proved by the standard deviation $\sigma_2$.

The BvG method also successfully fitted measured data. However, it was derived empirically and fitting parameters did not have physical significance, the process was complicated, and artificial selection of the junction point was somewhat arbitrary. All these shortcomings limit its application. Unlike the BvG model, the fitting procedure of the BLN model is easier and all parameters have physical significance that can be related to physical properties of the materials. These physically based parameters could be used to more intuitively analyze the measured data. After comparing the coefficients of determination ($R^2$) of the two methods, it can be concluded that the BLN function fits more accurately than the BvG function.
CONCLUSIONS

In this study, mathematical functions for both bimodal and multimodal SWCCs have been proposed. The proposed equations are defined by parameters that have physical significance that can be related to the properties of the materials. Experimental dataset fittings and parametric analyses were used to illustrate the fitting capability of the proposed equations. The proposed approaches make the treatment of SWCC data easier and resulted in good agreement between measurement and simulation. These functions can potentially be used as effective tools for indentifying hydraulic porosities in a medium with structures.

ACKNOWLEDGEMENTS

This research was supported by the Grant-in-Aid for Scientific Research (A) Ministry of Education, Culture, Sports, Science and Technology, Japan, Grant number: 22246064.

REFERENCES


Spohrer, K., Herrmann, L., Ingwersen, J. & Stahr, K. 2006
Applicability of uni- and bimodal retention functions for
water flow modeling in a tropical acrisol. Vadose Zone J.
5 (1), 48–58.
Šimínek, J., Kodešová, M. M., Gribb, M. & van Genuchten, M. T.
1999 Estimating hysteresis in the soil water retention function
from cone permeameter experiments. Water Resour. Res. 35
(5), 1329–1345.
Tuller, M. & Or, D. 2002 Unsaturated hydraulic conductivity of
structured porous media: A review of liquid configuration-
based models. Vadose Zone J. 1 (1), 14–37.
Van Genuchten, M. Th. 1980 A closed-form equation for
predicting the hydraulic conductivity of unsaturated soils.
Wilson, G. V., Jardine, P. M. & Gwo, J. P. 1992 Modeling the
56 (6), 1731–1737.
Zhang, L. M. & Chen, Q. 2005 Predicting bimodal soil–water
characteristic curves. J. Geotech. Geoenviron. Eng. 131
(5), 666–670.
Zhang, L. M. & Li, X. 2010 Microporosity structure of coarse granular

First received 10 July 2012; accepted in revised form 4 December 2012