Assessment of systematic errors in the surface gravity anomalies over North America using the GRACE gravity model

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SUMMARY

The surface gravity data collected via traditional techniques such as ground-based, shipboard and airborne gravimetry describe precisely the local gravity field, but they are often biased by systematic errors. On the other hand, the spherical harmonic gravity models determined from satellite missions, in particular, recent models from CHAMP and GRACE, homogenously and accurately describe the low-degree components of the Earth’s gravity field. However, they are subject to large omission errors. The surface and satellite gravity data are therefore complementary in terms of spectral composition.

In this paper, we aim to assess the systematic errors of low spherical harmonic degrees in the surface gravity anomalies over North America using a GRACE gravity model. A prerequisite is the extraction of the low-degree components from the surface data to make them compatible with GRACE data. Three types of methods are tested using synthetic data: low-pass filtering, the inverse Stokes integral, and spherical harmonic analysis. The results demonstrate that the spherical harmonic analysis works best. Eighty-five per cent of difference between the synthetic gravity anomalies generated from EGM96 and GGM02S from degrees 2 to 90 can be modelled for a region covering North America and neighbouring areas. Assuming EGM96 is developed solely from the surface gravity data with the same accuracy and GGM02S errorless, one way to understand the 85 per cent difference is that it represents the systematic error from the region of study, while the remaining 15 per cent originates from the data outside of the region.

To estimate systematic errors in the surface gravity data, Helmert gravity anomalies are generated from both surface and GRACE data on the geoid. Their differences are expanded into surface spherical harmonics. The results show that the systematic errors for degrees 2 to 90 range from about \(-6\) to \(13\) mGal with a RMS value of \(1.4\) mGal over North America. A few significant data gaps can be identified from the resulting error map. The errors over oceans appear to be related to the sea surface topography. These systematic errors must be taken into consideration when the surface gravity data are used to validate future satellite gravity missions.

Key words: Satellite geodesy; Gravity anomalies and Earth structure.

1 INTRODUCTION

On and near the Earth’s surface, three different techniques, in terms of platform type, are routinely used to measure the Earth’s gravity field: ground-based, shipboard and airborne gravimetry. For the convenience of discussion here, we refer to these techniques collectively as surface techniques. Accordingly, the data from these techniques are referred to as surface data. Surface gravity anomalies describe the Earth’s gravity field spectrum from the wavelength corresponding to twice the measurement spacing to the wavelength equal to the dimension of the data region in terms of the sampling theorem. They are relatively precise, but they are often contaminated by systematic errors such as datum errors, position (mainly elevation) errors, reduction errors, geodynamic effects, instrumental errors, etc. Heck (1990) reviewed all possible error sources affecting gravity anomalies. It is generally implausible to estimate and correct most of the errors if not all due to the lack of quantitative information about these errors.

Recent revolutionary development from the CHAMP and GRACE satellite gravity missions is driving scientists to find their applications in many fronts of Earth sciences. The upcoming GOCE mission will push gravimetry progress further, consequently making broader and more profound impacts. One of the primary products from these missions is a static Earth’s gravity model down to a spatial resolution of about 100 km at accuracy better than 1 mGal.
One question naturally arises: can the satellite data completely replace the surface data? The answer is clearly negative because these satellite missions do not precisely measure the gravity field at the same high spatial resolution as the surface techniques. The satellite and surface data are complementary in the determination of the Earth’s gravity field. Furthermore, the satellite gravity data can be used to assess systematic errors in the regional gravity field.

A few authors have conducted relevant studies on this topic. Mainville & Rapp (1985) and Pavlis (2000) estimated systematic errors in the global surface gravity data using tracking data from earlier satellite missions based on spherical harmonic analysis. Véronneau & Huang (2003) tried to correct systematic errors in the surface gravity data using tracking data with the latest GRACE gravity model. The primary objective of this study is to estimate the systematic error from spherical harmonic degrees 2 to 90.

### 2 METHODS

The Earth gravity field determined by satellites (SG) can be modelled by spherical harmonics

\[
\Delta g^{SG} = \sum_{n=2}^{L} g_n + \epsilon_n^{SG},
\]

where \(\Delta g^{SG}\) stands for gravity anomaly; \(L\) stands for maximum degree of the satellite gravity spherical harmonic model; \(g_n\) stands for the spherical harmonic component of degree \(n\) and \(\epsilon_n^{SG}\) stands for the commission error associated with the satellite model.

Similarly, we can express the surface (or terrestrial) gravity anomaly \(\Delta g^{TG}\) into a spherical harmonic form

\[
\Delta g^{TG} = \sum_{n=L+1}^{\infty} g_n + \epsilon_L^{TG} + \epsilon_H^{TG} + \epsilon_u,
\]

The second and third terms on the right hand side stand for the low- and high-degree systematic errors, respectively. The last term \(\epsilon_u\) stands for the random error. By subtracting eq. (1) from eq. (2), we have

\[
\Delta g^{TG} - \Delta g^{SG} = \sum_{n=L+1}^{\infty} g_n + \delta \epsilon_L + \epsilon_H^{TG} + \epsilon_u,
\]

where

\[
\delta \epsilon_L = \epsilon_L^{TG} - \epsilon_L^{SG}.
\]

If we assume that the error of the satellite models is much smaller than the systematic error in the surface gravity data, \(\delta \epsilon_L\) approximately gives the low-degree systematic error in the surface data below degree \(L\). Eq. (3) can now be rewritten as

\[
\delta \epsilon_L = \Delta g^{TG} - \sum_{n=L+1}^{\infty} g_n - \epsilon_H^{TG} - \epsilon_u - \Delta g^{SG}.
\]

For estimating \(\delta \epsilon_L\), \(\Delta g^{TG}\) must be smoothed in order to remove the high degree components, and high degree systematic and random errors.

### 2.1 Low-pass filters

A straightforward method to smooth the gravity field is low-pass (LP) filtering, that is, the associated systematic error can be estimated by

\[
\delta \epsilon_L^{LP} = F \left[ \Delta g^{TG} - \Delta g^{SG} \right],
\]

where \(\Delta g\) represents the gravity anomaly downward continued onto the geoid that is approximated by a reference ellipsoid for satellite data.

The \(F\) operator is defined as

\[
F[y] = \int_{\Omega} w(\psi) y d\Omega,
\]

where \(y\) is a function defined in the integration domain \(\Omega\), \(\psi\) is the spherical angular distance between the computational point and a running point, \(w\) is the normalized weight function which can be written as

\[
w(\psi) = \frac{w(\psi)}{\int_{\Omega} w(\psi) d\Omega}.
\]

where \(d\Omega\) is the surface integration element.

To show the relation between \(y\) and \(F[y]\), we can expand \(y\) into surface spherical harmonics

\[
y = \sum_{n=2}^{\infty} y_n,
\]

then

\[
F[y] = \sum_{n=2}^{\infty} \beta_n y_n,
\]

where \(\beta_n\) are eigenvalues (or spectral values) for the \(F\) operator. \(y_n\) is the spherical harmonic component of degree \(n\). \(\beta_n = 1\) corresponds to the Dirac delta weight function. However, the Dirac delta function does not work as a filter function because it only regenerates the function \(y\). Generally, \(\beta_n\) of a filter function are not equal to 1.

Three spatial filter functions, namely Pellinen’s, Gaussian, and ideal, are used to do the LP filtering. Jekeli (1981) discusses their application in smoothing the Earth’s gravity field in detail. They are summarized below.

#### Pellinen’s weight function

\[
w_p(\psi) = \left\{ \begin{array}{ll} 1 & 0 \leq \psi \leq \psi_0 \\ 0 & \psi_0 < \psi \leq \pi. \end{array} \right.
\]

#### The Gaussian weight function

\[
w_g(\psi) = e^{-a(1-\cos \psi)} \quad \frac{1}{2} \leq \psi \leq \pi,
\]

where \(a\) is dimensionless parameter that defines a Gaussian filter. Finally, the ideal filter weight function can be written as

\[
w_i(\psi) = \left\{ \begin{array}{ll} \frac{1}{L+1} \sum_{n=0}^{L} (2n+1) P_n(\cos \psi) & 0 \leq \psi \leq \psi_0 \\ 0 & \psi_0 < \psi \leq \pi. \end{array} \right.
\]

where \(L\) is the upper limit degree. It is ideal only if \(\psi_0 = \pi\).

Jekeli (1981) derived the expressions of \(\beta_n\) for Pellinen’s and Gaussian weight functions. The recursive formula for \(\beta_n\) of Gaussian weight function is numerically divergent at higher degrees. A numerical integration algorithm is implemented to compute \(\beta_n\) in
this study instead. We have also derived the expression of $\beta_n$ for the ideal filter weight function as

$$\beta_n^i = \frac{1}{A_w} \sum_{k=0}^{L} (2k + 1) \overline{R}_{ak}(t_0),$$

where

$$A_w = \sum_{n=1}^{L} [P_{n-1}(t_0) - P_{n+1}(t_0)] + (1 - t_0)$$

and

$$\overline{R}_{ak}(t_0) = \int_{-\pi}^{\pi} P_a(t) P_k(t) dt.$$  \hfill (16)

In the equations above, $t_0 = \cos \psi_0$. Comparing eqs (9) and (10), we can see that the LP filters rescale signals of interest at each degree by eigenvalues $\beta_n$. The systematic error from eq. (6) is distorted, and tends to be smaller than what it is because $\beta_n$ vary between $-1$ and $1$. It is necessary to mention that there is an optimal filter in the sense of Wiener–Kolmogorov (the collocation filter). However, this type of discussion is outside the scope of this study.

2.2 The inverse Stokes integral

The systematic gravity error can be computed in terms of the inverse Stokes integral (Véronneau & Huang 2003):

$$\delta \epsilon_{LW} = \frac{\gamma}{4\pi R} \int_{\Omega'} S_{N}^{-1}(\psi) \left[ N_{TG} - N_{L}^{S} \right] d\Omega',$$

where $\Omega'$ is the region on which we have data; $\gamma$ is the normal gravity and $R$ is the mean Earth radius. $N_{TG}$ is the geoid height computed from the surface gravity data. $N_{L}^{S}$ is the geoid height from a satellite gravity model. The inverse Stokes kernel $S_{N}^{-1}(\psi)$ to degree $L$ is defined as

$$S_{N}^{-1}(\psi) = \sum_{n=2}^{L} (n-1)(2n+1) P_n(\cos \psi),$$

where $P_n$ is the Legendre polynomial. The residual geoid is evaluated by

$$N_{TG} - N_{L}^{S} = \frac{R}{4\pi \gamma} \int_{\Omega'} S_{DB}^{-1}(\psi) \left[ \Delta g_{TG} - \Delta g_{S}^{L} \right] d\Omega',$$

where $S_{DB}^{-1}$ is the degree-banded Stokes kernel (Huang & Véronneau 2005).

It is noted that this method recovers the systematic error from spherical harmonic degree 2 to $L$ only, though $N_{TG}$ contains the geoid components above degree $L$. Since the inverse Stokes integration is performed only within the region of study, harmonic orthogonality no longer holds. Thus this method is subject to truncation error and is approximate. It is theoretically necessary to perform LP filtering to the geoid height before the inverse Stokes integration to avoid aliasing errors. Considering eqs (9) and (10), a modified inverse Stokes integral can be derived as

$$\delta \epsilon_{LW} = \frac{\gamma}{4\pi R} \int_{\Omega'} S_{N}^{-1}(\psi) F \left[ N_{TG} - N_{L}^{S} \right] d\Omega',$$

where

$$S_{N}^{-1}(\psi) = \sum_{n=2}^{L} (n-1)(2n+1) \beta_n P_n(\cos \psi).$$

Eq. (20) estimates the systematic errors from the LP filtered geoid height. Eq. (21) is the de-smoothing inverse Stokes kernel. The use of $\beta_n$ undoes the smoothing effect caused by the LP filter in eq. (10).

2.3 Spherical harmonic analysis

A surface spherical harmonic model can be applied to fit a regional gravity field by assuming the gravity anomalies for the rest of the Earth be zero. When the differences between the regional surface and satellite gravity anomalies are expressed by a surface spherical harmonic model, the low-degree systematic error of the surface gravity anomaly can be estimated approximately from the model

$$\delta \epsilon_{L}^{SH} = \sum_{n=0}^{L} \delta \epsilon_{a}^{TG},$$

where $\delta \epsilon_{a}^{TG}$ are harmonic components of the systematic error. In theory, eq. (22) can approximate the regional field with any accuracy, given sufficiently large $L$ (Walsh 1929). However, $L$ has to be chosen in terms of the quality of the GRACE model for the purpose of estimating the systematic error. To reduce the aliasing error from higher degrees of gravity components, the surface gravity data need be first filtered via a LP filter similar to the inverse Stokes method. The eigenvalues $\beta_n$ must be accounted for in estimating the coefficients to undo the smoothing effect in filtering.

3 NUMERICAL COMPARISONS USING SYNTHETIC FIELDS

The three methods described in Section 2 are evaluated against synthetic data for the North American region. They are compared in terms of how well they estimate the low-degree systematic error with the presence of high-degree components in a region. The region covers North America and neighbouring areas to simulate the real data domain, and is defined by the boundaries: latitude: 20°N–84°N; longitude: 10°W–170°W. Two sets of synthetic data are generated: surface and satellite. The surface gravity anomalies encompass gravity components with higher spherical harmonic degrees than those from the satellite data. First, we simulate the surface data using EGM96 (Lemoine et al. 1998) from degree 2 to degree 360 with a 2° spacing on the reference ellipsoid. The random error is not simulated because it is not significant compared with the systematic error in the surface gravity data over North America. Second, the GGM02S (Tapley et al. 2005) is used as the satellite model from spherical harmonic degrees and orders 2 to 90. By assuming GGM02S errorless, we aim to estimate the systematic errors of EGM96 from degree 2 to 90 with respect to GGM02S in the region of study using the methods discussed in Section 2.

Table 1 shows the synthetic gravity data from EGM96 and GGM02S. The magnitude of the components higher than degree 90 ($\Delta g_{S}^{90}$) is far greater than that of the synthetic data. Considering eqs (9) and (10), a modified inverse Stokes integral can be derived as

$$\delta \epsilon_{LW} = \frac{\gamma}{4\pi R} \int_{\Omega'} S_{N}^{-1}(\psi) F \left[ N_{TG} - N_{L}^{S} \right] d\Omega',$$

where

$$S_{N}^{-1}(\psi) = \sum_{n=2}^{L} (n-1)(2n+1) \beta_n P_n(\cos \psi).$$

Eq. (20) estimates the systematic errors from the LP filtered geoid height. Eq. (21) is the de-smoothing inverse Stokes kernel. The

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Table 1. Statistical information of synthetic gravity anomalies over the study region (latitude: 20°N–84°N; longitude: 10°W–170°W) with a 2′ spacing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta g_{\text{EGM96}})</td>
<td>-230.181</td>
<td>376.659</td>
<td>0.761</td>
<td>29.511</td>
<td>29.520</td>
</tr>
<tr>
<td>(\Delta g_{\text{EGM96}})</td>
<td>-108.403</td>
<td>82.201</td>
<td>0.762</td>
<td>21.579</td>
<td>21.593</td>
</tr>
<tr>
<td>(\Delta g_{\text{GGM02S}})</td>
<td>-109.635</td>
<td>81.923</td>
<td>0.767</td>
<td>21.582</td>
<td>21.596</td>
</tr>
<tr>
<td>(\Delta g_{\text{EGM96}}) - (\Delta g_{\text{GGM02S}})</td>
<td>-193.508</td>
<td>341.074</td>
<td>-0.001</td>
<td>20.352</td>
<td>20.352</td>
</tr>
<tr>
<td>(\Delta g_{\text{EGM96}}) - (\Delta g_{\text{GGM02S}})</td>
<td>-197.545</td>
<td>346.260</td>
<td>-0.007</td>
<td>20.467</td>
<td>20.467</td>
</tr>
<tr>
<td>(\Delta g_{\text{EGM96}}) - (\Delta g_{\text{GGM02S}})</td>
<td>-12.146</td>
<td>18.892</td>
<td>-0.005</td>
<td>2.206</td>
<td>2.206</td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

Figure 1. Weight versus spherical distance.

conserve the components up to degree 90 as much as possible in the LP filtering. Therefore, \(\psi_0 = 2^\circ\) has been chosen in terms of sampling theorem. The parameter \(a = 2903.970\) for Gaussian filter has been chosen so that its weight function value is equal to that of the ideal weight function for \(L = 90\) at \(\psi_0 = 2^\circ\). Fig. 1 illustrates values of the three weight functions with respect to the spherical distance.

Fig. 2 shows how the eigenvalues \(\beta_n\) decrease with respect to spherical harmonic degree. It can be seen that the LP filters significantly down-weigh spherical harmonic components above degree 90 while they also suppress the magnitude of components below degree 90. Pellinen’s filter shows the strongest oscillation among the three filters while the responses of the Gaussian and ideal filters are similar.

Table 2 gives the differences of the filtered results from the synthetic systematic errors using \(\Delta g_{\text{EGM96}}\) - \(\Delta g_{\text{GGM02S}}\) as input according to eq. (6). As the eigenvalues \(\beta_n\) of Fig. 2 imply, the high-degree components have basically been filtered out. However, these differences reach the same magnitude as the synthetic ones in the first row \(\delta \epsilon_n^{\text{Syn}} = \Delta g_{\text{EGM96}} - \Delta g_{\text{GGM02S}}\), that is, the errors from filtering are too large to estimate systematic errors.

The differences stem from distortion of the averaging processes and the aliasing from the high degree components as Fig. 2 implies. Assuming \(\delta \epsilon_L\) represents the true systematic gravity error, the differences shown in Table 2 can mathematically be separated as

\[
\delta \epsilon_L^{\text{LP}} - \delta \epsilon_L = F \left[ \delta \epsilon_L^{\text{LP}} \right] - \delta \epsilon_L + F \left[ \Delta g_L^{\text{Eig}} \right].
\]

The difference between the first and second terms on the right side of eq. (23) represents the distortion from filtering. The third term introduces the aliasing. Tables 3 and 4 show the distortion and aliasing over the region of study, respectively. While the major aliasing can be eliminated through pre-removal of the high-degree gravity components from the surface gravity data using a combined model such as EGM96 as demonstrated by Véronneau & Huang (2003), the distortion can not be avoided in filtering.
low-degree gravity components makes for the worst case scenario. The fact that the truncation error is largely caused by the Stokes integral and its inverse are defined as global.

3.2 Tests of the inverse Stokes integral

The use of the inverse Stokes integral is approximate in theory in recovering the long-wavelength systematic error in regional data because the Stokes integral and its inverse are defined as global integrals. Like the Stokes integral, the inverse Stokes integral is subject to truncation error when the integration is performed regionally. The fact that the truncation error is largely caused by the low-degree gravity components makes for the worst case scenario when using this method.

![Figure 2. The eigenvalues for the three low-pass filters versus spherical harmonic degree.](https://academic.oup.com/gji/article-abstract/175/1/46/720169)

Table 2. Differences between the synthetic systematic errors and the low-pass averaging results over the study subregion (latitude: 22°N–82°N; longitude: 20°W–160°W) with a 2′ spacing. $\psi_0 = 2°$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta \epsilon^S_{L}$</td>
<td>$-12.146$</td>
<td>18.892</td>
<td>0.026</td>
<td>2.258</td>
<td>2.259</td>
</tr>
<tr>
<td>$\delta \epsilon^L_{\text{Gaussian}} - \delta \epsilon^S_{L}$</td>
<td>$-15.000$</td>
<td>11.727</td>
<td>$-0.006$</td>
<td>1.903</td>
<td>1.903</td>
</tr>
<tr>
<td>$\delta \epsilon^L_{\text{Ideal}} - \delta \epsilon^S_{L}$</td>
<td>$-11.928$</td>
<td>13.046</td>
<td>$-0.003$</td>
<td>2.297</td>
<td>2.297</td>
</tr>
<tr>
<td>$\delta \epsilon^L_{\text{Ideal}} - \delta \epsilon^S_{L}$</td>
<td>$-11.191$</td>
<td>12.307</td>
<td>$-0.003$</td>
<td>2.183</td>
<td>2.183</td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

Table 3. Estimation of the distortion for the systematic errors due to filtering over the study subregion (latitude: 22°–82°N; longitude: 20°W–160°W) with a 2′ spacing. $\psi_0 = 2°$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{\text{Pellinen}} [\Delta \epsilon^S_{\text{G}}]$</td>
<td>$-9.684$</td>
<td>11.170</td>
<td>0.003</td>
<td>1.457</td>
<td>1.457</td>
</tr>
<tr>
<td>$F_{\text{Gaussian}} [\Delta \epsilon^S_{\text{G}}]$</td>
<td>$-11.932$</td>
<td>12.917</td>
<td>0.003</td>
<td>2.100</td>
<td>2.100</td>
</tr>
<tr>
<td>$F_{\text{Ideal}} [\Delta \epsilon^S_{\text{Ideal}}]$</td>
<td>$-11.170$</td>
<td>12.719</td>
<td>0.003</td>
<td>1.964</td>
<td>1.964</td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

Table 4. The aliasing error over the study subregion (latitude: 22°N–82°N; longitude: 20°W–160°W) with a 2′ spacing. $\psi_0 = 2°$.

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<thead>
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<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{\text{Pellinen}} [\Delta \epsilon^S_{\text{G}}]$</td>
<td>$-12.146$</td>
<td>11.704</td>
<td>0.058</td>
<td>2.018</td>
<td>2.019</td>
</tr>
<tr>
<td>$F_{\text{Gaussian}} [\Delta \epsilon^S_{\text{G}}]$</td>
<td>$-10.725$</td>
<td>11.266</td>
<td>0.062</td>
<td>1.902</td>
<td>1.902</td>
</tr>
<tr>
<td>$F_{\text{Ideal}} [\Delta \epsilon^S_{\text{Ideal}}]$</td>
<td>$-2.155$</td>
<td>1.778</td>
<td>0.004</td>
<td>0.495</td>
<td>0.495</td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

Table 5. The truncation errors of the inverse Stokes results in the region of latitude: 26°N–78°N; longitude: 30°W–150°W with a 2′ spacing.

<table>
<thead>
<tr>
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</tr>
</tbody>
</table>

Note: Unit: mGal.

The truncation errors from the inverse Stokes method are shown in Table 5. In this numerical test, the synthetic values $\delta \epsilon^L_{\text{Pellinen}}$ were first converted to the geoid using eq. (19), then $\delta \epsilon^L_{\text{Gaussian}}$ was estimated by eq. (17). A 6 arc-degrees integration radius was used. It can be seen that the truncation errors $\epsilon$ amount to about 25 per cent of the synthetic values in terms of RMS.

The systematic errors were estimated in terms of eqs (17), (19) and (20). In this numerical test, the synthetic residual surface data $\Delta \epsilon^S_{\text{G}}$ were first converted to the geoid using eq. (19), then $\delta \epsilon^L_{\text{Ideal}}$ was estimated by eq. (20). The results are listed in Table 6. Besides the truncation errors shown in Table 5, these results also include the aliasing from the gravity components above degree 90 which increases the total estimation errors up to
75 per cent of the synthetic values. Surprisingly, these results suggest that the LP filters do not improve the estimates.

In practice, we can pre-remove the high-degree components using a combined model such as EGM96 in order to eliminate most of the aliasing. Therefore, this method gives meaningful estimates of the systematic error only when applying the pre-removal process demonstrated by Véronneau & Huang (2003).

### 3.3 Tests of the spherical harmonic analysis

The algorithm and software for spherical harmonic analysis developed by Maimville (1987) are used in this study. As discussed earlier, the spherical harmonic analysis confined to a region suffers from the representation error due to a limited $L$. The representation error was analysed and the results are summarized in Table 7. In this numerical experiment, the synthetic values $\delta \epsilon_L^{\text{Syn}}$ in the North America region (latitude: 20°–84° N; longitude: 10° W–170° W) are expanded into a spherical harmonic model from degrees 2 to 90. The predicted values from this model were compared to the synthetic ones in a subregion to reduce the edge effect. The differences $(\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}})$ amount to about 15 per cent of the synthetic values in terms of RMS. Also they are leaked into the high-degree components. In other words, the synthetic errors over the region affect the full harmonic spectrum when they are expanded into a spherical harmonic model. Eighty-five per cent (85 per cent) of the synthetic errors are represented through the harmonic components from degrees 2 to 90. Assuming EGM96 is based solely on the surface gravity data, one way to understand the differences is that they represent the systematic errors from this region, while the remaining 15 per cent originates from the rest of the Earth. By increasing the maximum expansion degree, the differences decrease. However, in the presence of high-degree surface gravity components, their aliasing brings errors into the estimation that can be improved only by using higher degree satellite gravity models.

Numerical results containing the joint effect of the representation and aliasing are listed in Table 8. In this numerical test, the synthetic residual surface data $\Delta \epsilon_{\text{SG}}^{\text{EGM96}} - \Delta \epsilon_{\text{SG}}^{\text{GGM02S}}$ were used as input. The total differences $\epsilon$ amount to more than 50 per cent of the synthetic values in the subregion of comparison in terms of RMS. Again, contrary to general understanding again, the LP filters do not improve the estimation.

To reduce the effect of aliasing, the high degrees of components must be pre-removed as completely as possible. Table 9 shows that more than 80 per cent of the systematic errors can be recovered once the components of degrees 91 to 180 are pre-removed. Again, we can pre-remove the high-degree gravity components up to degree 360 or higher using a combined model from the surface gravity data so that the aliasing becomes even smaller to lead the better estimation of the systematic error.

Overall, with proper pre-removal, the spherical harmonic analysis best recovers the systematic error among the three methods. This method will be used to determine the systematic error in the surface gravity data over North America in the subsequent error estimation.

### Table 6. The inverse Stokes estimates of the systematic errors and their differences from the synthetic values without and with low-pass filters in the region of latitude: 26°N–78°N; longitude: 30°W–150°W with a 2’ spacing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Filter</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta \epsilon_L^{\text{Syn}}$</td>
<td>−12.146</td>
<td>11.704</td>
<td>0.058</td>
<td>2.018</td>
<td>2.019</td>
<td></td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>−12.130</td>
<td>11.605</td>
<td>0.055</td>
<td>1.970</td>
<td>1.971</td>
<td></td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−1.341</td>
<td>1.454</td>
<td>−0.003</td>
<td>0.327</td>
<td>0.327</td>
<td></td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

### Table 7. The representation errors of spherical harmonic expansion in the region of latitude: 26°N–78°N; longitude: 30°W–150°W with a 2’ spacing.

<table>
<thead>
<tr>
<th>Parameter</th>
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<td>1.970</td>
<td>1.971</td>
</tr>
<tr>
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<td>−1.341</td>
<td>1.454</td>
<td>−0.003</td>
<td>0.327</td>
<td>0.327</td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

### Table 8. The spherical harmonic estimates of the systematic errors and their differences from the synthetic values without and with low-pass filters in the region of latitude: 26°N–78°N; longitude: 30°W–150°W with a 2’ spacing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Filter</th>
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<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta \epsilon_L^{\text{Syn}}$</td>
<td>−12.146</td>
<td>11.704</td>
<td>0.058</td>
<td>2.018</td>
<td>2.019</td>
<td></td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>None</td>
<td>−11.466</td>
<td>11.913</td>
<td>0.052</td>
<td>2.210</td>
<td>2.211</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−8.225</td>
<td>6.539</td>
<td>−0.006</td>
<td>1.158</td>
<td>1.158</td>
<td></td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>Pellinen</td>
<td>−11.659</td>
<td>11.850</td>
<td>0.047</td>
<td>2.228</td>
<td>2.228</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−10.409</td>
<td>9.098</td>
<td>−0.011</td>
<td>1.212</td>
<td>1.212</td>
<td></td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>Gaussian</td>
<td>−11.522</td>
<td>11.872</td>
<td>0.048</td>
<td>2.202</td>
<td>2.202</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−8.555</td>
<td>6.571</td>
<td>−0.010</td>
<td>1.147</td>
<td>1.147</td>
<td></td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>Ideal</td>
<td>−11.528</td>
<td>11.869</td>
<td>0.048</td>
<td>2.201</td>
<td>2.201</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−8.573</td>
<td>6.581</td>
<td>−0.010</td>
<td>1.147</td>
<td>1.147</td>
<td></td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

### Table 9. The spherical harmonic estimates of the systematic errors and their differences from the synthetic values after pre-removing high-degree components (91–180) from the residual gravity anomaly. The region is of latitude: 26°N–78°N; longitude: 30°W–150°W with a 2’ spacing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta \epsilon_L^{\text{Syn}}$</td>
<td>−12.146</td>
<td>11.704</td>
<td>0.058</td>
<td>2.018</td>
<td>2.019</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>None</td>
<td>−12.089</td>
<td>11.534</td>
<td>0.055</td>
<td>1.972</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−1.675</td>
<td>1.486</td>
<td>−0.004</td>
<td>0.344</td>
<td>0.344</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>Pellinen</td>
<td>−12.165</td>
<td>11.620</td>
<td>0.053</td>
<td>1.974</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−2.775</td>
<td>2.309</td>
<td>−0.006</td>
<td>0.398</td>
<td>0.398</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>Gaussian</td>
<td>−12.121</td>
<td>11.589</td>
<td>0.054</td>
<td>1.970</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−2.020</td>
<td>1.561</td>
<td>−0.004</td>
<td>0.354</td>
<td>0.354</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}}$</td>
<td>Ideal</td>
<td>−12.122</td>
<td>11.591</td>
<td>0.054</td>
<td>1.970</td>
</tr>
<tr>
<td>$\delta \epsilon_L^{\text{SH}} - \delta \epsilon_L^{\text{Syn}}$</td>
<td>−2.048</td>
<td>1.576</td>
<td>−0.004</td>
<td>0.356</td>
<td>0.356</td>
</tr>
</tbody>
</table>

Note: Unit: mGal.
4 ESTIMATION OF THE SYSTEMATIC ERRORS

4.1 Surface gravity anomaly

The Canadian surface gravity data were mainly collected by ground surveys over the past 60 yr. They represent some 715,000 observations with an average spacing of about 9 km over land and oceans (see Fig. 3). The precision of most ground observations ranges between 0.01 and 0.1 mGal, while that of the shipboard observations is about 1–3 mGal. These gravity data are routinely used to compute different types of gravity anomalies such as free-air, Bouguer and isostatic anomalies for various purposes. The height reductions such as, free-air and Bouguer plate corrections, are standard. A significant part of the error in the gravity anomalies originates from the inaccuracy of the station heights. This inaccuracy can be estimated between 3 and 4 m, but it can reach tens to hundreds of metres under extreme cases for the earlier surveys. We have also included some 1.5 million observations over the United States obtained from National Geospatial-Intelligence Agency (NGA), and US National Geodetic Survey (NGS). Their precision ranges from 1 to 10 mGal with an average spacing of 5 km over the continental US.

Before comparing the surface and GRACE gravity anomalies, they must be reduced to the same type of gravity anomaly and a common reference surface. In addition, the surface data need to be interpolated to a regular geographic grid for the spherical harmonic analysis. In principle, no matter which type of gravity anomalies is used, we should obtain the same result for the systematic errors because common reductions are applied to both the surface and GRACE data. We use the geoid as the reference surface following general convention, and choose the Helmert gravity anomaly for the purpose of the comparison. The Helmert anomaly is a simple realization of isostatic reduction in which the topography is compressed onto a surface mass layer on the geoid. It is somewhat close to the free-air anomaly, but can be computed precisely following the approach described by Huang & Veroné (2005). The Helmert gravity anomaly was originally introduced for the purpose of geoid modelling (see, e.g. Vaniček et al. 1999).

The Helmert anomaly data set used for the Canadian Gravimetric Geoid 2000 (CGG2000) (Veronneau 2002) is adopted here. In this data set, the satellite altimetry derived gravity anomalies were used to fill gaps over large part of oceans where no shipboard data are available. In particular, for the Foxe Basin and Ungava Bay, there were no surface measurements available for CGG2000. These two areas were covered by the gravity anomalies computed from EGM96. It should be noted that downward continuation correction for the refined Bouguer anomalies was not accounted for in the process of generating the Helmert anomaly data. It amounts to 0.9 mGal RMS over the Rocky Mountains, and is generally smaller than 0.3 mGal RMS in other areas. Furthermore, since only the long-wavelength error is estimated, the effect of downward continuation is predictably far smaller than the values above, and is included in the estimation of the systematic errors.

4.2 GRACE gravity anomalies

A recent global gravity model GGM02C (Tapley et al. 2005) is used to compute the GRACE gravity anomalies in this study. It is a combined model of GGM02S, a GRACE-only gravity model, and the global surface gravity data that were used for the development of EGM96. GGM02S is one of the representative GRACE-only gravity models depicting the Earth’s gravity field at least 10 times better than the pre-GRACE gravity models for harmonic gravity field below degree 90. It dominates GGM02C from degrees 2 to 90. Additionally, the J2 term in GGM02C is from Satellite Laser Ranging (SLR) and is believed to be more reliable. The accumulative RMS error of GGM02C from degree 2 to 90 is 0.25 mGal. From degrees 91 to 110, GGM02C is from Satellite Laser Ranging (SLR) and is believed to be more reliable. The accumulative RMS error of GGM02C from degree 2 to 90 is 0.25 mGal. From degrees 91 to 110, GGM02C blends GGM02S with the global surface gravity data. From degree 111 to 200, the surface data dominates GGM02C. Above degree 200, GGM02C has been derived to transition smoothly to EGM96.
Assessment of systematic errors in the surface gravity anomalies

Table 10. The surface and GRACE Helmert gravity anomalies in the region is of latitude: 20° N–84° N; longitude: 10° W–170° W with a 2° spacing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta g_{\text{H}}^{\text{GGM02C}})</td>
<td>-240.669</td>
<td>541.208</td>
<td>0.976</td>
<td>32.874</td>
<td>32.888</td>
</tr>
<tr>
<td>(\Delta g_{\text{H}}^{\text{GRACE}})</td>
<td>-229.434</td>
<td>365.031</td>
<td>0.763</td>
<td>29.261</td>
<td>29.271</td>
</tr>
<tr>
<td>(\Delta g_{\text{H}}^{\text{GGM02C}} - \Delta g_{\text{H}}^{\text{GRACE}})</td>
<td>-201.773</td>
<td>425.542</td>
<td>0.214</td>
<td>15.562</td>
<td>15.564</td>
</tr>
</tbody>
</table>

Note: Unit: mGal.

We need to compute the Helmert gravity anomalies from GGM02C to compare them to the surface Helmert gravity anomalies. The method proposed by Vaníček et al. (1995) is used to compute the Helmert anomalies. In order to reduce the aliasing from high degree components, the Helmert anomalies are computed from degree 2 to 360 from GGM02C and EGM96 to cancel the components of degrees 91 to 360 as much as possible in the surface Helmert gravity anomalies.

4.3 Estimation of the systematic error

The surface and GRACE Helmert gravity anomalies, and their differences are statistically described in Table 10. The differences are primarily caused by the high-degree components in the surface anomalies. They were used to estimate the systematic errors in the surface gravity anomalies using spherical harmonic analysis. The results are shown in Table 11 and Fig. 4.

It can be seen that the results with and without filtering, and from any type of filter, do not differ from each other significantly. This implies that the spherical harmonic analysis alone works well in extracting the long-wavelength information. Over the whole region of study, the low-degree error is about 1.4 mGal in RMS if the error estimate for GGM02C is realistic. This value gives us quantitative information about quality of the surface data.

Fig. 4 reveals the areas where the systematic errors are significant. Origins of the errors at some locations can be identified. The high error values over the Southern Alaska, Foxe Basin, Ungava Bay, and Greenland are due to complete lack or sparseness of gravity data. New data are needed to cover these regions. Those features over Hudson Bay and North Atlantic may relate to the sea surface topography (SST) effect on the surface gravity anomalies. The areas showing significant low-degree systematic errors are most probably where high-degree systematic errors exist. Therefore, this map will help us to plan for new gravity survey projects. Conversely, the surface gravity data are often used to validate satellite data. This type of validation should consider the existence of systematic errors in the surface gravity data.

5 CONCLUSIONS

Accurate determination of the lower-degree Earth’s gravity field from the GRACE mission enables us to detect the lower-degree systematic error in the surface gravity data over North America. Three different methods are tested using synthetic gravity data: LP filtering, the inverse Stokes integral, and spherical harmonic analysis. The tests suggest that the spherical harmonic analysis works best, and can recover up to 80 per cent of the synthetic systematic

Note: Unit: mGal.

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Note: Unit: mGal.
error. To improve the estimation, higher resolution satellite gravity models will be required.

To compare the surface and GRACE gravity data, we have computed the surface and GRACE Helmert gravity anomalies on the geoid. Using the spherical harmonic analysis, the systematic errors from spherical harmonic degrees 2 to 90 have been estimated for the surface gravity data over North America. They range from about $-6$ to $13$ mGal with a RMS value of about 1.4 mGal. These estimates provide us the quantitative information on the quality of the data. A few locations with large systematic errors reveal the existence of data gaps. Certain systematic features over the oceans could be attributed to the SST. The error map provides supportive evidence for new gravity survey projects. It also suggests that the systematic errors must be considered when the surface gravity data are used to validate satellite gravity data.

ACKNOWLEDGMENTS

We are grateful to Joe Henton and Pierre Heroux from Geodetic Survey Division, CCRS, Natural Resources Canada for their efforts in improving this manuscript. We also want to thank two reviewers for their critical and constructive comments. The GMT package by Wessel and Smith (1998) has been used to generate the maps in this paper.

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