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Sensitivity Analysis of Groundwater Flow

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A numerical model developed for sensitivity analysis of groundwater flow is presented. Sensitivity analysis is a useful complementary aid for groundwater flow modelling to assess the importance of various governing flow parameters to the behaviour of any specific flow problem. Two different methods are considered: One is called the direct method and the other the adjoint method. In the direct method the sensitivity equations are obtained by directly differentiating the flow equations with respect to the parameters, while in the adjoint method they are obtained by a variational technique. The numerical method for solving the groundwater flow equation and the sensitivity equations are based on the Galerkin finite element method. The sensitivity model developed was applied to a simple flow problem, in which the sensitivity of the piezometric head as well as various flux performance measures to perturbations of the permeability of various layers of the flow domain were analyzed. The following performance measures were considered: The piezometric head, the Darcy flux at selected regions and the total influx into a tunnel system.

Introduction

In general, sensitivity analysis is a study of the rate of changes in model output results to the changes in model input parameters. It makes it possible to quantify how sensitive model results are to perturbations of the model parameters. Sensitivity techniques have been used in groundwater flow modelling to describe the variability of the solution parameters in response to changes of the input parameters such as the hydraulic conductivity, the storativity, the initial and boundary conditions, the interior and boundary flux. It may be used to study the behaviour and structure of geohydrologic systems and to assess the importance of various governing flow parameters to the behaviour of any specific flow problem. Venuri *et al.* (1969) and McElwee (1978) introduced sensitivity theory to groundwater flow. They derived the partial differential equations for the sensitivity coefficient by taking the partial derivative of the flow equation with respect to parameters such as transmissivity and storativity. They applied sensitivity analysis to calibrate their simulation models and to establish tolerances on transmissivity and storativity for a given tolerance of the error in the hydraulic head and to estimate the variance and confidence intervals for the hydraulic heads. Sensitivity analysis is an ingredient of uncertainty analysis, in which the uncertainty of the estimate of the performance measure is assessed through the measurement of the sensitivity of the performance measure to the parameter values (Thomson and Sykes 1986). It is also a useful tool for guiding the adjustment of the model parameters, thus measuring the importance of the parameters to aid in determining the order in which the parameter values in a model should be reconsidered to improve the accuracy in the solutions. Sensitivity analysis is also relevant to solving inverse problems and for parameter identification and optimization problems. In solving such problems it is usually necessary to determine the derivatives of the function which relates the predicted and measured piezometric heads to the hydraulic conductivity of the aquifer. Thus avoiding time consuming trial-and-error processes. Neuman (1980) and Townley and Wilson (1985) solved parameter estimation problems for transient confined groundwater flow models. They applied sensitivity methods to compute the gradients of the least squares objective functions with respect to the parameters of interest. Sun and Yeh (1985) used the variational method to evaluate sensitivity coefficients for the identification of the parameter structure, using the sensitivity coefficients in the Gauss-Newton algorithm elements of the Jacobian matrix. The sensitivity theory has also been used extensively in many other engineering disciplines such as in nuclear reactor design (Oblow 1978a, 1978b; Cacuci *et al.* 1980), in electrical power network design and optimization (Director and Rohrer 1969). Alsmiller *et al.* (1984) applied adjoint sensitivity theory to investigate the importance of various parameters of a liquid supply model. Chen *et al.* (1974) and Chavent *et al.* (1975) used an adjoint sensitivity equation of the linear oil flow equation to compute the gradients of a residual objective function for a history matching application. Lasdon *et al.* (1986) derived sensitivity equations of the nonlinear gas flow equation to determine the sensitivity of the maximum gas production objective function to changes in the pumping rate.

Mathematical Statement

The main approaches to sensitivity analysis are based either on pertubation or on variational theory. They are referred to as "forward sensitivity formalism" and "adjoint sensitivity formalism", respectively by Cacuci (1981). In the former approach the performance measure sensitivity equation can be derived by direct differentiation of the primary flow equations. The latter approach involves the evaluation of an arbitrary function (adjoint function) by an adjoint sensitivity equation based on variational theory. We will begin the theoretical presentation of the sensitivity theory by deriving the sensitivity equations for a general physical problem. We limit the derivation to the algebraic equations obtained from the numerical discretization.

Firstly, a general system state equation which relates parameters (independent variables) and state variables (dependent variables) is considered

$$V(\{p\},\{\alpha\}) = 0 \tag{1}$$

where V is a vector ($n \times 1$) of state functions (*e.g.* algebraic equations numerically obtained from the discrete form of the flow equation), $\{p\}$ is a vector ($n \times 1$) of state variables (*e.g.* piezometric head) and $\{\alpha\}$ is a vector ($m \times 1$) of all of the parameters in the system (*e.g.* permeability). m is the number of components of $\{\alpha\}$ and n is the number of components of $\{p\}$. For all of the components in $\{\alpha\}$ for which sensitivity information is needed, we will assume that, for a specific choice of $\{\alpha\}$, a unique solution of Eq. (1) exists and is represented by $\{p\}$. Thus, $\{p\}$ is a function of $\{\alpha\}$, but its dimension is not related to the dimension of $\{\alpha\}$.

Secondly, a specified function of $\{p\}$ and $\{\alpha\}$ is considered. It is referred to as a performance function or a performance measure

$$P = P(\{p\},\{\alpha\}) \tag{2}$$

where $\{p\}$ is a vector ($n \times 1$) of state variables, *e.g.* piezometric head values and $\{\alpha\}$ is a vector ($m \times 1$) of system parameters such as permeability and porosity. The performance measure P is a scalar that may be calculated from Eq. (2) when the state variable $\{p\}$ has been obtained from Eq. (1) for a specified $\{\alpha\}$. The total sensitivity of the performance function with respect to every parameter can be derived according to the definition of the total derivative

$$\frac{dP(\{p\},\{\alpha\})}{d\{\alpha\}^T} = \frac{\partial P(\{p\},\{\alpha\})}{\partial\{\alpha\}^T} + \frac{\partial P(\{p\},\{\alpha\})}{\partial\{p\}^T}\frac{\partial\{p\}}{\partial\{\alpha\}^T} \tag{3}$$

where the superscript T indicates the transposition of a vector or a matrix.

There are two methods for carrying out the calculation of $\partial\{p\}/\partial\{\alpha\}^T$. One is to derive a state sensitivity equation to directly obtain the state sensitivity matrix $\partial\{p\}/\partial\{\alpha\}^T$. The other is to eliminate $\partial\{p\}/\partial\{\alpha\}^T$ from Eq. (3) by means of defining an adjoint sensitivity matrix $[\psi^*]$, which is obtained by solving the adjoint sensitivity equation.

The first method, usually referred to as the direct method, uses Eq. (4) together with the state sensitivity Eq. (5)

$$\frac{dP(\{p\},\{\alpha\})}{d\{\alpha\}^T} = \frac{\partial P(\{p\},\{\alpha\})}{\partial\{\alpha\}^T} + \frac{\partial P(\{p\},\{\alpha\})}{\partial\{p\}^T}[\psi] \qquad (4)$$

$$\frac{\partial V(\{p\},\{\alpha\})}{\partial\{p\}^T}[\psi] + \frac{\partial V(\{p\},\{\alpha\})}{\partial\{\alpha\}^T} \equiv 0 \qquad (5)$$

where $[\psi] = \partial\{p\}/\partial\{\alpha\}^T$.

The second method, usually referred to as the adjoint method, uses Eq. (6) together with the adjoint sensitivity Eq. (7)

$$\frac{dP(\{p\},\{\alpha\})}{d\{\alpha\}^T} = \frac{\partial P(\{p\},\{\alpha\})}{\partial\{\alpha\}^T} + [\psi^*]^T \frac{\partial V(\{p\},\{\alpha\})}{\partial\{\alpha\}^T} \qquad (6)$$

$$\left[\frac{\partial V(\{p\},\{\alpha\})}{\partial\{p\}^T}\right]^T [\psi^*] + \left[\frac{\partial P(\{p\},\{\alpha\})}{\partial\{p\}^T}\right]^T = 0 \qquad (7)$$

The major computational work to compute the performance function sensitivity is to solve the set of linear equations defined by Eqs. (5) and (7). For Eq. (5) we need to solve $(m \times n)$ systems of linear equations and for Eq. (7) we need to solve $(l \times n)$ systems of linear equations, where m is the number of parameters, n is the number of state variables and l is the number of performance functions. This implies that if the number of parameters exceeds the number of performance functions, then the second method is preferable to the first method. Conversely, if the number of performance function exceeds the number of parameters, then the first method is preferable.

The performance function defined in Eq. (2) can be any general function of model outputs for which sensitivities are sought. For sensitivity analysis of groundwater problems, a general performance function can be written as

$$P = \int_0^\tau \int_\Omega f(\{\alpha\},p,t)\, d\Omega\, dt \qquad (8)$$

where τ is the final time, $f(\{\alpha\},p,t)$ is an arbitrary function of the system state such as the pressure p and the system parameters $\{\alpha\}$ at a time interval. $\{\alpha\}$ represents a column vector of the system parameters which may be permeability k, porosity ϕ, compressibility c, the recharge or discharge Q, the prescribed head boundary condition $\hat{p}$, flux boundary condition $\hat{q}$, and initial condition p_0 for transient problem

$$\{\alpha\}^T = \{k, \phi, c, Q, \hat{p}, \hat{q}, p_0\} \qquad (9)$$

Examples of various forms of $f(\{\alpha\}, p, t)$ are

$$f(\{\alpha\},p,t) = g(x_i,t)p(x_i,t) \tag{10a}$$

$$f(\{\alpha\},p,t) \equiv \sqrt{\sum_{i=1}^{3} q_i^2 \, g(x_i,t)} \tag{10b}$$

$$f(\{\alpha\},p,t) = \sum_{n=1}^{m} q_n g(x_i,t) \tag{10c}$$

$$\int_0^\tau \int_\Omega f(\{\alpha\},p,t)\,d\Omega dt = \sum_{t=1}^{T} \sum_{i=1}^{n} [p(\underset{\sim}{x_i},t) - p'(\underset{\sim}{x_i},t)]^2 \tag{10d}$$

where x_i is a location vector $x_i = \{x_1, x_2, x_3\}$ and $g(x_i)$ is defined as $g(x_i) \equiv \delta(x_i - x_i)$ $\delta(t - t')$

Eq. (10a) is used when the performance measure is pressure at certain points at a given time. For some specific groundwater flow problems we may consider a performance function of the form of Eq. (10b) expressing the magnitude of the Darcy flux at selected points defined by the weighting function $g(x_i, t)$. Eq. (10c) is considered in problems where the flow rate through a specific region or a specific boundary is of interest. Eq. (10d) represents a function, which is often used in general optimization problems, given as the sum of the squares of the difference between predicted and measured p' piezometric head values over the time interval. Response performance functions in the form of Eq. (10d) are also used for parameter estimation problems, for example when the minimum residual formulation and the Gauss-Newton algorithm are used (Ahlfeld *et al.* 1988).

Application of the Sensitivity Model

A numerical model developed for sensitivity analysis of groundwater flow (Thunvik and Bao 1989) is used to roughly analyze the flow conditions around a tunnel system located in a hard rock formation at a depth of about 500 metres below the ground surface. The sensitivity of the piezometric head distribution and the sensitivity of the flux around the tunnel system due to perturbations of the permeability in various layers are analyzed. The direct and adjoint methods are both applied for solving various problems. The following performance measures are considered: (i) the piezometric head in the vicinity of the tunnel system, (ii) the Darcy flux in the vicinity of the tunnel and (iii) the total flux into the tunnel system. The distribution of the piezometric head in the flow domain are also calculated and the mass transport balance in the system is checked.

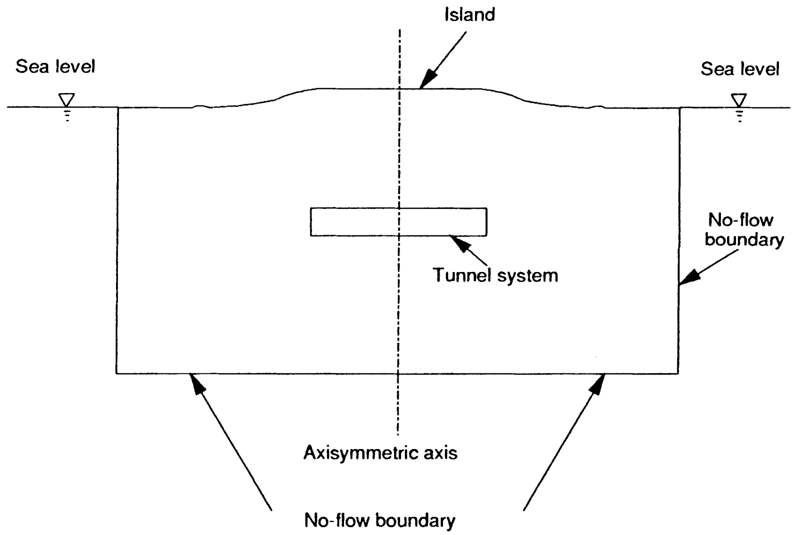

Fig. 1. Schematic illustration of the flow domain.

The Flow Problem

The basic geometric configuration and hydrologic data of the flow domain in the present study are relevant to the conditions at the Swedish Hard Rock Laboratory site located at Äspö (Svensson 1990). The flow domain is assumed to be axisymmetric with a lateral extent of 2 km and a depth of 1 km (Fig. 1). The tunnel system is geometrically simplified by a disc which is located at the centre of the island at depth of about 500 metres below the ground surface. The lateral extent of the tunnel system is 600 metres. The bottom and the vertical boundaries are assumed to be impermeable (no-flow boundaries). On top of the island, the part above the sea level is assumed to be a prescribed influx boundary with an average infiltration rate of 2×10^{-9} m/s (63 mm/year). The remaining part below the sea level is considered to be a hydrostatic pressure boundary. The boundary along the tunnel is assumed to be at atmospheric pressure. The permeability of the rock of the island is decreasing with the depth. The flow domain is simplified to be divided into four layers with different permeabilities (see Table 1). The parameter values

Table 1 – Distribution of permeabilities in the system.

The depth of the layers (m)	The permeabilities (m^2)
0 - 120	$2x10^{-15}$
120 - 500	$5x10^{-16}$
500 - 750	$2x10^{-16}$
750 - 1000	$2x10^{-17}$

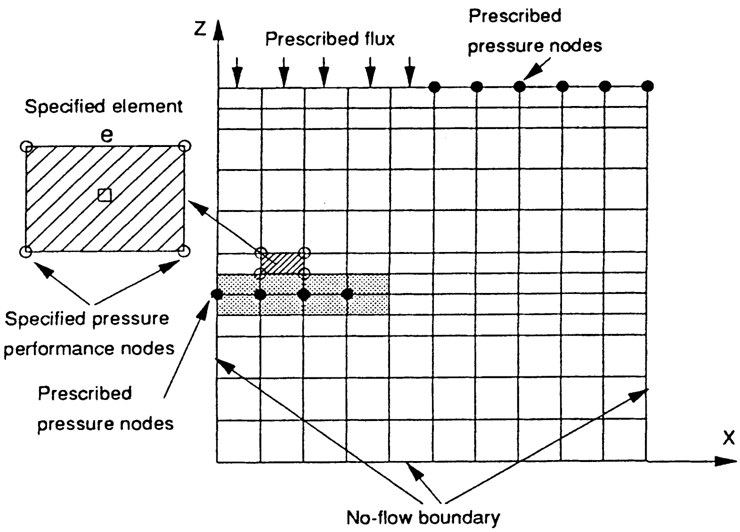

Fig. 2. The element mesh of the flow domain.

used in the calculations are presented in Table 2. The flow domain is discretized into 120 elements of 8-noded quadrilateral. The element mesh is displayed in Fig. 2.

The governing equations (Thunvik and Braester 1988) are

$$\phi\rho(c^f+c^r)p_{,t} - \left[\rho\frac{k_{ij}}{\mu}\left(\frac{\partial p}{\partial x_j} - \rho g_j\right)\right]_{,i} + Q = 0 \tag{11a}$$

Darcy's law is

$$q_i \equiv -\frac{k_{ij}}{\mu}\left(\frac{\partial p}{\partial x_j} - \rho g_j\right) \tag{11b}$$

Table 2 – Parameter values used in the calculations.

Symbol	Parameter	Value	Unit
ρ	Fluid density	998	kg/m^3
μ	Dynamic viscosity	0.001	Pas
c^f	Fluid compressibility	4.10^{-10}	1/Pa
c^r	Rock compressibility	0.0	1/Pa
ϕ	Porosity of the rock	0.001	-
g	Gravity	9.81	m/s^2

The boundary and initial conditions are

$$p(x_i, t) = \hat{p}(x_i, t) \qquad x_i \in \Gamma_1 \qquad (12a)$$

$$-\frac{k_{ij}}{\mu}\left(\frac{\partial p}{\partial x_j} - \rho g_j\right) n_i = \hat{q}(x_i, t) \qquad x_i \in \Gamma_2 \qquad (12b)$$

$$p(x_i, 0) = p_0(x_i, 0) \qquad x_i \in \Omega \qquad (12c)$$

where c is the total compressibility $(c^f + c^r)$, c^f is the compressibility of the fluid, c^r is the compressibility of the rock, $\hat{p}$ is prescribed pressure on boundary Γ_1, $\hat{q}$ is prescribed flux normal to boundaryΓ_2 (as designated by the components of the unit inward normal), n_i is an inward normal vector, $\Gamma = \Gamma_1 + \Gamma_2$ represents the external boundary of the flow domain Ω and p_o is the initial pressure over the flow domain Ω.

The numerical solutions are worked out using the Galerkin finite element method. For convenience the solutions are presented as piezometric head instead of pressure in Figs. 3 and 4. The piezometric head h is defined as $h = p/\varrho g + z$.

Mass Balance Calculation

According to the law of the conservation of mass, in the present case, the total flux through the top boundary should be equal to the total flux into the tunnel. The the total influx is obtained by integrating Darcy's law Eq. (11b) on the top boundary according to the axi-symmetric flow system as below

$$Q = 2\pi \int_{\Gamma} -\frac{k_{ij}}{\mu}\left(\frac{\partial p}{\partial x_j} - \rho g_j\right) r n_i \, dr \qquad (13)$$

Eq. (13) can also be used to calculate the flux into the tunnel system. The results of the mass balance calculation are shown in Table 3.

Table 3 – The result of mass balance calculation.

Total influx (m/s)	Total outflux (m/s)	Error (%)
0.5177×10^{-2}	0.5371×10^{-2}	2.63 %

Calculation of the State Sensitivities

The sensitivities of the piezometric head to permeability changes, *the state sensitivities,* are solved by the direct equation method

$$(\phi \rho c)\frac{\partial \psi}{\partial t} - \frac{\partial}{\partial x_i}\left[\left(\rho \frac{k_{ij}}{\mu}\right)\frac{\partial \psi}{\partial x_j}\right] + D \equiv 0 \qquad (14)$$

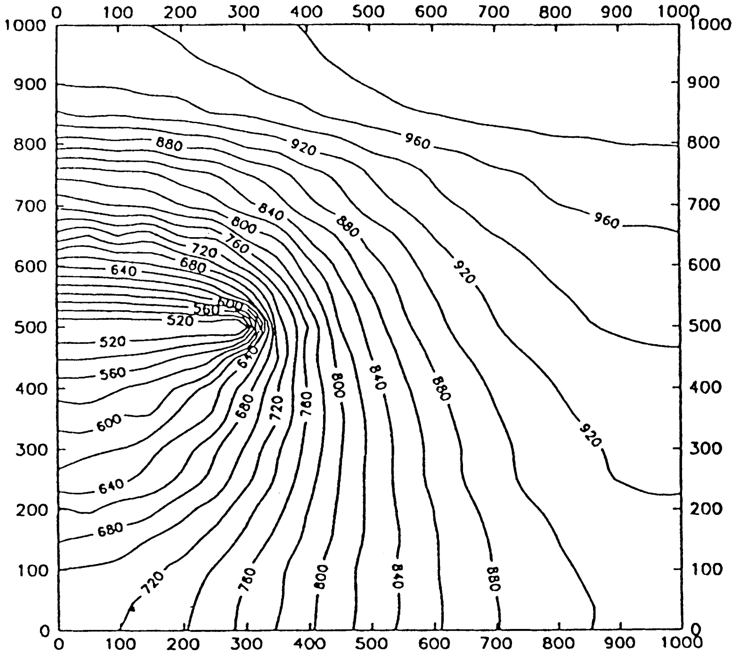

Fig. 3. Contour map of the piezometric heads.

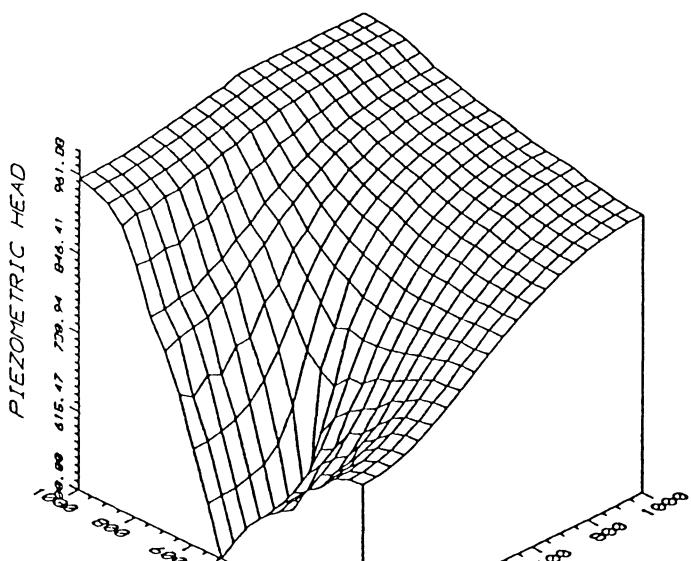

Fig. 4. Perspective plot of the piezometric heads.

where $\psi = \partial p/\partial k_l$ and

$$D = -\frac{\partial}{\partial x_i}\left[\frac{\partial[\rho(k_{ij}/\mu)]}{\partial k_l}\left(\frac{\partial p}{\partial x_j} - \rho g_j\right)\right] \tag{14a}$$

The initial and boundary conditions associated with Eq. (14) are

$$\psi(\psi, 0) \equiv p_0 \qquad \in \Omega, t = 0 \tag{15a}$$

$$\psi(\Gamma_1) \equiv 0 \qquad \in \Gamma_1 \tag{15b}$$

$$-\frac{\partial(k_{ij}/\mu)}{\partial k_l}\left(\frac{\partial p}{\partial x_j} = \rho g_j\right) n_i = \left(\frac{k_{ij}}{\mu}\right)\frac{\partial \psi}{\partial x_j} n_i \equiv 0 \in \Gamma_2 \tag{15c}$$

Solution of Eq. (14) gives a direct measure of state sensitivities for each point in the domain. The distributions of the state sensitivities caused by the perturbations in each of the four layers respectively are showed in Figs. 5 to 8.

Sensitivity of the Piezometric Head Performance Function

The following performance function is considered

$$P = \int_\Omega g(x_i)\, p(x_i)\, d\Omega \tag{16}$$

where x_i is a location vector $x_i = \{x_1, x_2, x_3\}$.

Eq. (16) is used when the performance measure is the pressure, with $g(x_i)$ being an arbitrary weighting function identifying the region of importance. Weight $g(x_i) = 1$ at the node points x_i' and $g(x_i) = 0$ at all other node points. In this case the marginal sensitivities of the performance measure are calculated by the following relationship

$$\frac{\partial P}{\partial k_l} \equiv \int_{\Omega_i} -\frac{\partial \psi^*}{\partial x_i}\frac{\partial[\rho(k_{ij}/\mu)]}{\partial k_l}\left(\frac{\partial p}{\partial x_j} - \rho g_j\right) d\Omega_i \tag{17}$$

where Ω_i is the subdomain around the selected nodes and ψ^* is the solution of the adjoint equations which are defined as below

$$(\phi\rho c)\frac{\partial \psi^*}{\partial t} - \frac{\partial}{\partial x_j}\left[\left(\rho\frac{k_{ij}}{\mu}\right)\frac{\partial \psi^*}{\partial x_i}\right] + \frac{\partial f(\{\alpha\}, p)}{\partial p} \equiv 0 \tag{18}$$

$$\psi^*(\Omega, \tau) \equiv 0 \qquad \in \Omega, t \equiv \tau \tag{19a}$$

$$\psi^*(\Gamma_1, t) = 0 \qquad \in \Gamma_1 \tag{19b}$$

$$\frac{k_{ij}}{\mu}\frac{\partial \psi^*}{\partial x_i} n_j = 0 \qquad \in \Gamma_2 \tag{19c}$$

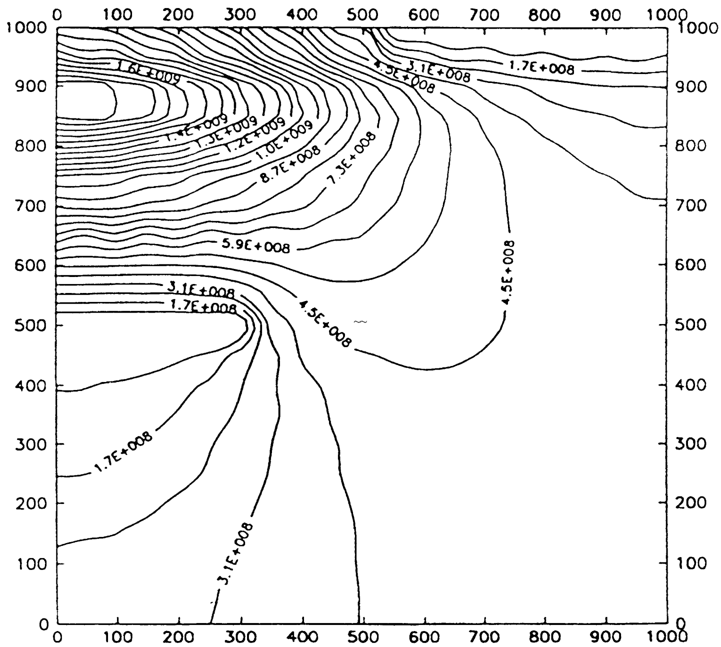

Fig. 5. Contour map of state sensitivity for perturbing the first layer.

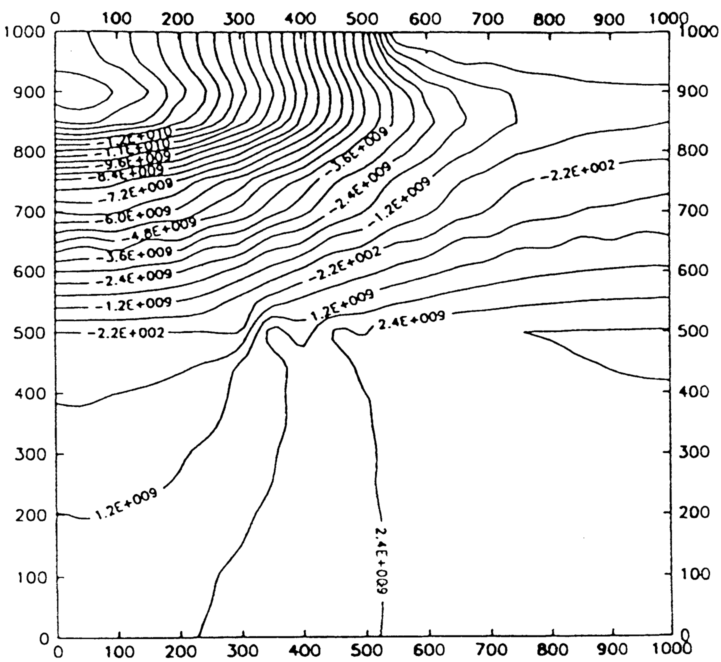

Fig. 6. Contour map of state sensitivity for perturbing the second layer.

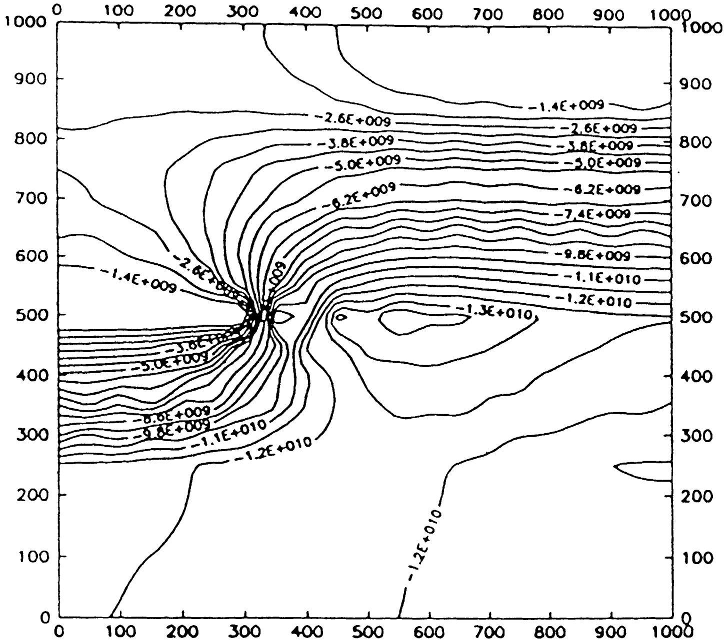

Fig. 7. Contour map of state sensitivity for perturbing the third layer.

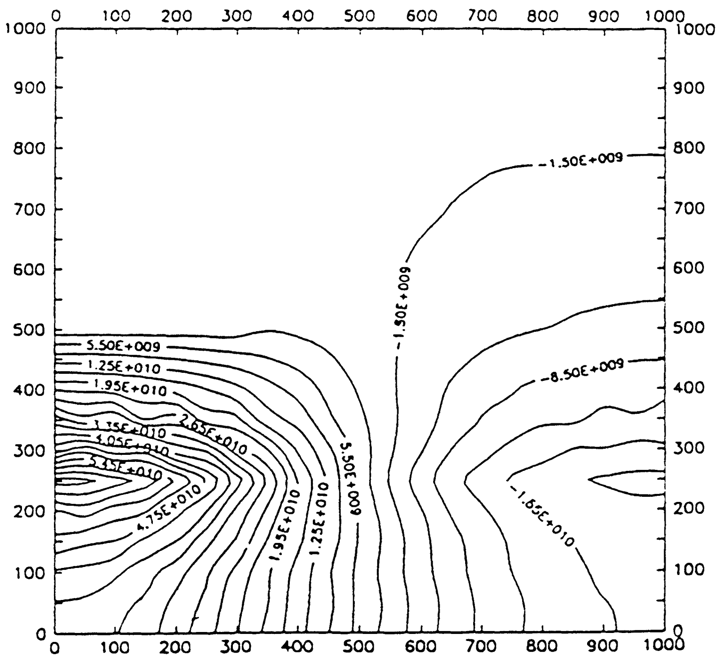

Fig. 8. Contour map of state sensitivity for perturbing the fourth layer.

where $f(\{\alpha\}, p) = g(x_i)p(x_i)$ in this case.

The third term of the adjoint equation becomes one on the nodes for which sensitivities are sought and becomes zero on all other nodes and the first term becomes zero for the steady state case. Then Eq. (18) becomes

$$-\frac{\partial}{\partial x_j}\left[(\rho\frac{k_{ij}}{\mu})\frac{\partial\psi^*}{\partial x_i}\right]+g(x_i)\equiv 0 \qquad (20)$$

If we consider the piezometric head a performance measure instead of the pressure then the sensitivity of the piezometric head performance function is obtained by dividing the results from Eq. (17) by ρg according to the relationship

$$\frac{\partial h}{\partial k_l}=\frac{1}{\rho g}\frac{\partial p}{\partial k_l} \qquad (21)$$

In the present case we specify 4 nodes adjacent to the tunnel (see Fig. 2). For convenience a dimensionless normalized sensitivity is defined as $S_k = dP\,k_l/dk_lP$ which describes the ratio of the relative change of performance measure P to the relative change of permeability k. The solutions of the sensitivity of this performance function are presented in Table 4.

Table 4 - Solutions of the head performance function.

Perturbation region (depth m)	Sensitivities of head performance function dP/dk($ML^{-3}t^{-2}$)	Normalized sensitivity of head performance function S_k(%)
0 - 120	-0.752×10^{9}	0.374 %
120 - 500	0.371×10^{10}	0.492 %
500 - 750	0.453×10^{10}	0.241 %
750 - 1000	-0.127×10^{9}	0.0006 %

Sensitivity of the Flux into the Tunnel System

The sensitivity of the flux into the tunnel is obtained as below

$$\frac{\partial Q_{out}}{\partial k_l}=-2\pi\int_\Gamma\left[\frac{1}{\mu}\frac{\partial k_{ij}}{\partial k_l}(\frac{\partial p}{\partial x_j}-\rho g_j)+\frac{k_{ij}}{\mu}\psi,_j\right]rn_i\,d\Gamma \qquad (22)$$

where pressure p and the state sensitivity ψ are obtained from Eq. (11). A dimensionless normalized sensitivity is defined as $S_k = dQk_l/dk_lQ$ which describes the ratio of the relative change of influx Q to the relative change of permeability k. The solutions are presented in Table 5.

Table 5 – Solutions of the sensitivities of the outflux.

Perturbation region (depth m)	Sensitivities of outflux $dQ_{out}/dk(\mathrm{Lt}^{-1})$	Normalized outflux sensitivities $S_k(\%)$
0 - 120	8950	3.3 %
120 - 500	563844	52.5 %
500 - 750	487688	18.2 %
750 - 1000	115841	0.4 %

Sensitivity of the Flux Performance Function to Permeability

A flux performance function may be defined as below

$$P = \int_{\Omega} \sqrt{\sum_{m=1}^{3} q_m^2}\, g(x_i)\, d\Omega \qquad (23)$$

which is the magnitude of the Darcy flux at a point x_i' located at the centre of the specified element (see Fig. 2).

The marginal sensitivity of this performance function is obtained by the following relationship

$$\frac{\partial P}{\partial k_l} = \int_{\Omega_i} \left[- \frac{1}{\mu\sqrt{\sum_{m=1}^{3} q_m^2}}\, q_i \frac{\partial k_{ij}}{\partial k_l} \frac{\partial p}{\partial x_j} - \frac{\partial \psi^*}{\partial x_i} \frac{\partial[\rho(k_{ij}/\mu)]}{\partial k_l} \left(\frac{\partial p}{\partial x_i} - \rho g_j \right) \right] d\Omega_i$$

$$\forall x_i \equiv x_i' \qquad (24)$$

where ψ^* is called the adjoint state sensitivity obtained from Eq. (18). The first term of Eq. (24) is non-zero only when k_l represents the value of k_{ij} within the element e. Since

$$f = \sqrt{\sum_{m=1}^{3} q_m^2}\, g(x_i)$$

for this performance function the third term of the adjoint Eq. (18) can be calculated by the following equation

$$\frac{\partial f}{\partial p_e} = - \frac{1}{\mu\sqrt{\sum_{m=1}^{3} q_m^2}}\, q_i \frac{\partial[k_{ij}(\partial p/\partial x_j)]}{\partial p_e} \qquad \forall\ e \text{ on the element} \quad (25)$$

$$\frac{\partial f}{\partial p_e} = 0 \qquad \forall\ e \text{ not on the element} \quad (26)$$

Table 6 – Solutions of the Darcy flux performance function.

Perturbation region (depth m)	Sensitivities of velocity performance measure dP/dk (Lt^{-1})	Normalized sensitivities of velocity performance measure S_k(%)
0 - 120	-18.14	4.9 %
120 - 500	-1377.4	93.7 %
500 - 750	100.2	0.54 %
750 - 1000	-2.77	0.008 %

where pressure p is obtained from Eq. (11). The solutions of this performance measure are given in Table 6.

Conclusions and Discussion

The purpose of the present investigation is to analyze the sensitivity of the piezometric head and the sensitivity of the flux for a tunnel system with respect to perturbations of the permeabilities. The calculation results of the state sensitivity distributions respect to permeabilities in four different layers show that the peaks of the sensitivity coefficients are situated mostly in the area around the tunnel in the flow domain. The solution of the piezometric head performance sensitivity show that the piezometric head is quite sensitive to perturbations of permeability in the area where the selelcted node is located but practically insensitive to perturbations of permeability in the bottom area. The total flux into the tunnel and the Darcy flux in vicinity of the tunnel are both most sensitive to the perturbations of permeability in the layer next to the top layer but are practically insensitive to perturbations of the permeability in the bottom layer. The application of the sensitivity analysis described in the present study is a simple uncertainty analysis for a simulated flow system. By selecting performance measures of interest it is possible to obtain some useful information about the sensitivities of those performance measures, which are significant to assessing the measurements of the input system parameters and investigating the behaviours and structures of a geohydrologic system.

The object of the present study is to give a tutorial example of a large scale sensitivity analysis for demonstration of the model developed by the authors. In order to facilitate the interpretation of the results as well as to reduce the investment in the time needed for the preparation of the input data and for the processing of the output data from the model, several simplifying assumption of the study case were made on the interior structure as well as on the boundary conditions of the flow domain: (i) The flow domain was treated as axi-symmetric. This assumption is considered fairly acceptable with regard to the actual shape of the flow domain, (ii) the tunnel system was geometrically represented by a disc. This assumption is unrealistic from the point of view that the actual shape of the tunnel system will be rectangular and that the individual tunnels will be separated by rock. A consequence of this assumption is that the effect of the tunnel system will be exaggerated and (iii) The rock permeability structure is described by only four layers. This simplification is a consequence of the fact that permeability is observed to generally decrease with depth at the study site. The assumption that the permeability distribution could be described by only four layers is somewhat unrealistic. A more realistic modelling should also account for local fracture zones as well as other major heterogeneities in the flow domain. It may be pointed out that the simplified assumptions mentioned above are not dictated by the actual capability of the model developed and that the on-going research work is directed towards to more complex flow problems.

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