$\Gamma = C_{p0}/C_p$. The synchronously reduced damping coefficient in the current analysis was developed in equations (35)–(51) with ν equal to the shaft rotational speed ω, and is nondimensionalized according to equation (34). Figure 5 shows the comparison of the nondimensional synchronously reduced damping coefficient $\bar{C}_{\xi\xi}$ versus $(1-\bar{h}_p)$ from Lund's analysis with the current approach. The agreement is good, further confirming Lund's trend showing decreased damping with decreased pad stiffness. Similar agreement was observed in the synchronously reduced stiffness coefficient and the load bearing capacity.

Summary and Conclusions

A method of incorporating pad flexibility effects, without the usual beam assumptions, into isothermal tilting pad bearing analysis has been developed. The method uses two-dimensional finite elements to determine pad deformations due to fluid film pressures. The pad assembly approach is extended to include single pad deformations, thereby eliminating iterations on the individiual pad angles in the assembled bearing analysis. The pad deformations are represented by a single degree of freedom, the change in the pad radius of curvature, thus the current method is an approximate approach. A method of calculating frequency reduced coefficients is presented. Synchronously reduced coefficients are in agreement with previous curved beam approximate methods as well as more rigorous iterative approach. The finite element pad model employed here provides more versatility in modeling circumferential nonuniform pad cross sections and the skewed and distributed boundary conditions which occur at the pivot-pad interfaces.

Acknowledgment

This work was funded by the Turbomachinery Research Consortium in the Dept. of Mechanical Engineering at Texas A&M University. The authors are grateful for their support of bearing simulation work.

References

Allaire, P. E., Parsell, J. K., Barrett, L. E., 1981, "A Pad Perturbation Method for the Dynamic Coefficients of Tilting-Pad Journal Bearings," *Wear*, Vol. 72, Oct., pp. 29–44.

Allaire, P. E., Nicholas, J. C., and Gunter, E. J., 1977, "Systems of Finite Elements for Finite Bearins," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 99, No. 2, Apr., pp. 187–197.

Barrett, L. E., Allaire, P. E., and Wilson, B. W., 1987, "The Eigenvalue Dependence of Reduced Tilting Pad Bearing Stiffness and Damping Coefficients," STLE Preprint No. 87-TC-3A-1, Presented at the STLE/ASME Tribology Conference, San Antonio, Texas, Oct. 5–8.

Bathe, K. J., 1982, *Finite Element Procedures in Engineering Analysis*, Chapter 5, Prentice-Hall, Inc., Englewood Cliffs, N.J.

Branagan, L. A., 1988, *Thermal Analysis of Fixed and Tilting Pad Journal Bearing Including Cross-Film Viscosity Variations and Deformations*, Ph.D. dissertation, University of Virginia, June.

Hashimoto, H., and Wada, S., 1985, "Performance of Large Scale Tilting-Pad Journal Bearings," *Bulletin of JSME*, Vol. 28, No. 242, Aug., pp. 1761–1767.

Kirk, R. G., and Reedy, S. W., 1988, "Evaluation of Pivot Stiffness for Typical Tilting-Pad Journal Bearing Designs," *Journal of Vib., Ac., Stress, and Rel. in Des.*, Vol. 110, pp. 165–171.

Kirlan, P., 1977, "The Effect of Pivot Stiffness on System Rotor Dynamics," Presented at the Eleventh Semiannual Meeting of the University of Virginia Industrial Sponsored Research Program on Rotor Bearing Dynamics, August.

Lund, J. W., 1964, "Spring and Damping Coefficients for the Tilting-Pad Journal Bearing," *ASLE Transactions*, Vol. 7, No. 4, pp. 342–352.

Lund, J. W., 1968, "Calculation of Stiffness and Damping Properties of Gas Bearings," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 90, pp 793–803.

Lund, J. W., and Pedersen, L. B., 1986, "The Influence of Pad Flexibility on the Dynamic Coefficients of a Tilting Pad Journal Bearing," ASME JOURNAL OF TRIBOLOGY, Vol. 109, No. 1, pp. 65–70.

Nilsson, L. R. K., 1978, "The Influence of Bearing Flexibility on the Dynamic Performance of Radial Oil Film Bearings," *Proceedings of the Fifth Leeds-Lyons Symposium on Tribology*, pp. 311–319.

Oh, K. P., and Huebner, K. H., 1973, "Solution of the Elastohydrodynamic Finite Journal Bearing Problem," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 95, pp. 342–352.

Parsell, J. K., Allaire, P. E., and Barrett, L. E., 1982, "Frequency Effects in Tilting-Pad Journal Bearing Dynamic Coefficients," *ASLE Transactions*, Vol. 26, pp. 222–227.

Rouch, K. E., 1983, "Dynamics of Pivoted-Pad Journal Bearing, Including Pad Translation and Rotation Effects," *ASLE Transactions*, Vol. 26, pp. 102–109.

Shapiro, W., and Colsher, R., 1977, "Dynamic Characteristics of Fluid-Film Bearings," *Proceedings of the Sixth Turbomachinery Symposium*, Texas A&M University, pp. 39–53.

Wilson, B. W., and Barrett, L. E., 1985, "The Effect of Eigenvalue-Dependent Tilt Pad Bearing Characteristics on the Stability of Rotor Bearing Systems," Report No. UVA/643092/MAE85/321, University of Virginia.

DISCUSSION

L. A. Branagan[1]

The authors have addressed an important refinement to the isothermal analysis of tilting pad bearings for industrial applications. They are to be complimented for the completeness and clarity of the derivation and for the inclusion of dimensional design criteria. I have a concern with the dynamic coefficient reduction, especially with regards to equation (18) which calculates the total pad radial stiffness as the sum of the film stiffness and structural stiffness. The model implied here is of springs in parallel, where the stiffest element governs the total stiffness. My conception of this calculation is that a springs in series model should be used, where the weakest element governs the total stiffness. I would be interested in the authors rationale for their calculation.

I have a second question concerning the application of the refinements possible with a pad finite element procedure. The magnitude of the changes introduced by variable pad thickness or by skewed boundary conditions would be helpful in determining whether the simpler curved beam analysis is sufficient. Of related interest would be some comments as to the limitations introduced by the "three point method" which reduces the complexity of the pad geometry and boundary conditions to a single number.

[1]Technical and Ecological Services, Pacific Gas and Electric Co., San Ramon, CA 94583.

Authors' Closure

The authors wish to thank Dr. Branagan for his response and insightful questions. In regard to the series versus parallel models of the pad curvature changes, if the fluid film is removed, a force is still required to change the radius of the pad. Consequently, the stiffness of the fluid film (related to the pad's curvature changes) is considered to be in parallel with the pad's structural stiffness, as indicated by equation (18). Placing these components in series would imply that with the removal of the film no force would be required to change the pad's radius of curvature.

Since the agreement was good between the Lund approach and the current method using a pad of constant cross section and a single point pivot (Figs. 4, 5), constant cross-section

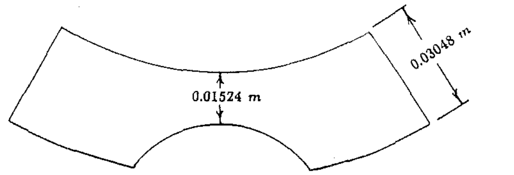

Fig. A1a Original geometry of industrial pad model

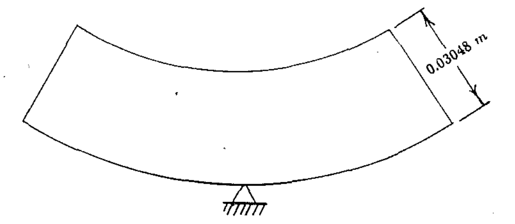

Fig. A1b Thick pad model of constant cross section

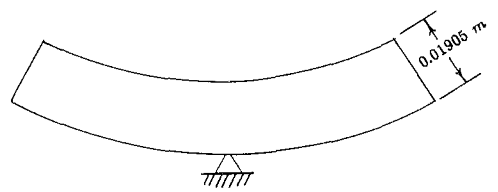

Fig. A1c Thin pad model of constant cross section

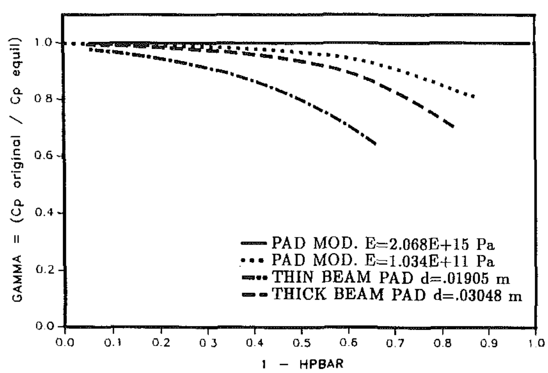

Fig. A2 Comparison of the change in pad radial clearance caused by static load, for a single pad of the industrial bearing with uniform pad models of different radial thicknesses and single point pivots, 60 deg pad, L/D = 0.5966, pivot offset 0.5

models of the industrial pad from Part II of this paper were developed rather than programing Lund's analysis. The comparison is made between two pad models with different constant cross sections and single point pivots with offsets of 0.5. Figures A1a, A1b, and A1c show the original pad model with the skewed boundary conditions at the pad-pivot interface, a thick beam-type model with a radial thickness of 0.03048 meters, and a thin beam-type model with a thickness of 0.01905 meters. A modulus of elasticity of E = 1.034E + 11 Pa. was used in the beam-type pad models, and the pad's were of constant thickness equal to the length of the bearing (0.09525 m). Both beam-type pad models exhibit more pad flexibility

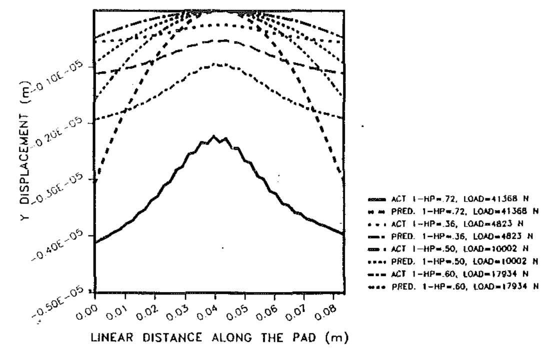

Fig. A3 Comparison of the finite element calculated and kinematic predicted displacements at lower loads for a single pad of the industrial bearing, 60 deg pad, L/D = 0.5966, pivot offset 0.5

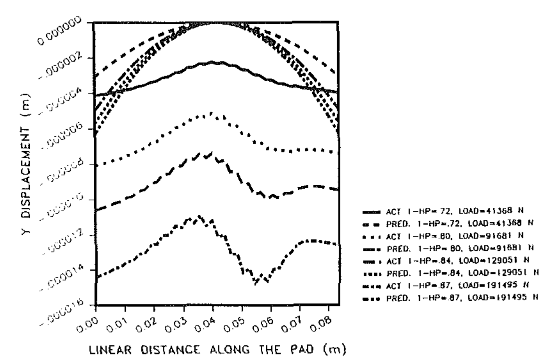

Fig. A4 Comparison of the finite element calculated and kinematic predicted displacements at higher loads for a single pad of the industrial bearing, 60 deg pad, L/D = 0.5966, pivot offset 0.5

than the current analysis as shown in Fig. A2. This is due to the additional support provided by the actual pivot compared to a single point pivot.

Another advantage of the present approach over the curved beam method is the ability to provide the analyst with information for discerning when the assumption of a "circular deformed pad" is violated. Figure A3 is a plot of Y-displacements of the bronze industrial pad (from Part II of this paper) calculated from the finite element pad model compared to the kinematically predicted Y-displacements for $1-\bar{h}_p$ from 0.60 to 0.72. The total load on the bearing was 41368 N. The calculated curves maintain a shape similar to the kinematically predicted curves. The kinematic equations predict zero displacement at the pivot, and the pad's upper surface is actually experiencing a "crush" Y-displacement at the pivot. The current analysis does not take this "crush" into account. Figure A4 shows these same calculated and predicted displacements as the load is increased. Note the change in scale of the vertical axis between Figs. A3 and A4. The calculated curves for $1-\bar{h}_p \leq 0.8$ maintain a shape similar to the kinematically predicted curves. However, for larger loads the calculated displacements indicate significant local deformations due to large pressure gradients, and the pad's deformation pattern is too complex to be described by a circular fit.