Discussion

Stresses in a Notched Strip Under Tension[1]

R. E. Peterson.[2] For design purposes considerable use has been made of the Neuber values of stress-concentration factor. For this reason it may be of interest to superpose a curve of such values on the author's Fig. 3; this is shown in Fig. 1 of this discussion. For $\lambda = 0$ the Neuber value is 3, the same as for a hole in an infinitely wide plate. Approaching $\lambda = 1$, the Neuber values are for a deep hyperbolic notch. An arbitrary function connects these end conditions, so that in the general region of $\lambda = 0.1$ to 0.4 the Neuber values are particularly questionable. It is not the intention of this discussion to attempt to draw any conclusions by comparison with experimental points; it is merely intended to show the differences between the author's results and those which we have been using.

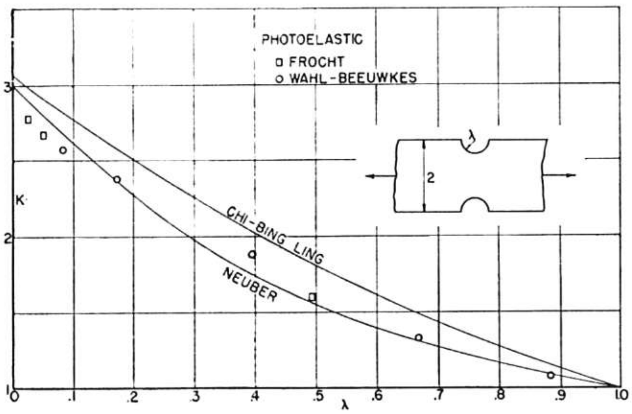

Fig. 1 Stress-Concentration Factor for a Notched Strip in Tension

Author's Closure

The author acknowledges with appreciation the discussion of Mr. Peterson.

In Fig. 1 of his discussion, the Neuber curve appears to be considerably lower than the author's theoretical curve. Also, save at both ends, this curve for its greater portion lies below the experimental points obtained by Frocht, Wahl, and Beeukes. It is not the intention of the author to reaffirm here the theoretical curve he obtained, but merely to indicate that Neuber's curve indeed underestimates the stress-concentration factor of the notched strip.

In experimental measurement, it is often found exceedingly difficult to obtain a very precise determination of maximum stress in a plate and usually a lower value is obtained, particularly when the stress in the plate drops rapidly from the point of maximum stress. For instance, a stress-concentration factor as low as 2.5 has been found photoelastically by Capper[3] in the case $\lambda = 0$, as compared with the theoretical value 3.065. This leads us to think that Neuber's curve probably underestimates the factor concerned.

To establish the statement positively, let us consider an infinite number of such notched strips matched sidewise and cemented together. We then have an infinite plate perforated by a series of equal and equally spaced circular holes. Such a plate has been investigated theoretically by Howland.[4] He gave a maximum stress $3.24T$ in the case $\lambda = {}^1/_2$ when the plate is under a tension T, acting transversely to the line of holes. Together with the end values, the stress-concentration factor of the plate is as follows:

λ	0	$^1/_2$	1
Stress-concentration factor	3	1.62	1

It is curious to note that such end values are the same as Neuber's, and besides, the value 1.62 is equal very nearly to Neuber's 1.56 and slightly higher. Thus Neuber's curve would indeed give closely, though perhaps slightly nonconservatively, the stress-concentration factor of such a plate. Now consider strips to be cut off one by one from both sides of the plate along the cementing lines until finally only a single strip is left. Each cut increases the maximum stress at the rim of the notch in the last strip by some amount; the nearer the cut, the larger will be the effect. This leads us to infer that Neuber's curve definitely underestimates the factor.

---

[1] By Chih-Bing Ling, published in the December, 1947, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 69, p. A-275.

[2] Mechanics Department, Westinghouse Research Laboratories, East Pittsburgh, Pa.

[3] "Photoelasticity," by E. G. Coker and L. N. G. Filon, Cambridge University Press, London, England, 1931, p. 560.

[4] "Stresses in a Plate Containing an Infinite Row of Holes," by R. C. J. Howland, Proceedings of the Royal Society of London, England, vol. 148, 1935, pp. 471–491.

Note on the Tightness of Expanded Tube Joints[1]

R. G. Lloyd[2] and G. J. Schoessow.[3] A new paper on expanded tube joints is always welcome, and the present one offers new analytical methods for consideration. In this paper particular reference is made to the paper on holding power and tightness by Messrs. Goodier and Schoessow.[4] The present paper claims a considerably simplified method of obtaining like results.

The author's results are apparently obtained solely from elastic theory with no use made of plasticity considerations. The paper is based upon his Equation [1], which is obtained from the elastic equation of equilibrium for an element in a cylindrical body by neglecting differentials of higher order and simplifying. This equation is then integrated for the specified boundary conditions. Together with the elastic equation for radial stress in a plate, these equations are combined to give a function which, when plotted, has some agreement with results from the Goodier-

---

[1] By G. Sachs, published in the December, 1947, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 69, p. A-285.

[2] Engineer, The Babcock & Wilcox Company, Barberton, Ohio. Jun. ASME.

[3] Engineer, The Babcock & Wilcox Company, Barberton, Ohio. Mem. ASME.

[4] "The Holding Power and Hydraulic Tightness of Expanded Tube Joints," by J. N. Goodier and G. J. Schoessow, Trans. ASME, vol. 65, 1943, pp. 489–496.

Schoessow paper. The extent of agreement may be seen by the graphs, Fig. 1 of this discussion. Data from the Goodier-Schoessow paper were converted to a comparable abscissa basis of b/a.

This same equation just discussed also appears in the Appendix of the Goodier-Schoessow paper in slightly different form, but may be made to read the same by a simple transposition. The equation is basic for any investigation of this nature. In the Goodier-Schoessow paper, the plastic stress functions are then used to obtain a different integral solution of the equation, these results then being plotted. This serves as the basis of theoretical discussion in the paper.

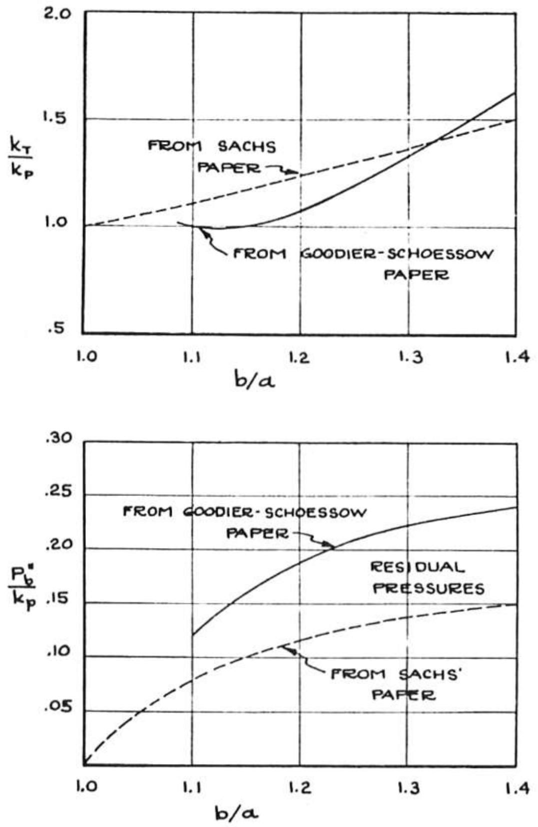

FIG. 1

In the present paper the Goodier-Schoessow calculations are described as being a rather elaborate analysis. If the mathematics alone is considered, that in each paper seems very comparable regarding length and complexity. The Goodier-Schoessow paper devotes some of the text to development of calculations for frictional holding power, as well as discussion of important results arising from considerations based upon plastic theory. For example, plastic flow begins at the hole when $s_0/\sqrt{3}$ is reached, s_0 being yield stress. As pressure increases the plastic zone spreads, but does not do so indefinitely. At $1.75r$ of the hole, the plastic zone no longer spreads but comes to a stop, any further increases being expended in the form of axial extrusion. This would seem to be one worth-while result in considering the effect of rolling an adjacent tube closely spaced to one already rolled, since it is well known that rolling one tube can loosen an adjacent one already made tight.

Other knowledge developed from the plastic basis of investigation would be lost, it seems, if only elastic considerations are used. In other words, the present paper has apparent brevity but is not as comprehensive as might be desired, nor does the method seem applicable to new problems and test results now before us, which will almost certainly call for plasticity considerations for solution and interpretation. To obtain the particular results in the papers in question, it is doubtful whether the one method is more advantageous or not.

These papers, and the literature in general, have about the same assumptions. Both papers make the assumption of uniform internal pressure. Also, the effect of strain hardening is usually neglected although effects of the omission are discussed. Neither are cases considered where the initial yielding due to rolling is so great that plastic yielding is encountered after elastic unloading as pressure is removed. However, the material and methods of the Goodier-Schoessow paper would furnish the necessary basis for such inquiries whereas the elastic method of the present paper would not lend itself to these inquiries.

One further assumption that is usual is the neglect of stress and plastic-deformation phenomena in the axial direction of the tube, although the Goodier-Schoessow paper discusses briefly the effect of friction in resisting this deformation. It may well be that some of these assumptions call for reconsideration, particularly the latter one, since current work and new results seem to indicate new importance of this factor. Any future work in this direction would most certainly be based upon plastic-deformation theory.

It is pleasing to receive a new paper on this very important subject, and every effort to contribute to the existing literature is commendable. It is to be hoped that future papers will be forthcoming and that with increasing discussion still more valuable information may be brought to the field.

AUTHOR'S CLOSURE

Referring to discussion by Messrs. Lloyd and Schoessow, the author is at a loss to understand the basis of their objections. Their claim that "the author's results are apparently obtained solely from elastic theory" is evidently wrong. Equation [1] of the paper, used also by Goodier and Schoessow, is not restricted to elastic equilibrium but is the basic condition for any equilibrium of forces in a cylindrical body. It does not neglect differentials of higher order and it is not simplified in any respect.

The difference between Goodier-Schoessow's approach and that by the author results from the different conditions of plasticity used. The only advantage the author claims is that simple analytical expressions have been obtained for the desired quantities.

Neither paper considers that the actual conditions may deviate considerably from that of plane stress upon which the calculations are based. Dependent upon the dimensional ratios, the actual pressures may be considerably higher than those calculated, because of the presence of axial stresses in both the plate and the tube. It appears that an experimental investigation of this problem would be timely.

Approximate Solutions for Symmetrically Loaded Thick-Walled Cylinders[1]

J. H. HITCHCOCK.[2] It has been indicated that a rigorous mathematical solution of this interesting problem is possible, and that the authors' recourse to empirical methods of evaluating coefficients is to be deplored. Lest the merit of the authors' work be obscured by this academic consideration, and from the viewpoint of the machine designer, the writer would like to state that

---

[1] By C. W. MacGregor and L. F. Coffin, Jr., published in the December, 1947, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 69, p. A-301.

[2] Director of Research, Morgan Construction Company, Worcester 5, Mass. Mem. ASME.