Conclusions

A method of calculating the effect of cooling on the efficiency of a multistage turbine has been presented and found to give results in substantial agreement with accurate calculations. The results show the importance of the effect in turbines with blades cooled substantially below the gas temperature, a 4–5 per cent reduction in turbine efficiency being obtained in the example quoted.

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3 "Some Factors in the Use of High Temperatures in Gas Turbines," by T. W. F. Brown, Proceedings of The Institution of Mechanical Engineers, vol. 162, 1950, p. 167.

4 "A New Method of Calculation of Reheat Factors for Turbines and Compressors," by J. Kaye and K. R. Wadleigh, *Journal of Applied Mechanics*, Trans. ASME, vol. 73, 1951, p. 387.

Discussion

L. S. Dzung.[6] Professor Hawthorne's papers are certainly a much welcomed contribution to a timely subject. The treatment is accurate and the assumptions and limitations are well recognized and defined.

The assumption of constant friction coefficient is justified on account of its simplification. The mathematics could be further simplified by making another assumption, which is perhaps just as good, namely, by assuming constant polytropic efficiency.

Using the author's symbolism (except the obvious difference in sign of heat and work) one may write in differential form the First Law ($K = c^2/2$ is the kinetic energy)

$$dh + dK = dw + dq$$

the Second Law (d_is is the internal irreversible entropy production)

$$Tds = dq + Td_is$$

the Gibbsian fundamental equation

$$dh = Tds + vdp$$

and define for a steady-flow process the three inexact differentials, i.e., the hydraulic work (per unit mass)

$$dy = v\,dp \qquad [26]$$

the internal energy transformation

$$dz = dw - dK \qquad [27]$$

the dissipation

$$dj = Td_is \qquad [28]$$

which is essentially positive. In case of turbine, it is more convenient to define y and z with the opposite sign, so that

$$dz = dy \pm dj$$

where the minus sign is used for turbine.

The polytropic efficiency for turbine may be defined as

$$\eta = dz/dy \qquad [29]$$

Now assume

$$dq = \vartheta dj \qquad [30]$$

ϑ is negative in case of cooling. The polytropic coefficient φ, defined by[7]

$$\varphi = dh/dy \qquad [31]$$

can be expressed as

$$\varphi = \eta + (1 - \eta)\vartheta \qquad [32]$$

which is identical with Equation [22] of author's second paper.

The "reheat factor," being the ratio of hydraulic work (integration of Equation [26] along actual state path) to isentropic work (integration along isentropic path) can be written as

$$F = \frac{\nu(1 - r^{1/\nu})}{\lambda(1 - r^{1/\lambda})}$$

$\lambda = k/(k-1)$, k is isentropic exponent, ν depends on λ and φ.[7] For ideal gas, this formula reduces to Equation [10] in author's second paper. The present formula is applicable to a stage or to the whole turbine. Correction to finite number of stages is not necessary. This formula would be useful if φ and ϑ were constant along the whole turbine, which is seldom the case. Actually, it is necessary to take the variation of ϑ and φ into account as follows.

The change of state in a cooled turbine with constant blade wall temperature T_w may be obtained by writing

$$\vartheta = \frac{bc_p(T - T_w)}{2K'} + \frac{br'}{2}, \quad b = \frac{\mathrm{St}}{(f/2)} \qquad [33]$$

b takes care of cases where Reynolds' analogy does not strictly apply. St is Stanton's number defined by the left-hand side of author's Equation [32], first paper. K' is the kinetic energy of the relative flow, r' is the recovery factor.

Substitution of Equation [33] into [30], [31], [32] and using ideal-gas equations $pv = RT$, $dh = c_pdT$ result in

$$\frac{d\tau}{\tau[\gamma + \beta(\tau - 1)]} = \frac{\eta}{\lambda}\frac{dp}{p} \qquad [34]$$

where

$$\tau = T/T_w$$

$$\beta = bc_pT_w\left(\frac{1}{\eta} - 1\right)\Big/2K'$$

$$\gamma = 1 + \left(\frac{1}{\eta} - 1\right)\frac{br'}{2}$$

This integrates readily between inlet 1 and exhaust 2, assuming constant β, into

$$\log\frac{\beta\tau_1 - (\beta - \gamma)}{\beta\tau_2 - (\beta - \gamma)} - \log\frac{\tau_1}{\tau_2} = (\beta - \gamma)\frac{\eta}{\lambda}\log\frac{p_1}{p_2}\ (\beta \neq \gamma) \qquad [35]$$

This gives a relation between temperature and pressure along the state path.

The hydraulic work can be obtained by integrating Equation [26] using [34]; the energy transformation is then obtained by multiplication by η. The result is

[6] Brown, Boveri & Company, Ltd., Baden, Switzerland.

[7] "Thermostatische Zustandsänderungen des trockenen und des nassen Dampfes," by L. S. Dzung, *Zeitschrift für angewandte Mathematik und Physik*, vol. 6, 1955, p. 207.

$$\frac{z}{c_p T_w} = \frac{\eta y}{c_p T_w} = \frac{1}{\beta} \log_e \frac{\beta\tau_1 - (\beta - \gamma)}{\beta\tau_2 - (\beta - \gamma)} \quad (\beta \neq 0) \ldots [36]$$

The turbine work w is just z corrected to change of kinetic energy between inlet and exhaust.

Equations [35] and [36] together give the turbine work as function of pressure ratio with one of the τ's as parameter. The special case of $\beta = \gamma$ in Equation [35] and $\beta = 0$ (adiabatic) in Equation [36] can be easily deduced.

By nondimensionalizing the turbine work as in Equation [36] it is emphasized that the work is increased by cooling at constant blade wall temperature and hence constant thermal stress. Author's (adiabatic) efficiency makes this point obscure.