Finally, it is proposed for practical applications that the heat-transfer rate is proportional to $[\text{Pr}\ h_{fg}/c_p(t_s - t_w)]^{1/4}$, i.e.,

$$\text{Nu}/C^{1/4} = K[\text{Pr}\ h_{fg}/c_p(t_s - t_w)]^{1/4} \tag{49}$$

The constant K has been determined empirically from the numerical results for three cases:

K = 1.8465, upper stagnation point
K = 1.7354, average over the upper hemisphere
K = 1.3499, average over the entire sphere

The uncertainty of equation (49) is within 2 percent of the range of $c_p(t_s - t_w)/h_{fg}$ from 0.001 to 1 and Pr > 1.

Equation (49) can be written in an alternate form. For the Nusselt number at the upper stagnation point it is

$$\text{Nu} = 1.098\left[\frac{g(\rho - \rho_v)h_{fg}D^3}{k\nu(t_s - t_w)}\right]^{1/4} \tag{50}$$

The average Nusselt number over the upper hemisphere is

$$\overline{\text{Nu}} = 1.032\left[\frac{g(\rho - \rho_v)h_{fg}D^3}{k\nu(t_s - t_w)}\right]^{1/4} \tag{51}$$

The average Nusselt number over the entire sphere is

$$\overline{\text{Nu}} = 0.803\left[\frac{g(\rho - \rho_v)h_{fg}D^3}{k\nu(t_s - t_w)}\right]^{1/4} \tag{52}$$

It is interesting to compare the overall Nusselt number of a sphere, equation (52), with that of a vertical plate and a horizontal cylinder. From [1, 2] we have

$$\overline{\text{Nu}} = 0.943\left[\frac{g(\rho - \rho_v)h_{fg}L^3}{k\nu(t_s - t_w)}\right]^{1/4} \tag{53}$$

for a vertical plate of height L, and

$$\overline{\text{Nu}} = 0.733\left[\frac{g(\rho - \rho_v)h_{fg}D^3}{k\nu(t_s - t_w)}\right]^{1/4} \tag{54}$$

for a horizontal cylinder of diameter D. Therefore, for a given Prandtl number and subcooling, the average Nusselt number of a sphere of diameter D is 8.8 percent higher than that of a horizontal cylinder of the same diameter D, and is 17.6 percent lower than that of a vertical plate of height D.

It should be pointed out that there are certain conditions for which the analysis does not apply. For example the surface-tension term which is neglected in the momentum equation can be important for small spheres. Bromley [2] has indicated that the surface tension also produces droplets covering appreciable areas on the lower stagnation region.

References

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Discussion

J. H. Lienhard[2] and V. K. Dhir[3]

We wish to point out a prior publication on this subject. In 1970 we solved the problem of laminar condensation on a sphere.[4] That paper showed how to replace g with an effective gravity, g_{eff}, in the Nusselt–Rohsenow expression for condensation on a vertical plate. Thus the expression

$$\text{Nu} = 0.707\left[\frac{g_{\text{eff}}(\rho - \rho_v)h_{fg}'x^3}{k\nu(t_s - t_w)}\right]^{1/4} \tag{55}$$

where

$$g_{\text{eff}} \equiv x(gr)^{4/3}\left[\int_0^x g^{1/3}r^{4/3}\,dx\right]^{-1} \tag{56}$$

gave the condensing heat transfer on any axisymmetric (or plane, if $r \to \infty$) body with any variation of gravity, $g(x)$, along the x coordinate. We worked out eight examples, including the sphere, for which the result was

$$\overline{\text{Nu}} = 0.785\left[\frac{g(\rho - \rho_v)h_{fg}'D^3}{k\nu(t_s - t_w)}\right]^{1/4} \tag{57}$$

Three points should be made in comparing this result with Professor Yang's:

1 His equation (52) is 2.3 percent above our result. One might feel that his full boundary-layer treatment has provided a minute improvement over our simple adaptation of the Nusselt–Rohsenow computation.

2 However, in finally obtaining equation (52), he has neglected the sensible heat absorbed by the film. The latent heat for condensation should be corrected to

$$h_{fg}' \equiv h_{fg} + 0.68c_p\Delta T \tag{58}$$

so part of his improvement has already been lost in this omission.

3 The only real ground gained by the present analysis would then appear to be that it can be used when Pr is well below unity—a regime in which the Nusselt–Rohsenow analysis fails. However, even that is misleading. As we pointed out in our paper, this region is one in which neither analysis can be used because of temperature-decrement problems that enter at the outer interface when Pr is small.

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[4] Dhir, Vijay, and Lienhard, John, "Laminar Film Condensation on Plane and Axisymmetric Bodies in Nonuniform Gravity," JOURNAL OF HEAT TRANSFER, TRANS. ASME, Series C, Vol. 93, No. 1, Feb. 1971, pp. 97–100.

Author's Closure

The author would like to thank Drs. Lienhard and Dhir for their comments on the average Nusselt number over a sphere. The agreement or disagreement between the full boundary-layer treatment and the Nusselt–Rohsenow treatment is well established for the cases of a flat plate and a horizontal cylinder. Condensation on spheres is under the same situation. It should be pointed out that the boundary-layer treatment possesses a certain advantage in describing the local variations of heat and mass transfer rate, particularly near the upper stagnation point. For example, the assumption that $\delta = 0$ at $x = 0$ is not required in the boundary-layer treatment but is necessary for the Dhir-Lienhard analysis. This assumption is not realistic and will introduce a certain error on the average Nusselt number.

The correction of the latent heat for condensation is necessary only if the subcooling parameter $(c_p\Delta T)$ is comparable with h_{fg}. However, in most cases the sensible heat absorbed is much less than the latent heat transfer. Under these conditions, the correction does not provide significant improvement.