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A Simple Approximation of the Hydrograph Downstream of a Flooded Area

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A simple model for estimating the hydrograph at the outlet of a reach crossing a flood-prone area is investigated. The first part of the model involves routing of the inflow hydrograph through an ideal reach with infinite vertical extension of the walls. The second part routes the flow which determines a two-dimensional flooding (outbank flow) through a linear reservoir whose storage coefficient is linked with geometric features of the flood plain. The model was applied to various geometric configurations representative of real areas in Central Italy. Its calibration and validation were carried out by comparison of the results of a diffusion hydrodynamic model. The hydrodynamic model incorporates two-dimensional overland flow and one-dimensional open channel flow. The two models were in substantial agreement, as revealed by similar shape characteristics of the corresponding hydrographs and by the values of the peak runoff error which were lower than 15% of the maximum outbank flow. Model validity was also confirmed by a comparison of its results with observed outflow hydrographs.

Introduction

The main emphasis in basin runoff models is usually on synthesizing the flood hydrograph at the outlet, and little consideration is given to local flooding within certain portions of the basin. There exists a multitude of cases where this local phenomenon is important and may also significantly influence the flood hydrograph at the outlet. Sometimes these cases may be associated with failure of a small dam or breach of a levee. Frequently such cases are linked with a channel geometry characterized by too restricted sections after the main junctions, substantial contraction of sections and levees constructed intermittently along channel banks. Many basins in Central Italy where real time flood forecasting is needed to mitigate flood damages exhibit such characteristics. When a large flood event occurs and a channel overflows, the outbank flow undergoes transformation when travelling through and flooding the plains, and then returns to the channel. Modeling of this local phenomenon constitutes the subject matter of this study.

In a recent study Hromadka and Yen (1986) developed a diffusion hydrodynamic model (DHM) coupling two-dimensional overland flow and one-dimensional open channel flow. Similar two-dimensional models were used by Xanthopoulos and Koutitas (1976) and Hromadka *et al.* (1985). The DHM can be used to describe local flooding. However, in the context of real time flood forecasting, where adaptive estimation of model parameters is required (O'Connell and Clarke 1981, Kitanidis and Bras 1980, Reed 1984, Corradini and Melone 1986), the DHM may not be suitable in its present form. Its two-dimensional formulation with explicit integration scheme, when incorporated in rainfall-runoff models, would require prohibitively large computational time because of the adaptive estimation of parameters at each time origin of forecast. The use of a two-dimensional approach with an implicit integration technique of the diffusion approximation (Akan and Yen 1981) of the momentum equations reduces the computational time but would still remain impractical for real time flood forecasting. Furthermore, the implicit technique requires a matrix solution which, because of the two-dimensional grid, would involve large dimensions requiring an even greater computer memory. Therefore, what is needed is a simpler approach which, although somewhat less accurate, is efficiently adaptable for real time flood forecasting. In principle one could select a multilinear approach (Keefer and McQuivey 1974, Becker 1976) with bankful level used as a threshold in modeling the significant differences in flow characteristics of the channel and the adjacent inundation area. This approach has been used in real- world cases where historical time series of inflow-outflow data were available for model calibration (Becker and Kundzewicz 1987). However, most cases involve ungaged reaches and therefore, the model parameters must be estimated from physical consideration (Kundzewicz 1986).

The objective of this paper is to investigate the performance of a simple multilinear hydrologic model coupling overland and channel flows. The model was calibrated and validated by expressing its parameters in terms of physical system characteristics and by comparing with the results of the DHM. The choice of the DHM was supported by satisfactory comparison of one-dimensional models based on the complete dynamic wave equations and on the diffusion wave approximation (Akan and Yen 1981, Hromadka and Yen 1986), from which only minor discrepancies were found for a wide range of slopes (0.0005-0.01). Both the models were applied using reaches and adjacent plains, as well as input hydrographs and rating curves, whose main features were of principal interest. The outflow hydrographs computed by the DHM and the proposed model are designated henceforth as the actual and approximated hydrographs, respectively.

Statement of the Problem

A Short Description of the DHM Model

The two-dimensional DHM model is based on the assumptions of constant fluid density, absence of sources or sinks in the flow field, negligible effect of the Coriolis force, hydrostatic pressure in the vertical cross section and small slope of the bed surface with cos θ ≅ 1. In addition it ignores the inertial terms of the momentum equations. With reference to a Cartesian coordinate system, the DHM is formulated as (Hromadka and Lai 1985, Hromadka and Yen 1986)

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} \equiv \frac{\partial H}{\partial t} \qquad (1)$$

where H is the water surface elevation, t is the time and K_x and K_y are conduction parameters defined through the Manning formula and computed, in the metric units, as

$$K_x = \frac{1/n\,(h^{5/3})}{|(1/\alpha)\,\partial H/\partial x|^{1/2}} \qquad (2)$$

$$K_y = \frac{1/n\,(h^{5/3})}{|(1/\beta)\,\partial H/\partial y|^{1/2}} \qquad (3)$$

where h is the water depth, n is the Manning roughness coefficient, and α and β are quantities dependent on the flow direction but assumed equal to unity in computations. Flow in a reach is further simplified using a one-dimensional approach, and is represented in the x-direction as

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} \equiv \frac{\partial A_x}{\partial t} \qquad (4)$$

where A_x is the cross-sectional area of flow and K_x is given by

$$K_x \equiv \frac{1/n\,(A_x R^{2/3})}{|\partial H/\partial x|^{1/2}} \qquad (5)$$

with R being the hydraulic radius. The solution procedure is based on an explicit integration scheme employing an algorithm for the self-selection of the time step which can vary during the routing procedure.

The Proposed Model

We consider a reach of length L, width B and slope S, where flow may be assumed to be one-dimensional. We denote the quantities at the upstream end of the reach by the upper index i. If inflow, Q^i, is known at each time t, in the commonly used approximation of steady uniform-flow the corresponding water depth may be derived by the rating curve expressed by Manning's equation, as

$$h^i = \left(\frac{Q^i n}{B S^{1/2}} \right)^{3/5} \tag{6}$$

For $h^i \leqslant h_c$, with h_c channel depth, Q^i may be routed through the channel, whereas for $h^i > h_c$ h^i is divided into two parts h_c and the outbank water depth h_e^i. The corresponding inflows, $Q^i(h_c)$ and $Q^i(h_e^i)$, are defined using the rating curve, with the latter flow expressed as

$$Q^i(h_e^i) = Q^i(h^i) - Q^i(h_c) \tag{7}$$

In order to derive the outflow hydrograph, $Q(t)$, from $Q^i(t)$ the input hydrograph is cut at the flow value $Q^i = Q^i(h_c)$ and two input hydrographs are considered. The first, $Q_c^i(t)$ approximating the inbank flow, is routed through the channel by a mechanism synthesized by a function $f(t\text{-}\tau)$ which denotes the response of the channel to an input at its upstream end occurring at time τ and which is represented by the Dirac delta function. The corresponding flow at the outlet, $Q_c(t)$, for a linear channel is expressed by

$$Q_c(t) = \int_0^t Q_c^i(\tau) f(t-\tau) d\tau \tag{8}$$

The remaining hydrograph $Q_e^i(t)$, in principle, should be routed through the flood plain surrounding the channel. However, this procedure is cumbersome and is simplified by estimating the flow at the outlet $Q_e(t)$ by first routing through the channel and then routing through a linear storage element placed between the end of the channel and the outlet. The quantity $Q_e(t)$ may be written as

$$Q_e(t) = \int_0^t g(\eta) d\eta \int_0^{t-\eta} Q_e^i(\tau) f(t-\eta-\tau) d\tau \tag{9}$$

with $g(\eta)$ being the response of the storage element to an input flow at the time η=0 represented by the delta function. Using Eqs. (8) and (9) $Q(t)$ may be computed as

$$Q(t) = Q_c(t) + Q_e(t) \tag{10}$$

with the functions f and g appropriately specified. Many approaches may be used for *f*. A translation routing may be adequate for short reach lengths and therefore *f* in this case is expressed by

$$f(\tau) = \delta(\tau - L\,C^{-1}) \tag{11}$$

where *C* is the celerity given by $C = (3/2)v$ with *v* water velocity. For this parameter an average value in the range of the flows involved may be used.
The function *g* is represented by

$$g(\eta) = K^{-1}\,\exp(-\eta K^{-1}) \tag{12}$$

where *K* is the storage coefficient, and is influenced by many factors. We evaluate *K* by relating it to flood plain geometric characteristics as

$$K \equiv a\,A\,S^{-1}\,L^{b-1} \tag{13}$$

where *A* denotes the flood plain area, *S* is its slope assumed equal to the channel slope; and *a* and *b* are coefficients determined by calibration. The choice of Eq. (13) was supported by a preliminary analysis of the results obtained by the DHM.

Case Studies and Parameter Estimation

Straight reaches, without levees, of depth 5 m and with rectangular flood plains on one side were selected. Each reach was assumed to have uniform slope in the range 0.0005-0.002. Reach length and flood plain area were considered in the range 1-4 km and 1-16 km^2, respectively, while a constant reach width of 40 m was used. For each slope, two input hydrographs involving maximum outbank water depths of 1 m and 2 m, respectively, were selected. These maximum outbank depths, together with the above geometric features, may be considered representative of real areas in Central Italy. In fact, for example, the Topino River, which is the main tributary of the Upper Tiber River, involves a levee structure practically discontinuous with height increasing upstream from zero to 2 m in an area of average slope 0.0011. Furthermore, flood-prone plains are frequently located on one side of river. The main characteristics of the input hydrographs for a few of the slopes investigated here are summarized in Table 1. The cases characterized by slope 0.001 and peak flow 884 m^3s^{-1}, forming the calibration set, were used for estimating the model parameters, while the remaining ones were utilized for testing the model. Four parameters were fixed in advance: the Manning roughness coefficient, *n*, the celerity, *C*, of the translation routing equation and the parameters *a* and *b* of Eq. (13). The parameter *n* was assumed to be 0.04 for flood plains and 0.03 for reaches. The last value produced theoretical rating curves that matched the experimental ones determined in a few sections of the Upper Tiber River. The value for flood plains was selected on the basis of the vegetation cover existing on the flood-prone areas

Table 1 – Some characteristics of inflow hydrographs. Q_c^i and t_c^i denote the flow producing the beginning of flooding and its corresponding time of occurrence, respectively.

Slope	Peak flow (m^3s^{-1})	Time to peak (h)	Q_c^i (m^3s^{-1})	t_c^i (h)
0.0005	500	8.5	376	4.0
	625	8.5	376	4.0
0.0010	720	9.5	531	4.8
	884	9.5	531	4.8
0.0020	993	11.0	751	6.7
	1,250	11.0	751	6.7

adjacent to the same river. The parameter C was roughly equated to the velocity estimated by the Manning approximation for $h = h_c$. The remaining parameters, a and b, were determined from calibration. In particular, for each case of calibration an optimal value of the storage coefficient K, K_i, was first estimated through a trial-and-error procedure. This procedure was employed by minimizing an objective function represented by the sum of the squares of the errors in flow at the outlet associated with the use of the proposed model instead of the DHM. Then, optimal values of a and b were computed by applying the same procedure and minimizing a similar objective function referring to the errors between each K_i and the corresponding value estimated by Eq. (13). With K in h, L in km and A in km^2, the values of 1.22×10^{-3} and 0.2 for a and b, respectively, were determined.

Model Verification

Computations of flow hydrographs downstream of flood areas were performed for each of the selected cases except those belonging to the calibration set. For computations by the proposed model a 30-min time step was found adequate, while for the DHM a time step ranging from 0.1 to 20 s had to be used together with a uniform grid size of 150 m. Minimum values larger than 0.1 s caused unacceptable errors in the computed flood depths. The above values of time step and grid size resulted in a very long computational time for the DHM on Hewlett Packard HP1000 computer system. For example, for a square flood plain of dimension 4 km and slope 0.001 the computational time was comparable with the duration of inflow

Table 2 – Peak flow and time to peak for a slope of 0.002. Comparison of values computed by the diffusion hydrodynamic model and the proposed model.

Flood plain		DHM model		Proposed model	
Width	Length	Peak flow (m^3s^{-1})	Time to Peak* (h)	Peak flow (m^3s^{-1})	Time to peak* (h)
		Peak value of inflow 993 (m^3s^{-1})			
1	1	988	5.6	988	5.6
1	2	988	5.6	988	5.6
1	4	989	6.1	984	6.1
2	1	970	6.1	980	6.1
2	2	972	6.4	975	6.1
2	4	972	7.1	969	7.1
4	1	938	6.1	956	7.1
4	2	940	6.6	947	7.1
4	4	932	7.6	939	8.1
		Peak value of inflow 1,250 (m^3s^{-1})			
1	1	1,234	5.8	1,240	5.3
1	2	1,233	5.8	1,235	5.3
1	4	1,236	6.3	1,233	5.8
2	1	1,194	6.5	1,222	6.2
2	2	1,198	6.8	1,213	6.3
2	4	1,202	7.3	1,198	6.8
4	1	1,110	7.0	1,153	7.3
4	2	1,118	7.3	1,155	7.3
4	4	1,120	8.0	1,134	7.7

* Values referred to t_c^i of Table 1.

hydrograph. An evaluation of the performance of the proposed model was made using errors on peak runoff, ε_{Qp}, and time to peak, ε_{tp}, together with a visual examination of the approximated and actual hydrographs. These errors (throughout the text computed ignoring algebraic signs) were defined through quantities characterizing the beginning of flooding, namely $Q^i(h_c)$ and its corresponding time of occurrence, t_c^i, respectively. In particular, ε_{Qp} was referred to the difference of the actual peak flow, Q_p^a, and $Q^i(h_c)$, and ε_{tp} to that of the actual time to peak, t_p^a and t_c^i. Details of model results are given in Table 2 for the maximum slope of 0.002. Their behaviour with changes in flood plain width, B_F, and in input hydrograph is well representative of that obtained for the other slopes investigated here. For a fixed value of the length of flood plain and $S = 0.002$ a reduction in Q_p^a from a few per cent to about 25 % of Q_e^i, almost independent of peak flow input, was obtained with increases in width from 1 to 4 km. Such a reduction was found to increase up to 45 % for $S = 0.001$ (see also Table 3) and to 75 % for $S = 0.0005$. On

Table 3 – Extreme values of the errors on peak flow and time to peak for different slopes and input hydrographs. Reduction of peak flow is also given.

Slope	Peak value of inflow (m^3s^{-1})	Peak reduction by DHM model* (%)	Peak error* (%)	Time to peak error* * (%)
0.0005	500	14.5-64.3	1.4- 6.2	0.0-28.6
	625	13.7-75.3	0.9-12.1	0.0-12.5
0.0010	720	5.3-44.5	0.0-14.6	0.0-13.9
0.0020	993	1.7-25.2	0.0- 9.6	0.0-16.4
	1,250	2.8-28.1	0.4-12.0	0.0- 8.6

* values referred to Q_c^i of Table 1.
** values referred to t_c^i of Table 1.

the other hand, for a fixed value of the width an increase in length from 1 to 4 km produced no significant variation of peak flow reduction with $S = 0.002$ and only a modest increase of the same with $S = 0.0005$. These results support the choice of Eq. (13) for representing the storage coefficient which determines the peak flow reduction.

The results obtained by the proposed model were very similar to the actual ones. The values of ε_{Qp} and ε_{tp} were found to be rather limited and nearly independent of the length and width of flood plains. From the results of Table 2 and referring to the inflow hydrograph with the peak of 1,250 m^3s^{-1}, the ε_{Qp} values in per cent averaged over the cases with the same width (length), $<\varepsilon_{Qp}>$, range from 0.7 for B_F = 1 km (1,8 for L = 4 km) to 8.6 for B_F = 4 km (6.5 for L = 1 km) and the ε_{tp}, $<\varepsilon_{tp}>$, from 2.7 for B_F = 4 km (5,3 for L = 2 km) to 8.4 for B_F = 1 km (6.2 for L = 4 km). For other slopes the behaviour of the errors with increases in B_F and L were not well-defined while, as shown in Table 3, their magnitude was found to be up to 14.6 % and 28.6 % for ε_{Qp} and ε_{tp}, respectively. However, for the quantities $<\varepsilon_{Qp}>$ and $<\varepsilon_{tp}>$ referring to the ensemble of the cases investigated for each inflow hydrograph, the computed values were in the rather narrow ranges 3.0-7.6 % and 3.6-8.7 %, respectively. The shape characteristics of the outflow hydrographs were appropriate. This may be observed in Figs. 1-3, where a comparison of actual and approximated hydrographs is shown for three sample cases. The approximated curves substantially match the actual ones. Minor discrepancies are associated with the lower slopes, as illustrated in Fig. 1 where for the larger widths the approximated hydrographs experience a faster recession. Because the model accuracy appears to be unrelated with the geometric features of the flood plains, it follows that the proposed model appropriately incorporates them. In addition, the slight

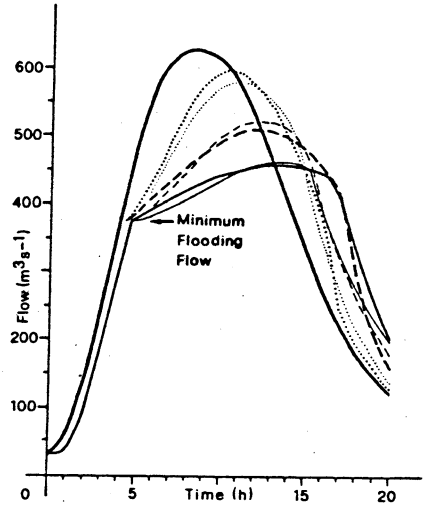


Fig. 1. Slope 0.0005 and input peak flow 625 m^3s^{-1}.

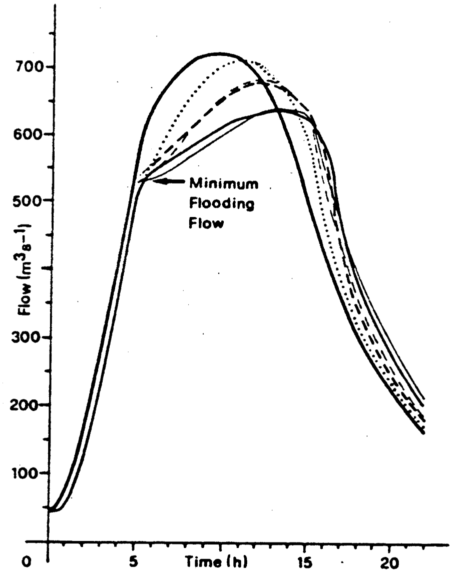
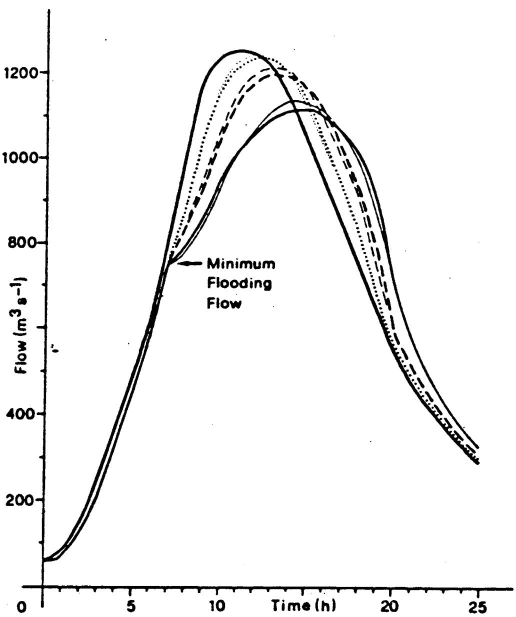


Fig. 2. Slope 0.001 and input peak flow 720 m^3s^{-1}.



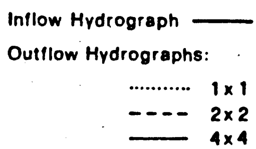


Fig. 3. Slope 0.002 and input peak flow 1,250 m^3s^{-1}.

Comparison of actual hydrographs derived by the diffusion hydrodynamic model (heavier lines) and approximated hydrographs computed by the proposed model for various widths and lengths, in km, of flood plain.

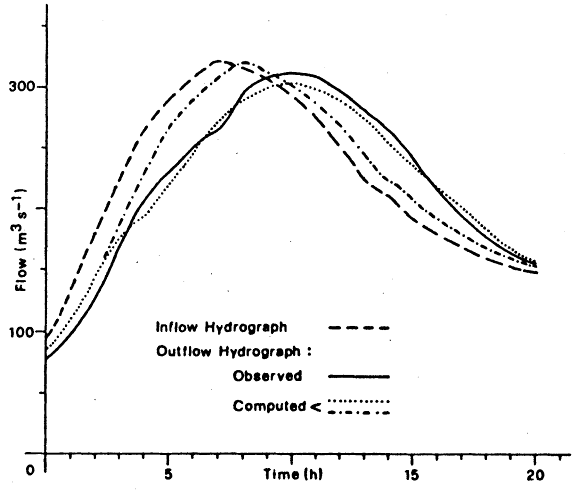


Fig. 4. Comparison of observed and computed outflow hydrographs for the event of February 25, 1984 on the Topino River at Bettona. Curves computed by the proposed model (.......) and considering a linear channel without flooding (-·-·-) are shown.

variation in accuracy computed for different inflow hydrographs applied to the same geometric characteristics further supports the adoption of a storage coefficient of the linear reservoir independent of flow. An analysis of the changes in the outflow hydrographs produced by a distribution of the rectangular flood plain area on both sides of the reaches was also made. For fixed values of S, B_F and L only a slight modification of the actual hydrographs was found which indicates that the model proposed here, with K given by Eq. (13), is adequate also for this geometry.

Indications of model-structure adequacy were also derived from a comparison with observed data. The events of February 25, 1984 over the Topino River basin and February 1, 1986 over the basin of Tiber River at Torgiano were used for this objective. In the first event with $A = 1.8 \text{ km}^2$, $L = 1.5$ km and $S = 0.0011$, through Eq. (10), the value of K was of 1.5 h. For the reach of width 17 m and depth 4.5 m, according to its condition the value of n was assumed to be of 0.03. The inflow hydrograph was measured in a section (with levee height of 2 m) localized 5.5 km upstream of that (without levees) where the river started flooding. Fig. 4 shows that the model provided an appropriate shape of the outflow hydrograph together with a correct shift of the peak. The other event with $A = 2.9 \text{ km}^2$, $L = 1.5$ km and $S = 0.0013$ involved $K = 2$ h. Further, the reach was characterized by width 22 m, depth 7 m and $n = 0.03$. The model accuracy was found to be very similar to that obtained for the preceding event. Many other events with more considerable overflows occurred over the same basins, but they were not utilizable in this context because of the absence of appropriate measurements of inflow or outflow hydrographs.

Conclusions

1) The proposed model may be conveniently used for estimating the outflow hydrograph downstream of a flooded area, avoiding a solution of the problem based on the routing of the inflow hydrograph by a complex two-dimensional model. The simple model structure allows its incorporation in more general adaptive models of real time flood forecasting without significantly increasing computational effort.
2) The model allows a fast estimation of the changes in the hydrograph due to a rectangular plain whose flooded area is invariant with the elevation of water above its surface. Despite such an idealization of geometry through the slope, area and length of plain, the approach may be considered a good approximation for a large number of real configurations.

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