which is the same form as Equation [2] with η_p replaced by η_q and r_B by r_A.

Appendix 2

Derivation of Equation [5]

Hawthorne (3) suggests the following approximate analysis for determining the heat transfer per pound of fluid flowing across a turbine-blade row.

The Reynolds' analogy relating momentum transfer and heat transfer is

$$\frac{\alpha}{c_p G} = \frac{f}{2} \quad [20]$$

An energy balance for an element of length dx along the blade surface in the direction of flow is

$$dq = \alpha P(T_0 - T_w)dx = SGc_p dT_0 \quad [21]$$

Substituting Equation [20] into Equation [21] and integrating between points 1 and 2 at inlet and outlet of the blade passage results in

$$\frac{T_{01} - T_{02}}{T_{02}} = \frac{T_{02} - T_w}{T_{02}} \left(e^{\int_0^L \frac{fP}{2S} dx} - 1 \right) \quad [22]$$

Define a loss factor ξ_0 by

$$\xi_0 \equiv \int_{x=0}^{L} f \frac{P}{S} dx \quad [23]$$

Then since the heat transferred per pound of fluid is

$$Q = c_p(T_{01} - T_{02}) \quad [24]$$

Equation [22] with Equations [23] and [24] becomes

$$Q = c_p(T_{02} - T_w)(e^{\xi_0/2} - 1) \quad [25]$$

which is Equation [5].

Initial calculations for this study were made with a loss coefficient in the range $0.05 < \xi_0 < 0.08$. Because of the varying opinions as to the correct value of ξ_0 and because of its pronounced effect on cooled turbine performance, the range was extended to include $0.02 < \xi_0 < 0.08$. It is believed that this range will adequately bracket the actual value of ξ_0 for any particular blade design. It will also bring the data into closer agreement with more recent work completed on profile-loss factors as mentioned subsequently.

A discussion on the determination of ξ_0 is presented here to show some of the reasoning behind the variance of opinion and to verify, if possible, the use of a loss coefficient in the range $0.02 < \xi_0 < 0.08$.

For low Mach number and for the case in which (P/S) does not vary along the flow path, Hawthorne (3) shows that $\xi_0 = \xi_p$, the profile-loss factor. This would correspond to the case of impulse blading. For reaction blading and nozzles having decreasing area in the direction of gas flow, and for values of Mach number greater than zero, ξ_p becomes less than ξ_0.

Hawthorne (3) combined the following equation for the heat-transfer coefficient from gas to blade

$$\text{Nu} = 0.14(\text{Re})^{0.63}(\text{Pr})^{1/3} \quad [26]$$

with the heat-balance equation for a blade passage

$$\alpha A(T_0 - T_w) = S\rho V c_p(T_{01} - T_{02}) \quad [27]$$

and Reynolds analogy to obtain

$$(e^{\xi_0/2} - 1) = 0.14\,(\text{Re})^{-0.32}\,(\text{Pr})^{-0.67}\Gamma \quad [28]$$

where $\Gamma = A/S$ = heat-transfer surface/cross-sectional flow area.

Equation [28] yields $\xi_0 = 0.026 - 0.084$ for Reynolds number of $5 \times 10^5 > \text{Re} > 2 \times 10^4$ where Γ varies from $5 < \Gamma < 10$ for most blade shapes.

Subsequent to Hawthorne's analysis, Van Le (7) completed a study of loss coefficients in turbine passages. His results showed that

$$\xi_p = f(i, \alpha_1/\alpha_2, \theta, \text{Re}, S/C)$$

Data taken from charts in reference (7) show the following trends:

(a) ξ_p decreases as α_1/α_2, Re, and S/C increase.

(b) ξ_p increases as θ and i increase.

(c) ξ_p for reaction blades is less than ξ_p for impulse blades.

(d) For normal variations in turning angle θ, degree of reaction α_1/α_2, pitch chord ratio S/C, incidence angle i, and blade Reynolds number Re, the profile-loss coefficient was found to vary within the limits, $0.02 < \xi_p < 0.14$.

These values of ξ_p would seem to agree fairly well with Hawthorne's analysis that $\xi_0 = \xi_p$, particularly at the higher values of Reynolds number.

Smith and Pearson (8) define a heat-extraction coefficient ϵ for a turbine stage

$$\epsilon = \frac{\Delta H}{W} \frac{u^2}{Jgc_p(T_0 - T_w)} \quad [29]$$

By comparison with Equation [27] for a cascade of blades, where $K_1 = 4$ for impulse blades the following relation can be established

$$\epsilon = (e^{\xi_0/2 - 1}) = \xi_0/2 \quad [30]$$

Reference (7) includes plots of ϵ versus flow coefficient which show that ϵ varies between $0.01 < \epsilon < 0.03$. These values show good agreement with Hawthorne's values for $\xi_0/2$ between $0.012 < \xi_0/2 < 0.042$.

Appendix 3

Derivation of Equations [9] and [10]

The definition of stage efficiency is

$$\eta_{stg} = \frac{W_{stg}}{\Delta h_{ss}} = \frac{\Delta h_{stg} - Q}{\Delta h_{ss}} \quad [31]$$

if the difference between the kinetic energy of the gases entering and leaving the stage is negligible. Then

$$\eta_{stg} = \left(1 + \frac{Q}{W}\right) = \frac{\Delta h_{stg}}{\Delta h_{ss}} = \frac{c_p(\Delta T_{stg})}{c_p(\Delta T_{ss})} \quad [32]$$

Assuming equal pressure ratio per stage and the condition curve Equation [17] defining η_q, Equations [32] become Equation [10]. Then if Q/W is zero (uncooled stage) $\eta_q = \eta_p$ from Equation [15] and Equation [10] reduces to Equation [9].

Discussion

J. B. Esgar.[3] One of the main questions in determining the

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desirability of using turbine-blade cooling to permit increasing gas temperatures in gas-turbine engines is whether the quantity of heat removed from the cycle by cooling will cause losses that would cancel performance gains due to increased gas temperature. Most cycle analyses of cooled engines have required a knowledge of the engine configuration and rather tedious heat-transfer analyses to determine the quantity of heat removed from the cycle. The author presents a unique type of solution to determine the quantity of heat rejected by cooling without use of extensive heat-transfer analyses. To make intelligent use of his results, however, a knowledge of the magnitude of his loss factor ξ_0 is required. The range considered ($0.02 < \xi_0 < 0.08$) is probably greater than will be encountered on most cooled turbines.

Using methods of calculating heat-transfer rates for turbine blades that have been published in unclassified NACA reports, corresponding values of ξ_0 have been calculated at NACA using the relation between ξ_0 and heat-transfer rate presented by the author. These calculations were made for a wide variety of engines and it was found that ξ_0 was in the range 0.023 to 0.045. In most cases it was found that ξ_0 was less than 0.03. These results used in conjunction with the curves presented in the paper should give a reasonable indication of the effects of heat rejection on cooled turbine-engine performance.

It also should be stated that the analysis presented in the paper cannot be generalized to cover the effects of all types of turbine-cooling methods. The method applies only to the case where heat is removed from the turbine blades and is rejected to a sink that is external to the engine. A simple liquid-cooled system where the coolant circulates through the turbine blades and then carries the heat to a radiator or cooling tower would be one example of a type of cooled-turbine engine where this analysis is applicable. The analysis does not apply to air-cooling systems where air is bled from the compressor and used for cooling turbine blades. In general, air-cooling systems will result in somewhat poorer performance than presented in the paper.

In conjunction with the efficiency results presented for finite-staged turbines where "leaving losses" must be considered, it appears that some confusion might result owing to the fact that the efficiency levels are so very low compared with results previously presented. If the number of stages is greater than 2 or 3 the efficiency is probably approaching the efficiency that would be obtained with an infinite number of stages. The primary cause of the low efficiency shown for the finite-staged turbine is due to the fact that the curves are presented for very much lower pressure ratios than the results presented for turbines with an infinite number of stages.

In conclusion it can be said that the concept of the loss factor ξ_0 is not easy to understand completely, but by using the recommended values of this factor the paper presents a rather comprehensive picture of the effect of heat removal by turbine cooling on liquid-cooled gas-turbine power plants.

David G. Wilson.[4] In the past I have been alternately encouraged and disillusioned, during some elementary work on blade cooling, by people who could prove conclusively that turbine cooling could either endow prize-winning efficiencies or crippling losses on gas-turbine power plants. In this paper most disadvantages of cooling are taken into account; in fact, some assumptions, such as the compressor efficiency of 85 per cent, would seem pessimistic for design point values. However, the heat losses connected with duct cooling and the power losses due to coolant pumping had been ignored, which could be significant factors. On the other hand, it is conceivable that the heat losses could be turned partly to heat gains by transferring the heat removed from blades and ducts to the air stream before combustion; or power could be recovered by using a steam turbine with liquid cooling. I should much appreciate the author's views on the general magnitude of the favorable and unfavorable effects of both extremes on their conclusions.

Author's Closure

It is indeed gratifying to receive the discussions of Messrs. Esgar and Wilson, both of whom have devoted a considerable amount of energy to the practical solution of the turbine-cooling problem. When our analysis was begun some years ago the magnitude of ξ_0 was not well established; so we used a rather wide range of values, 0.02 to 0.08. It is quite pertinent to the interpretation of the present results to know the proper reasonable range, 0.023 to 0.045, as suggested by Mr. Esgar.

Mr. Esgar and I seem to have some disagreement regarding the efficiencies of the finite-staged turbine but I suspect they can be resolved when we see each other. The results in Figs. 13 and 14 are presented to show the effect of number of stages on turbine performance. To keep the calculations from being too lengthy a lower pressure ratio was chosen; so the level of cycle efficiency is lower than in the previous curves. Nevertheless, the effect of number of stages is clearly shown. As number of stages increases the leaving loss decreases but the heat-transfer loss in the cooled part increases. These two opposing effects result in a maximum efficiency at a particular value of N for given design conditions.

Some of the questions regarding auxiliary losses are discussed in a companion paper, reference (9). We have not investigated the suggestions of Dr. Wilson regarding the recovery of some of the heat removed in cooling. They obviously would increase the efficiency and should be investigated to determine their economic feasibility.

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