For buckets where the principal axes of moment of inertia of the cross section are appreciably skewed with respect to the tangential and axial directions or where the blade has a significant amount of twist along its length, the procedures of this paper are not directly applicable but may be extended to cover these situations. For banded groups of skewed or twisted buckets there will be coupling between both components of flexural motion and the torsional motion.

Additional coupling between flexure and torsion results from the dissymmetry of the bucket cross section with respect to the principal axes of inertia. This dissymmetry causes the center of twist to be displaced from the center of gravity. For medium-height buckets of customary proportions with negligible skew and twist of the principal axes of the cross sections, the center of twist will be displaced essentially in the tangential direction. The coupling effect will be negligible for tangential vibration and it is believed to be of minor significance for many of the axial and torsional modes of vibration.

It has been assumed in the analysis that each banded group can be treated as a separate entity. For axial modes of vibration the wheel on which the buckets are mounted will participate and there will be coupling between banded groups. This wheel action will be most pronounced at low frequencies. At frequencies of the order of the frequency of passing nozzles, wheel action is much less pronounced and the results obtained from the present analysis should be adequate for design purposes.

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Discussion

R. W. Nolan.[3] The modified Holzer technique used by the author appears to be well suited to digital calculations. The writer's company has found it to be economical for digital calculation of higher-order critical speeds of long overhung shafts. In the complicated problem handled by the author, this method should result in a considerable saving of engineering time.

The logarithmic decrement of many materials is not constant but varies with stress, usually increasing with increased stress. Furthermore, considerable damping is obtained in the root fastenings of turbine blades. The value of δ which is used in Equations [41], [46], and [50] will then be a function of the value Y_m. The writer is interested in the author's procedure for obtaining a value of δ which represents the average for the entire blade group.

[3] Engineering Service Division, E. I. du Pont de Nemours & Company, Wilmington, Del. Mem. ASME.

Abdon Rubio.[4] The problem of determining the natural frequencies and operating stresses of a banded group of turbine buckets has occupied the efforts of numerous investigators. The author is to be commended on the complete and systematic presentation he has made on a practical method of attack. Within the limitations set forth by the author, which are justifiable in the bucket sizes and constructions with which he is concerned, this paper is by far the most advanced of any published.

However, when the subject is a long, slender, highly twisted airfoil, grouped by numerous tie wires as well as cover bands, the arithmetic increases severely. This is due (as the author mentions) to the various coupling actions. The matrix size of the transfer functions required increases from approximately the number of buckets involved (s in the case of decoupled tangential vibration) to approximately two and a half times the number of buckets when all the necessary coupling actions are accounted for. In the case of ten of these slender buckets being grouped together, the determinant to be evaluated would be 25×25 in size. In order to achieve three-place accuracy in the final calculated frequency, the determinant evaluation would require double-precision and perhaps triple-precision calculations (24-decimal digits). The final calculated mode shape is an even more critical function of the determinant evaluation.

Also of vital importance in the calculation of the natural frequencies of long slender buckets is the action of the centrifugal field. Of lesser importance is the proper determination and evaluation of initial twist, camber, center of twist, dovetail fixity, and wheel action.

In closing, the writer would like to venture the opinion that this excellent paper will become a basic tool in the design of steam turbines.

Author's Closure

Mr. Nolan has raised some pertinent questions with regard to logarithmic decrement of damping. The decrement δ as used in the paper represents the total damping in the banded group of buckets. As Mr. Nolan suggests, two types of damping are present: (i) Material damping due to internal friction and (ii) structural damping due to mechanical fits and joints (for the type of buckets under consideration aerodynamic damping may be properly disregarded).

Measurements of material damping are available for standard 12 per cent chrome iron.[5,6,7] The material damping is a function of the vibratory stress level and, since this alloy exhibits magneto-mechanical effects, the damping is substantially reduced by the presence of steady centrifugal stresses. If the values of material damping are expressed in the form of damping energy per unit volume per cycle in the manner of Lazan,[6] then the total energy dissipation in the banded group may be obtained by integrating over the total volume for any specified distribution of vibratory and centrifugal stresses.

Relatively little is known regarding structural damping in turbine buckets under operating conditions. The evaluation of

[4] Buckets and Rotors Engineering, Large Steam Turbine, Generator Department, General Electric Company, Schenectady, N. Y. Assoc. Mem. ASME.

[5] "Effect of Static Stress on the Damping of Some Engineering Alloys," by A. W. Cochardt, Trans. ASM, vol. 47, 1955, pp. 440–450.

[6] "The Effect of Static Mean Stress on the Damping Properties of Materials," by Neal L. Person and Benjamin J. Lazan, presented at Annual Meeting of ASTM, June 17–22, 1956.

[7] "The Specific Damping of Fixed-Fixed Beam Specimens," by W. C. Hagel and J. W. Clark, *Journal of Applied Mechanics*, Trans. ASME, vol. 79, 1957, pp. 426–430.

structural damping is, for the present, largely a matter of a judicious engineering estimate. Further study of structural damping in dovetail and rivetted connections under steady loads corresponding to centrifugal forces is needed.

Since the total damping varies with the amplitude of vibratory stress, the value of logarithmic decrement δ used in Equations [44], [48], or [52] for evaluating resonant vibration stress must be consistent with the calculated stress level. This requires a trial-and-error process of calculation.

Mr. Rubio has mentioned some of the complications that arise when additional factors are included in the calculation method. The consideration of twist in the buckets represents a logical next step in the extension of the methods of the paper. As Mr. Rubio notes, the frequency determinant for a simple banded group of s twisted buckets will be of order $5\ s/2$ (exactly $5\ s/2$ for an even number of buckets in the banded group, and approximately so for an odd number if full advantage is taken of the symmetry of the banded group). The addition of a single tie wire to the banded group would double the order of the determinant, the addition of two tie wires would triple the order, etc.

The question of accuracy of solution is a most difficult one with which to deal and the author will confine his comments to actual experience with the computer program based on the method of the paper. Frequency determinants up to 18×18 in. size have been satisfactorily evaluated by single precision arithmetic (8 significant figures) for modes of vibration of the type shown in the companion paper.[8] No evidence of breakdown or significant loss of accuracy has been observed. The procedure used in evaluating the determinant is all important. The author has used the Gauss process[9] whereby elements in the matrix below the principle diagonal are successively reduced to zero. Each step in the reduction process involves division by a diagonal element and the key to successful reduction is the rearrangement of the matrix prior to each division to bring the numerically largest element still available into the given diagonal position. The results obtained with this system of determinant evaluation have far exceeded the author's original expectations.

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