Photodisintegration of the Deuteron near 11 Mev

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It is shown that we can account for the photodisintegration of the deuteron near 11 Mev in consistence with nucleon-nucleon scattering at the corresponding energy. Some features of nuclear forces are elucidated in this analysis. It is shown that the phase shift of the singlet even state is somewhat smaller than that previously supposed, and the phase shifts of the triplet odd state are not small at such an energy.

§ 1. Introduction

The investigation of photodisintegration of the deuteron is very important for understanding of nuclear forces. However, its complete treatment has not yet been performed. The characteristic feature in the photodisintegration of the deuteron is that the isotropic part in the angular distribution is large. Many authors have tried to account for the large isotropic part from different points of view. However, reasonable explanation has scarcely been given, and fair agreement between theoretical results and the experimental data is obtained only in a few treatments.

In this paper we shall show that we can account for the photodisintegration of the deuteron near 11 Mev in consistence with p-p scattering at 18.2 ± 0.2 Mev. Then some features of nuclear forces can be elucidated in our analysis, and discussions are made on these properties of nuclear forces.

In the photodisintegration of the deuteron, most of the contribution comes from the electric dipole (e. d.) and the magnetic dipole (m. d.) transitions. As is well known, the final states of e. d. and m. d. are $^3O$ (the triplet odd state) and $^1E$ (the singlet even state), and these states are just the only two states that are concerned with p-p scattering. Now the important point to be paid attention is that there exists large ambiguity in the determination of the phase shifts in p-p scattering, and on the other hand the phase shifts are very sensitive to the cross sections of photodisintegration of the deuteron. On account of this we make use of the phase shifts, which fit p-p scattering data, for the calculation of the photodisintegration of the deuteron. In this paper the photodisintegration of the deuteron at $E_\gamma = 11.3$ Mev corresponding to 18.2 Mev p-p scattering is treated.

In § 2 the formulas for computations are given, and our method is explained. In

* Small contributions from other states are neglected here.
§ 3 we show the calculated results and comparison with experiments. Then the features of $^1E$ and $^3O$ brought out by our calculation are illustrated in § 4. Finally the conclusions are summarized in § 5.

§ 2. Preliminary for our work

The symbols used in this paper are as follows.

$E_\gamma$ : Energy of the incident $\gamma$ ray in C. M. S.

$M$ : Mean mass of neutron and proton.

$\epsilon$ : Binding energy of the deuteron.

$k^2 = M(E_\gamma - \epsilon)/h^2$, $\alpha^2 = Me/h^2$, $k = mc/h$.

$\mu_p$ : Magnetic moment of proton.

$\mu_n$ : Magnetic moment of neutron.

$U_{31}$ : Radial wave function for $^3S_1$ state of the deuteron.

$w_{31}$ : Radial wave function for $^3D_1$ state of the deuteron.

$f_{31}$ : Radial wave function for $^3S_1 + ^3D_1$ state of the deuteron.

$U_0$ : Radial wave function for $^1S_0$ state.

$U_2$ : Radial wave function for $^1D_2$ state.

$U_{14}$ : Radial wave function for $^3P_4$ state.

$\delta_{LJ}$ : Phase shift for state of orbital angular momentum $L$ and total angular momentum $J$.

$\sigma_{ed}$ : Electric dipole cross section.

$\sigma_{md}$ : Magnetic dipole cross section.

$\sigma_{T}$ : Total cross section.

$\sigma_{ew}$ : Isotropic cross section arising from e. d.

$\sigma_{mew}$ : Isotropic cross section arising from m. d.

$\sigma_{sp}$ : Total isotropic cross section.

$D_{2}$ : Hard core radius of $^1E$.

At our energy the situation is rather simple (because the energy is low). A number of effects which give merely small contributions can be neglected. The electric quadrupole (e. q.) cross section is about 0.2% of $\sigma_{ed}$, and the magnetic quadrupole cross section is much smaller. Therefore, these two may be safely neglected.* In $\sigma_{md}$ the contribution from the exchange current is still small at $E_\gamma \simeq 10$ Mev, and the transition to the continuous $^3S_1 + ^3D_1$ state is only of the order of e. q. near our energy. Therefore the transitions to be considered are only e. d. corresponding to $^3S_1 + ^3D_1 \rightarrow ^3P_3$, $^3P_1$, $^3P_2$, and m. d. corresponding to $^3S_1 \rightarrow ^1S_0$, $^3D_1 \rightarrow ^1D_2$ without including the exchange current.

The formulas for these transitions are as follows.

* Although the interference term of e. d. with e. q. is not very small, its effect is unimportant as is seen in § 3.
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\[ \sigma_{\text{tot}} = \frac{1}{3} \cdot \frac{e^2}{b_c} \cdot \frac{M_E^+}{b^5} \cdot k \cdot \left( I_{10}^2 + 3 I_{11}^2 + 5 I_{12}^2 \right), \]  

(1)

\[ ds_{\text{tot}}(\theta) = \frac{1}{3} \cdot \frac{e^2}{b_c} \cdot \frac{M_E^+}{b^5} \cdot k \cdot \frac{d\Omega}{4\pi} \cdot \left[ \frac{1}{9} \left| e^{i\delta_1} I_{10}^1 - e^{i\delta_2} I_{12}^1 \right|^2 + \frac{1}{4} \left| e^{i\delta_1} I_{11}^1 - e^{i\delta_3} I_{13}^1 \right|^2 \right] \]

\[ + \frac{1}{2} \sin^2 \theta \left[ \frac{1}{4} \left| e^{i\delta_1} I_{11}^1 + 3 e^{i\delta_3} I_{13}^1 \right|^2 - \frac{1}{15} \left| e^{i\delta_1} I_{10}^0 - 5 e^{i\delta_2} I_{12}^0 \right|^2 + \frac{1}{15} I_{10}^2 \right], \]  

(2)

\[ I_{10} = \int d\theta U_{10}(U_d - \sqrt{2} \omega_d), \quad I_{11} = \int d\theta U_{11}(U_d + \frac{1}{\sqrt{2}} \omega_d), \]  

(3)

\[ \sigma_{\text{tot}}(\theta) = \frac{1}{3} (\mu_n - \mu_p)^2 \cdot \frac{e^2}{b_c} \cdot \left( \frac{\hbar}{M_c} \right)^2 \cdot \frac{M_E^+}{b^5} \cdot k \cdot \left( I_{10}^0 + I_{12}^0 \right), \]  

(4)

\[ ds_{\text{tot}}(\theta) = \frac{1}{3} (\mu_n - \mu_p)^2 \cdot \frac{e^2}{b_c} \cdot \left( \frac{\hbar}{M_c} \right)^2 \cdot \frac{M_E^+}{b^5} \cdot k \cdot \frac{d\Omega}{4\pi} \cdot \left[ \frac{3}{\sqrt{2}} \cos \left( \delta_1 - \delta_2 \right) I_{10} I_{12} \right. \]

\[ + \sin^2 \theta \left. \left[ \frac{3}{\sqrt{2}} \cos \left( \delta_1 - \delta_2 \right) I_{10} I_{12} + \frac{3}{4} I_{10}^2 \right] \right], \]  

(5)

\[ I_{10} = \int U_d U_d dr, \quad I_{12} = \int U_d \omega_d dr. \]  

(6)

In what follows we explain our method in detail.

I. Wave function of the deuteron.

In the calculation we used two sorts of wave functions of the deuteron. One is that obtained from phenomenological potential without hard core, which is given by Feshbach and Schwinger. Its D-state probability is 2.5%. The other one is that derived from the pion potential, and it has a small hard core, which is given by \( I = O - T - W \). The D-state probability of it is 6.4%.

The former is obtained by adjusting the parameters of the Yukawa type potential without hard core, so that it can account for the low energy and the deuteron data. The latter is derived from the pion potential at \( r > 1/k \) (because the inner potential is not well established, and the deuteron data depend on the behavior at \( r > 1/k \)), and it can reproduce the scattering and the deuteron data. It is interesting to see what sort of differences will appear between these two cases.

The approximate forms, which we used in the calculation, of the wave functions of the deuteron for the above two cases are as follows.

1) \( f_d \) of Feshbach and Schwinger.

\[ U_d = 1.042 \sqrt{k} \left( e^{-0.150k} - e^{-1.370k} \right), \]  

(7·a)

\[ \omega_d = 0.2524 \sqrt{k} \left( 1 - 0.0199k \right) e^{-0.437k} - 1.140 \sqrt{k} e^{-1.540k}. \]  

(7·b)
2) \( f_d \) of \( I-O-T-W \).

\[
U_d = 1.039 \sqrt{k} e^{-0.328kr} - 1.392 \sqrt{k} e^{-2.328kr}, \tag{8\cdot a}
\]

\[
\sigma_d = 0.02624 \sqrt{k} \left\{ 1 + 3/0.328kr + 3/(0.328kr)^2 \right\} e^{-0.328kr} - 1.298/rk \cdot e^{-0.926k}, \tag{8\cdot b}
\]

for \( r > 1/k \).

The approximation of \( \sigma_d \) of \( (8\cdot b) \) is not good at \( r < 1/k \), and we used a smooth curve, so that it falls to zero at \( r=0 \) in stead of \( (8\cdot b) \) at \( r<1/k \). (The results are scarcely changed by the form at \( r<1/k \) for low energy reactions.)

II. Calculation of e. d. and m. d.

Now let us elucidate the computation of e. d. and m. d.\(^{13}\)

1) E.D. We discuss the e. d. at first. If \( \delta_i \), \( \delta_i^1 \), and \( \delta_i^2 \) are given, \( \sigma_{ed} \) (and \( \sigma_{ed}^{(p)} \)) can be computed at once using the formulas \( (1) \sim (3) \). Because \( U_{it} \) is small at small distances, and because there is a factor \( r \) in \( I_i \), the contribution from small \( r \) is unimportant, and we may approximate \( U_{it} \) by

\[
kU_{it} = \frac{\sin (kr + \delta_i)}{kr} = \cos (kr + \delta_i). \tag{9}
\]

There may arise a question that, \( U_{it} \) of \( (9) \) tends to \( \infty \) at \( r=0 \), and therefore the approximation \( (9) \) may not be good enough. Nevertheless we can prove that the approximation \( (9) \) is not bad. (See Appendix A.)

2) M.D. In the computation of m. d. special attention is needed. As we can see from \( I_{00} \) of \( (6) \), the contribution from small distances is important for this case. \( U_6 \) at small distances is quite unknown, if only \( \delta_0^0 \) is given. However, we can estimate the lower and the upper limits of \( I_{00} \) (and therefore the lower and the upper limits of \( \sigma_{m6} \) and \( \sigma_{m6}^{(p)} \)).

The lower limit of \( I_{00} \) can be obtained, if \( D_i \) is given. (The relation between the phase shifts and the hard core radius will be discussed later), by putting \( U_6 \) as

\[
kU_6 = \sin (kr + \delta_0^0) \left( 1 - e^{-\zeta (r - D_i)} \right), \tag{10}
\]

and choose \( \zeta \) so that it satisfies the effective range theory.

The upper limit of \( I_{00} \) can be estimated as follows. The simplest way to compute the upper limit is to use

\[
kU_6 = \sin (kr + \delta_0^0). \tag{10}^{17}
\]

However, to reduce the indeterminacy it is better to use

\[
kU_6 = \sin (kr + \delta_0^0) \left( 1 - e^{-\eta (r - D_i)} \right) \tag{10}^{17}
\]

with suitable values of \( \eta \). We chosen \( \eta \) so that \( (10)^{17} \) and \( (10)'' \) are differed by 1% at \( r=1/k \). Values of \( \zeta \) and \( \eta \) for various \( D_i \) are given in Table 1 (See Appendix B).

Generally speaking, the smaller \( \delta_0^0 \) is, the larger \( D_i \) is. In the calculation we varied \( D_i \) for given values of \( \delta_0^0 \). If \( \sigma_{m6} \) (and \( \sigma_{m6}^{(p)} \)) varies quickly by the variation of \( D_i \) (for a given \( \delta_0^0 \)), then our method may become unreliable, because this means that \( \sigma_{m6}^{(p)} \) (and
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Table 1. Values of $\zeta$ of (10)' and $\eta$ of (10)'' as the function of $D_\nu$ is shown.

<table>
<thead>
<tr>
<th>$D_\nu (1/k)$</th>
<th>$\zeta (k)$</th>
<th>$\eta (k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.69</td>
<td>3.6</td>
</tr>
<tr>
<td>0.2</td>
<td>2.15</td>
<td>5.8</td>
</tr>
<tr>
<td>0.4</td>
<td>2.98</td>
<td>7.7</td>
</tr>
<tr>
<td>0.6</td>
<td>4.83</td>
<td>11.5</td>
</tr>
</tbody>
</table>

$\sigma_{md}$ cannot be determined only by $\delta_0^0$. (Note that $D_\nu$ cannot be known by $\delta_0^0$, although smaller $\delta_0^0$ corresponds to larger $D_\nu$.) Fortunately the situation is not so, as shown in the calculation in § 4. It should be noted that the magnitude of $\sigma_{md}^{lp}$ (and $\sigma_{md}$) is determined approximately by $\delta_0^0$. Also it is to be remarked that the actual $\sigma_{md}$ (and $\sigma_{md}^{lp}$) is rather close to the lower limit at $E_\gamma=11.3$ Mev, because the energy is not high.

Up to now we have discussed $I_{0\sigma}$. There is no difficulty in the calculation of $I_{0\alpha}$. We can do it at once only if $\delta_0^2$ is known (in our case $\delta_0^2 \simeq 0$), because the contribution from small distances is unimportant for such a case.

III. Phase Shifts of p-p Scattering at 18.2 Mev.

We used the phase shifts of p-p scattering at 18.2 Mev given by Clementel et al. and Iwadare et al. to the computation of the photodisintegration of the deuteron at 11.3 Mev. These phase shifts are given in Table 2. The six sets corresponding to

Table 2. Some sets of phase shifts (in unit of degree) obtained by Clementel et al. and Iwadare et al. to account for p-p scattering at 18.2 Mev.

<table>
<thead>
<tr>
<th></th>
<th>$\delta_0^0$</th>
<th>$\delta_1^0$</th>
<th>$\delta_1^1$</th>
<th>$\delta_1^2$</th>
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<tr>
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<td>43.7</td>
<td>19.4</td>
<td>-7.4</td>
<td>2.8</td>
</tr>
<tr>
<td>B</td>
<td>&quot;</td>
<td>-14.6</td>
<td>10.5</td>
<td>-1.5</td>
</tr>
<tr>
<td>C</td>
<td>&quot;</td>
<td>24.1</td>
<td>-4.3</td>
<td>-1.1</td>
</tr>
<tr>
<td>D</td>
<td>48.3</td>
<td>16.7</td>
<td>-3.3</td>
<td>1.4</td>
</tr>
<tr>
<td>A'</td>
<td>50.5</td>
<td>3.3</td>
<td>-4.3</td>
<td>1.5</td>
</tr>
<tr>
<td>B'</td>
<td>&quot;</td>
<td>-4.8</td>
<td>3.6</td>
<td>1.9</td>
</tr>
<tr>
<td>C'</td>
<td>&quot;</td>
<td>6.3</td>
<td>-3.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

$\delta_0^0=43.7^\circ$ and 50.5$^\circ$ were given by Clementel et al. by the pure phenomenological analysis of phase shifts. The one corresponding to $\delta_0^0=48.3^\circ$ was given by Iwadare et al. in the investigation of the pion potential. We do not discuss here the methods to obtain these phase shifts (The reader may refer to the original papers). In § 3, we see what results will be got from these sets of phase shifts, and what differences will appear among them.

§ 3. Calculated results

There is yet no good experiment at $E_\gamma \simeq 10$ Mev. The cross section at $E_\gamma=11$ Mev given by Allen is as follows.

$$\sigma_\nu = 1.20 \pm 0.15 \text{ mb}, \quad (11)$$
and the main part of the angular distribution is

\[ a + b \sin^2 \theta \]  

with

\[ a/b = 0.05 \pm 0.02. \]  

In what follows we show the calculated result, and compare it with the experiment. For convenience we denote \( f_a \) of (7) by \( f_a(\text{phe}) \), meaning that it is derived from phenomenological potential. Also we denote \( f_a \) of (8) by \( f_a(\pi) \), by which we mean that it is derived from the pion potential.

The calculated results, when we assume special values of \( D \) in the computation of \( \sigma_{\text{tot}} \) and \( \sigma_{\text{tot}} \), are shown in Table 3 and Table 4. Here we took \( D_a = 0 \) for \( f_a(\text{phe}) \) and \( D_a = 0.2/k \) for \( f_a(\pi) \). (Note that, the hard core radius of \( \delta \) is 0 for \( f_a(\text{phe}) \), and 0.2/k for \( f_a(\pi) \)). To show the situation more clearly the values of \( a/b \) of Table 3 and 4 are presented in Fig. 1 and Fig. 2.

In these tables and figures the relation between the phase shifts and the hard core radius is not taken into consideration. Although \( D_a \) in the case of \( \delta = 50.5^\circ \) may not be large, \( D_a \) in the case of \( \delta = 43.7^\circ \) might not be small. Therefore we over estimated \( \sigma_{\text{tot}} \) (and \( \sigma_{\text{tot}} \)) in the latter case. However, this causes only a slight effect on our final results, if we take the effect of hard core into consideration. This point is clarified in § 4. Therefore we may see the aspect by Tables 3, 4 and Figs. 1, 2.

From the calculated results we see, in spite of a large difference between \( f_a(\text{phe}) \) and \( f_a(\pi) \), the results of Table 3 and of Table 4 are quite similar.

From these tables we see at once that 1) although there exist some differences among the calculated values of \( \sigma_T \), all sets can account for \( \sigma_T \) within the experimental error,* 2) the term \( \sin^2 \theta \cos \theta \), which is due to the interference of e. d. and e. q., is not interesting** because the calculated results of the seven sets are very similar. Therefore we shall not touch these further in what follows.

The most important one to be paid attention is \( a/b \) which is strongly dependent on the features of the phase shifts. We may understand that special conditions are necessary to account for the experimental value of \( a/b \) (although the indeterminacy of \( a/b \) is still large). Concerning \( a/b \) we can comprehend the following properties of nuclear force. 1) \( a/b \)'s of the first three sets corresponding to \( \delta = 43.7^\circ \) fall into the required region, \( 0.05 \pm 0.02 \), while \( a/b \)'s of the last three sets corresponding to \( \delta = 50.5^\circ \) do not. Also it seems that the set \( D \) corresponding to \( \delta = 48.3^\circ \) would fail.

It is owing to the largeness of \( \sigma_{\text{tot}} \) and \( \sigma_{\text{tot}} \) in the first three sets, which are obtained with the smaller \( \delta \) and the larger \( \delta \), that the first three sets can account for the experimental \( a/b \) (within the experimental error). We shall discuss this point again in § 4.

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* There exists important relation between \( \sigma_T \) and the triplet odd potential, about which we shall illustrate in another paper.

** Except a case in which the theoretical value is found to be different from the (future) experimental value.
2) The calculated results are almost the same for both of \( f_d(\text{phe}) \) and \( f_d(\pi) \). Generally speaking, the cross sections for \( f_d(\pi) \) are a little larger than those for \( f_d(\text{phe}) \) at our energy. Because the difference is not large, it shows that the influence of the wave function of the deuteron is small at low energies, as it is expected.

3) From the dependence of \( \sigma_{0d}^{lp} \) (and \( \sigma_{0d}^{id} \)) on \( \phi_0 \) we see that, the smaller \( \phi_0 \) is, the larger \( \sigma_{0d}^{lp} \) (and \( \sigma_{0d}^{id} \)) is. It indicates that smaller \( \phi_0 \) is favourable to the photodisintegration of the deuteron. The physical meaning of this will be illustrated in § 4.

4) From the dependence of \( \sigma_{0d}^{lp} \) on \( \phi_0 \), we see that as \( \phi_0 \) are larger (in absolute value) and more separated, \( \sigma_{0d}^{lp} \) is larger. The physical meaning of this will also be discussed § 4.

### Table 3
The calculated result for the transitions from \( f_d(\text{phe}) \) at \( E_r = 11.3 \) Mev using the phase shifts of Table 2 is shown. In the calculation of m. d. we took \( D_x = 0 \). The upper half of the table is for the case when we estimate m. d. by its lower limit, and the lower half is for the case when m. d. is estimated by its upper limit. All cross sections are in unit of \( 10^{-29} \text{ cm}^2 \). Also in the last column the coefficient of \( \sin^2 \theta \cos \theta \) (\( \beta \)), which is given by the interference of e. d. and e. q., is given.

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_{ed} )</th>
<th>( \sigma_{md} )</th>
<th>( \sigma_{p} )</th>
<th>( \sigma_{lp} )</th>
<th>( \sigma_{lp} )</th>
<th>( \sigma_{sp} )</th>
<th>( a/b )</th>
<th>( \beta )</th>
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<tbody>
<tr>
<td>A</td>
<td>108.30</td>
<td>2.96</td>
<td>111.26</td>
<td>2.08</td>
<td>2.37</td>
<td>4.45</td>
<td>0.028</td>
<td>0.099</td>
</tr>
<tr>
<td>B</td>
<td>114.93</td>
<td></td>
<td>117.89</td>
<td>4.79</td>
<td></td>
<td>7.16</td>
<td>0.043</td>
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</tr>
<tr>
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<td></td>
<td>109.93</td>
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<td></td>
<td>4.85</td>
<td>0.031</td>
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</tr>
<tr>
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<td>113.51</td>
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<td>1.99</td>
<td>3.78</td>
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<td>0.100</td>
</tr>
<tr>
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<td>111.19</td>
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<td>2.85</td>
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</tr>
<tr>
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</tr>
<tr>
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<td></td>
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<td>6.26</td>
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<td>D</td>
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<td>1.44</td>
<td>3.32</td>
<td>4.76</td>
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<td>4.84</td>
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<tr>
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<td>112.47</td>
<td>0.93</td>
<td></td>
<td>4.01</td>
<td>0.025</td>
<td>0.101</td>
</tr>
</tbody>
</table>

**Fig. 1.** \( a/b \) of Table 3 is plotted.
Table 4. The calculated result for the transitions from $f_d(\pi)$ at $E_r=11.3$ Mev is shown. In the calculation of m. d. we took $D_\pi=0.2/k$. For other contents see the explanation of Table 3.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{md}$</th>
<th>$\sigma_{md}^{sp}$</th>
<th>$\sigma_{md}^{d}$</th>
<th>$\sigma_{min}$</th>
<th>$\sigma_{sp}$</th>
<th>$a/b$</th>
</tr>
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<td>A</td>
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<td>3.29</td>
<td>112.58</td>
<td>2.31</td>
<td>2.77</td>
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<td>&quot;</td>
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<td>4.82</td>
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<td>2.79</td>
<td>114.99</td>
<td>1.66</td>
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<tr>
<td>A'</td>
<td>111.10</td>
<td>2.56</td>
<td>113.66</td>
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<tr>
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In what follows we illustrate the properties of nuclear forces indicated by the calculation of § 3. At first we elucidate the feature of $\delta_5^l$ indicated by $\sigma_{md}^{sp}$, secondly of $\delta_3^l$ indicated by $\sigma_{md}^{d}$, and then about the relation with p-p scattering.

I. Feature of $\delta_5^l$ Indicated by m. d.

In § 3 we saw that the smaller $\delta_5^l$ is, the larger $\sigma_{md}^{sp}$ is. In what follows, we elucidate the reason, and explain the feature of $\delta_5^l$.

From (5) we see, $\sigma_{md}^{sp}$ is determined by

$$I_{00}^l - \sqrt{2} \cos (\delta_5^l - \delta_5^l) I_0 I_{00} + \frac{1}{2} I_{00}^2.$$  \(5')

From (6) we can understand that $I_{00}$ is much smaller than $I_{00}$ at low energies, because $U_2$ is small at $r<1/\alpha$, and $\varpi_d$ is smaller than $U_d$. Therefore the main part of (5)'}
is \( I_{00} \). From the first equation of (6) we see that, \( I_{00} \) comes from the overlap of \( U_0 \) and \( U_d \). From Fig. 3 we can see that \( I_{00} \) is determined by the first positive part and the first negative part of \( U_0 \) wave. If the overlap of the first positive part of \( U_0 \) wave with \( U_d \) is much larger than the overlap of the first negative part of \( U_0 \) wave with \( U_d \), \( I_{00} \) is very large. If it is not the case, \( I_{00} \) is small. The former corresponds to smaller \( \delta_0^0 \), and the latter to larger \( \delta_0^0 \).

To make the situation more clear, we show the dependence of m. d. cross sections on \( \delta_0^0 \) in Figs. 4, 5, and 6. In these figures the lower limit of \( \sigma_{md}^{sp} \) and \( \sigma_{md} \) are plotted as the function of \( \delta_0^0 \) and \( D_0 \). Here we used \( f_d \) (phe) as the wave function of the deuteron, and varied \( D_0 \) in \( 0 \sim 0.6/k \).

From these figures we can comprehend that, taking the existence of the hard core into consideration,* "If \( \delta_0^0 \) is smaller, the corresponding \( \sigma_{md}^{sp} \) (and \( \sigma_{md} \)) is larger", except a case in which \( \delta_0^0=\pi(D_0) \) is changed quite slowly with the variation of \( D_0 \) at our energy, although it does not seem to be the case (Even if \( \delta_0^0 \) varies so slowly that, when \( D_0 \) varies from 0 to 0.6/k, it decreases from 55°# to 45° at our energy,* and \( \sigma_{md}^{sp} \) is still increased by 20% more in this variation). Perhaps the shadowed part of Fig. 5 would represent the relation between the phase shifts and \( \sigma_{md}^{sp} \).

It seems that \( \delta_0^0 \) has been taken somewhat

* If \( \delta_0^0 \) is smaller, the corresponding \( D_0 \) is larger, generally speaking.

# The Yukawa type potential without the hard core gives \( \delta_0^0=54° \) at 18 Mev.\(^{10} \)
larger at energies higher than 10 Mev. Speaking in the words of potential, it seems that the singlet even potential would be rather repulsive than it has been supposed thus far (except at low energies).

II. Feature of $^3O$ indicated by e.d.

Although smaller $\delta_0^a$ is favourable for the photodisintegration of the deuteron, it is still insufficient by itself to account for the large experimental $a/b$. (See the computation in §3). Therefore we are forced to expect some part of the isotropic cross section to come from the e.d. transition.

From the formulas (2) and (3) we may see that, generally speaking, if the D-state probability is larger and $\delta_1^i$ are larger, $\sigma_{dd}^{lp}$ is larger. However, there is a limit which we can take about the D-state probability. Furthermore the detailed form of the wave function of the deuteron is unimportant at low energies as we see in §3. Therefore, to expect large $\sigma_{dd}^{lp}$, we infer that for 18 Mev nucleon-nucleon scattering $\delta_1^i$ are no longer small and somewhat separated from one another. In the words of potential we may say that the tensor force in $^3O$ is not small.

III. Relation with p-p Scattering

We have seen that if $\delta_0^a$ is smaller, $\sigma_{dd}^{lp}$ is larger, and that if $\delta_1^i$ are larger and more separated, $\sigma_{dd}^{dp}$ is larger. Summing up these two effects, we can account for the experimental value of $a/b$. Now we discuss the relation with p-p scattering. In p-p scattering $d\sigma(90^\circ)/d\Omega$ must be accounted for by $\delta_0^a$ and the difference among $\delta_0^a$, $\delta_1^i$, and $\delta_2^i$. Therefore if $\delta_0^a$ is smaller, $\delta_1^i$, $\delta_1^i$, and $\delta_2^i$ must be larger in the absolute value and more separated from one another. Thus we may conclude as follows,

"To account for both p-p scattering and the photodisintegration of the deuteron in
consistence with one another, \( \delta_0^0 \) would be smaller, and \( \delta_3^0, \delta_1^1, \delta_1^2 \) would be larger and more separated from one another, than they have been supposed up to today".

§ 5. Conclusion

We summarize the results obtained in this paper in the following.

1) We can account for the photodisintegration of the deuteron near \( E_r = 11 \) Mev in consistence with \( p-p \) scattering, if we take \( \delta_0^0 \) to be somewhat smaller than that previously supposed, and the phase shifts of \( ^3P \) states larger than those previously given.

2) Generally speaking, if \( \delta_0^0 \) is smaller, \( \sigma_{out}^{sp} \) is increased, and makes the isotropic part larger, which is favourable for the photodisintegration of the deuteron. It seems that \(^1E\) potential would be somewhat more repulsive than previously estimated at energies above 10 Mev for nucleon-nucleon scattering. From the analysis of § 3 and § 4 it seems to be \( \delta_0^0 \lesssim 48^\circ \) for 18 Mev nucleon-nucleon scattering.

3) Generally speaking, if the three phase shifts of \( ^3P \) are larger (in the absolute value) and more separated from one another, \( \sigma_{out}^{sp} \) is larger, and it is favourable for the photodisintegration of the deuteron. These phase shifts are no longer small for 18 Mev nucleon-nucleon scattering.

4) To account for both \( p-p \) scattering and photodisintegration of the deuteron consistently, \( \delta_0^0 \) would be smaller, and \( \delta_1^1 \) would be larger and separated from one another, than those previously given.

5) The influence of the wave function of the deuteron is small at \( E_r \simeq 11 \) Mev.

Similar analysis at higher energies to show up many properties of nuclear forces and further analysis near our energy to reveal a concrete shape of the nuclear potential are now being carried out and will be published in due time.

The author is much indebted to the financial support by the Yomiuri Yukawa fellowship.

Appendix A

\( U_{st} \) of (9) tends to \( \infty \) at \( r=0 \), whereas \( U_{st} \) must vanish at \( r=0 \). Therefore we need to examine the accuracy when we approximate \( U_{st} \) by (9). (9) can be rewritten as

\[
U_{st} = r \left( j_1(kr) \cos \delta_1^1 - n_1(kr) \sin \delta_1^1 \right) \tag{9}'
\]

To estimate the accuracy of approximation, we calculated the contribution of the last term of (9)' from \( r \leq 1/k \) to \( I_{st} \) of (6) (Note that, \( \lim_{r \to 0} j_1(kr) = 0 \), and \( \lim_{r \to 0} n_1(kr) = -\frac{1}{k^2 r^2} \)).

The integrated form of \( I_{st} \) can be written as

\[
I_{st} = G_s \cos \delta_1^1 + H_s \cos \delta_1^1 \tag{6}'
\]

Here \( G_s \) is due to \( j_1(kr) \), and \( H_s \) is due to \( n_1(kr) \) of (9)'. At our energy

\[
G_s \simeq 2H_s \tag{A}
\]

In \( H_s \) the contribution from \( r \leq 1/k \) (we denote it by \( H_s' \)) can be expressed by
From (6'), (A) and (B) we get

\[ \frac{H'}{\sin \delta / I_\text{H}} \lesssim 0.04, \text{ when } \delta = 20^\circ, \]

and

\[ \frac{H'}{\sin \delta / I_\text{H}} \lesssim 0.02, \text{ when } \delta = \pm 5^\circ. \]

Therefore the approximation of (9) is not bad.

### Appendix B

Applying the effective range theory, the singlet effective range \( r_s \) can be expressed as the function of the singlet scattering length \( a_s \), the hard core radius \( D_s \), and \( \zeta \) of (10) by

\[ r_s/2 = P + Q/\zeta + R/\zeta^2 + S/\zeta^3 \]

Here

\[ P = D_s + a D_s^2 + a^2 D_s^3/3, \quad Q = 3/2 \left( 1 + 2 a D_s + 2 a^2 D_s^2 \right), \]

\[ R = a^2 (7 + 13 a D_s), \quad S = 15 a^3/2, \quad \text{and} \quad a = -1/a_s \]

If \( r_s, a_s, \) and \( D_s \) are given, \( \zeta \) can be computed from (C) and (D). (See Table 1)

For \( r_s \) and \( a_s \) we used \( r_s = 2.7 \times 10^{-13} \text{cm} \), and \( a_s = -23.68 \times 10^{-13} \text{cm} \).

### References

5) For Brief explanation see S. H. Hsieh, Prog. Theor. Phys. 16 (1956), 68.
15) Lew Allen, Private Communication.