The magnitude of load capacity is shown in Fig. 2 with dashed lines. Both load capacity and frictional moment in this case decrease by 33 per cent of theoretical values considering no sliding.

Conclusions

The analysis of the fluid lubrication for roller bearings leads to the following conclusions.

(1) The character of non-Newtonian lubricant such as Bingham plastic is distinguished from Newtonian lubricant at low speed, especially for starting friction due to its yield stress. At high speed, load capacity and friction for non-Newtonian lubricant are almost equal to those of Newtonian lubricant which has the same amount of viscosity as the plastic viscosity of non-Newtonian lubricant.

(2) Under unsteady load condition, the effect of squeeze film is remarkable. Then, the load capacity tends to increase, while the variation of friction with its average value is nearly equal to that under the constant basic load which is the average value of vibrating load.

(3) The sliding between rollers and raceways cannot be avoided in the case of fluid lubrication. Considering the sliding of rollers, the frictional moment and load capacity in complete fluid lubrication of roller bearings show a decrease by 33 per cent of those in the case of assumed no sliding.

References

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2 J. F. Osterle, "Fluid Lubrication of Roller Bearings," *Wear*, vol. 2, 1959, p. 195.

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DISCUSSION

Fletcher Osterle[2]

In their analysis of the pressure distribution in the film between rotating cylinders, the authors employ the boundary condition that the pressure vanishes at the point of nearest approach. They claim that this condition is "often used in the study of lubrication theory." In point of fact it is generally accepted today that this boundary condition is incorrect and that the pressure vanishes at that point in the film where the pressure gradient vanishes. This is the so-called Reynolds' boundary condition and follows from a consideration of cavitation in the film. In the case of two identical cylinders rotating without sliding and employing a Newtonian lubricant, the load capacity calculated on the basis of the incorrect boundary condition is 18 per cent low.

At the end of Part I, the authors assert as their third conclusion that, in the case of perfect lubrication (by which they mean Newtonian), (1) the peripheral velocity of a cylinder driven by sliding friction is $^1/_3$ that of the driving cylinder, (2) the friction on the driver cylinder is $^4/_3$ the no-sliding case, and (3) the load capacity is $^2/_3$ the no-sliding case. These results are totally incorrect and as such invalidate their results (in Part II) with respect to the lubrication of roller bearings with sliding considered. Using the correct boundary condition, the discusser finds from his own paper on the hydrodynamic lubrication of roller bearings (reference [2], Part II) that the velocity ratio is 0.107 and not $^1/_3$, the friction ratio is 1.11 and not $^4/_3$, and the load ratio is 0.55 and not $^2/_3$. Even for their own boundary condition the authors' results are incorrect. For their boundary conditions (x_a infinite) the velocity of the driven cylinder is zero. Purday's book "An Introduction to the Mechanics of Viscous Flow" can be referred to for this result.

Authors' Closure

The authors wish to thank Professor Osterle for the contribution of the interesting comparisons with his theoretical research.

It may be noticed that there is another boundary condition by M. R. Hopkins.[3] This Hopkins' boundary condition is that the pressure vanishes at the point where the flow becomes discontinuous and the lubricant divides between two rotating cylinders. At this point, midway between two cylinders, the velocity is zero since a finite velocity there would mean a discontinuity in the vector velocity of a particle requiring infinite accelerations of an impossible nature.

According to this boundary condition, the pressure gradient is not zero but must be negative at the point where the pressure vanishes. Consequently, subambient pressure appears upstream from this point. In the case of two identical cylinders rotating without sliding and employing a Newtonian lubricant, the pressure distribution curve calculated on the basis of this boundary condition lies between two pressure distribution curves on the basis of the Reynolds' and the Gümbel's boundary conditions and is nearer to the pressure curve on the basis of the Gümbel's condition than that on the basis of the Reynolds' condition. At $x=0$, where the Gümbel's condition assumes $p=0$, the Reynolds' condition yields $p=0.0633\,\dfrac{12\mu\bar{U}\sqrt{2rh_0}}{h_0^2}$ and the Hopkins' condition $p=0.0183\,\dfrac{12\mu\bar{U}\sqrt{2rh_0}}{h_0^2}$. The load capacity calculated on the basis of the Hopkins' condition is 47.5 per cent lower than that on the basis of the Reynolds' condition because of the existence of the subambient pressure.

The Hopkins' boundary condition may be rigorous for the separation of lubricant between two rotating cylinders. The Reynolds' boundary condition gives the continuous pressure distribution but is not entirely correct. Therefore, the authors chose the Gümbel's boundary condition for convenience to analyze the fluid lubrication of roller bearings lubricated with Newtonian lubricant together with those lubricated with non-Newtonian lubricant. The authors needed also to use the Gümbel's boundary condition for the analysis of the case under unsteady load condition.

It may be better generally to use the Reynolds' boundary condition; however, the comparison between the results obtained on the basis of the Reynolds' and the Gümbel's boundary conditions, as Professor Osterle shows, seems to be necessary and useful for these problems of lubrication, since the actual boundary condition would be between them as the Hopkins' boundary condition.

The authors wish to read Purday's book and refer to it for their results, although they have not yet been able to get the book. At present, the authors do not think that the velocity of driven cylinder is zero for the Gümbel's boundary condition which they used.

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[3] M. R. Hopkins, "Viscous Flow Between Rotating Cylinders and a Sheet Moving Between Them," *British Journal of Applied Physics*, vol. 8, 1957, p. 442.