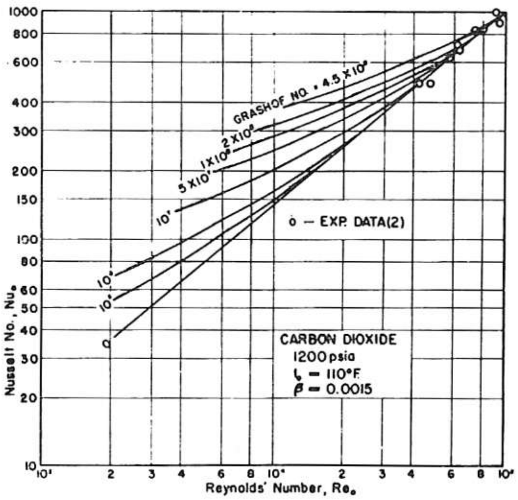

Fig. 6 Nusselt number versus Reynolds number with Grashof number as parameter

The density and viscosity values were obtained from Bringer [2]. Reference to Fig. 6 shows that natural convection would not effect the Nusselt number under these conditions, unless Re_o was less than about 20,000. Bringer's data were obtained at Re_o greater than 45,000. The experimental values are shown by the circled points in Fig. 6 and represent the same data as indicated in Fig. 4.

Conclusions

The proposed method of predicting the influence of density variations on the transfer of momentum and heat indicates a significant effect if the fluid is in the critical region. The result for heating is to flatten the velocity and temperature profiles and increase the heat-transfer coefficient at a given Reynolds number. The limited experimental data available indicate that the proposed method is an improvement over the constant-density approach. Additional measurements are desirable to solidify this conclusion.

The analysis proposed for the influence of natural convection shows that this effect also flattens the velocity profile and increases the heat-transfer coefficient. Again, the effect would be significant only when there is a large variation in density across the tube radius, as when the fluid is near its critical point. Experimental data are needed at relatively low Reynolds numbers and high values of the Grashof group in order to test the analysis presented.

Acknowledgment

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DISCUSSION

R. G. Deissler[3]

The authors' calculations on the effects of density variation and of free convection were of considerable interest to the writer. The results give an indication of the conditions under which free convection effects become important in comparison to forced convection for turbulent heat transfer, and thus should be of considerable value in the design of experiments.

The writer would prefer to write equation (1) as

$$\tau = \rho\epsilon dU/dy$$

rather than as

$$\tau = \epsilon d(\rho U)/dy$$

If we expand the latter equation, we obtain

$$\tau = \epsilon(\rho dU/dy + U d\rho/dy)$$

Thus according to that equation a turbulent shear stress will be produced by the density gradient even when the velocity is uniform. But there seems to be no physical mechanism by which that would occur. If two adjacent layers of fluid are moving at the same velocity then chunks of fluid which are thrown back and forth between the layers will produce no shear force between the layers regardless of whether or not the chunks coming from the two layers are at the same density.

Also, it appears that Prandtl's mixing length theory can be modified so as to apply to flow with a density gradient. Consider two fluid layers 1 and 2 which are separated by a distance of one mixing length. If the mean axial velocities and densities in the two layers are U_1, ρ_1, and U_2, ρ_2, then the rate at which the axial component of momentum leaves layer 1 per unit area is $\rho_1 U_1 v_1$, where v_1, is the transverse fluctuating velocity. Similarly, the rate at which momentum enters region 1 from region 2 per unit area $= \rho_2 U_2 v_2$. But by conservation of mass, $\rho_1 v_1 = \rho_2 v_2 = \rho v$, where density changes in the axial direction have been assumed small in comparison with those in the transverse direction. By conservation of momentum,

$$\tau = \rho_1 v_1 U_1 - \rho_2 v_2 U_2 = \rho v(U_1 - U_2)$$
$$= \rho v l \, dU/dy = \rho\epsilon dU/dy$$

where l is the mixing length, or the distance between the fluid layers 1 and 2. Thus according to the mixing length theory, it appears that the density ρ in the expression for turbulent shear stress should be outside the derivative sign. Although the foregoing arguments cannot be considered as a rigorous proof, they give plausible reasons for writing the shear stress as

$$\tau = \rho\epsilon \, dU/dy$$

It is for these reasons that the writer would choose this expression for the turbulent shear stress in preference to the one used in the paper.

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Kurt Goldmann[4]

This paper presents an attempt at refining an analytic method for predicting heat transfer and fluid friction with variable properties proposed by Deissler [4]. In a discussion of that reference, this writer commented upon two basic assumptions which may seriously limit the applicability of the analysis. The same two assumptions are used in the analysis presented in the present paper and comments concerning them appear to be as valid now as they were then.

The first assumption is that the empirical constants K and n, which have been obtained from isothermal tests, have the same values for nonisothermal flow of fluids with variable properties. There is no a priori justification for this assumption. Using a different approach, developed by this writer and briefly described in the discussion of reference [4] and presented in considerable detail in reference [14],[5] heat-transfer and fluid-friction parameters can be predicted without making this assumption. The validity of this approach can be demonstrated by observing the good agreement between predicted heat transfer and shear stress parameters from reference [14] and the recently declassified experimental results of reference [15].

The second assumption, made in all of these analyses, is that the mechanism of heat transfer is that usually observed in single phase turbulent flow. In the discussion of reference [4] this writer pointed to the possibility that a boilinglike phenomenon may provide the mechanism for heat transfer to supercritical fluids under certain conditions. As described in another recently declassified paper [16], an unusual mode of heat transfer has actually been observed with high velocity water at supercritical pressures during tests at high heat fluxes and other conditions under which such "boiling" would be expected to occur. Visual observations and photographic records made during a recent study of heat transfer to supercritical Freon 114A [17] also appear to confirm that boiling hypothesis.

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[5] Numbers in brackets from [14 to 17] designate References at end of discussion.

Authors' Closure

The authors wish to thank Messrs. Deissler and Goldmann for their interesting comments. In Mr. Deissler's comments the question is how the equation for shear stress should be written. As the authors pointed out in the text, the shear stress is the transfer of momentum. Thus it should be $\epsilon\partial\,(\rho U)/\partial y$. On the other hand, as shown in Mr. Deissler's discussion, the basis for writing $\tau = \rho\epsilon\partial U/\partial y$ is the assumption that $\rho U = \rho_1 U_1 = \rho_2 U_2$. This assumption implies the absence of mass flux in the transverse direction, a good assumption for the case of forced convection. However, with the change of the velocity profile into a flatter form or even with a dip at the center of the tube, as shown in Fig. 3, it is necessary that there be present a certain extent of internal circulation; therefore $\rho_1 U_1 \neq \rho_2 U_2$.

Mr. Goldmann's first comment is about using constants n and k for the nonisothermal case. It might be noted that, when Equations (10) and (11) replace Equations (8) and (9), the n^2 and k^2 are replaced, respectively, by $(1 + F_m)n^2$ and $(1 + F_m)k^2$. Both are variables. In fact, the introduction of F_m was aimed at taking into account variations in density, which are an important factor in the nonisothermal case.

Mr. Goldmann also suggested that supercritical heat transfer can be treated as analogous to boiling heat transfer. The authors agree that the mechanism of heat transfer in the immediate vicinity of the critical point may be different from the conventional ones in homogeneous fluids. A mechanism similar to boiling may be involved. In an attempt to verify this often suggested explanation for unusual heat-transfer results in the critical region, radial temperature and velocity profiles are now being studied at Northwestern University.