Inversion of the amplitude of the two-dimensional analytic signal of the magnetic anomaly by the particle swarm optimization technique

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Accepted 2010 April 16. Received 2010 April 13; in original form 2009 August 23

SUMMARY
Amplitude of the 2-D analytic signal of the magnetic anomaly profile is independent of the directions of the Earth’s magnetic field vector and remnant magnetization of the causative source. It exhibits peaks corresponding to the locations of the corners of a causative source, modelled by say a polygon. It also exhibits a peak corresponding to different idealized source geometries related to the structural indices. This amplitude is computed from the first-order horizontal and vertical derivatives of the observed magnetic anomaly and is relatively less noisy than second-order derivatives. The amplitude can also be computed directly from the measured derivatives. Particle swarm optimization (PSO)—a global optimization technique is applied to interpret this amplitude in terms of the horizontal location and depth, constant (related to magnetization) and various source geometries through structural indices. Applicability of the proposed technique is evaluated through the analyses of simulated magnetic anomalies (noise-free and corrupted with 20 per cent random noise) over different types of source geometries, namely, a thin dyke and a contact with high accuracy in parameter estimation. Studies on the choices of search parameter space reveal that a relatively wide search space can be assigned. Practical applicability of the proposed technique has been demonstrated through three magnetic anomaly profiles digitized from published literature. The results of PSO, Euler deconvolution, enhanced local wavenumber and drill hole are comparable. PSO results also seem to be more stable than other techniques.

Key words: Inverse theory; Geopotential theory.

1 INTRODUCTION
Nowadays, vast amount of magnetic data are collected for the environmental and geological applications, including mineral and oil exploration and groundwater investigations. A reliable depth estimate to the top of the causative source improves in budgeting and planning of drill holes and exploitation programme. In oil exploration, for example, reliable estimates of depth to magnetic basement are essential for better understanding of the critical first-order basin-exploration parameters (Li 2003). Magnetic data are also required for mapping of the sedimentary basins, faults, folds, channels and salt structures for oil exploration. Several automated techniques are available in the literature to estimate horizontal locations and depths, and geometries of the causative sources in a reasonable time and cost. These techniques help geophysicists to cope up with the ever-increasing size and resolution of aeromagnetic surveys having their own merits and demerits.

Geophysicists aim at deriving the best possible model from the observed magnetic anomaly. The quantitative interpretation of a potential field data involves estimation of the horizontal location and depth, the source geometry and contrast in the physical property. Such an interpretation suffers from an inherent ambiguity (Skeels 1947; Roy 1962) because there is lesser number of known quantities than the number of unknown parameters describing the geological model. Therefore, it is impossible to obtain all the information simultaneously without an a priori assumption. Nabighian et al. (2005) broadly classified these techniques in three categories, namely, (i) depth-to-source estimation, (ii) physical parameter mapping and (iii) inversion. The depth-to-source estimation can be further grouped into various techniques, namely, Werner deconvolution (Werner 1955; Ku & Sharp 1983), statistical methods (Spector & Grant 1970; Syberg 1972), symmetric and antisymmetric analysis (Naudy 1971), CompuDepth (O’Brien 1972), analytical signal (AS) (Nabighian 1972, 1974; Agarwal 1984; Hsu et al. 1996; Bastani & Pedersen 2001; Salem & Ravat 2003; Keating & Pilkington 2004; Salem et al. 2004; Salem 2005), resultant gradient from modified observed anomaly (Shaw & Agarwal 1997; Agarwal & Shaw 2005), Euler deconvolution (Thompson 1982; Keating & Pilkington 2004), source parameter imaging (Thurston & Smith 1997), local wavenumber (Smith et al. 1998; Thurston et al. 2002; Salem et al. 2005; Smith & Salem 2005) and continuous/discrete wavelet transform (Moreau et al. 1997; Hornby et al. 1999; Ridsdill-Smith & Dentith 1999; Fedi et al. 2004). The physical property mapping is either carried out by terracing (Cordell 1999; Ridsdill-Smith & Dentith 1999; Fedi et al. 2004). The physical property mapping is either carried out by terracing (Cordell 1999; Ridsdill-Smith & Dentith 1999; Fedi et al. 2004).
& McCafferty 1989) or susceptibility mapping (Grant 1973; Bhattacharyya & Chan 1977; Macdonald et al. 1980; Misener et al. 1984; Silva & Hohmann 1984). In inversion methods, a model is parametrized to describe either source geometry (Bott 1960; Whitehill 1973; Pedersen 1977; Pedersen 1979; Ballantyne 1980; Bhattacharyya 1980; Silva & Hohmann 1983; Pilkington & Crossley 1986; Pustisek 1990; Wang & Hansen 1990) or the distribution of a physical property such as magnetic susceptibility (Parker & Huestis 1974; Li & Oldenburg 1996; Pilkington 1997). Most of the inversion methods assume that the causative source is magnetized in the direction of the Earth’s magnetic field. As a result, their application is limited when strong remanent magnetization alters the total magnetization direction of the source and the computed model parameters are unstable. This problem could be tackled by either 3-D non-linear inversion to recover the magnetization magnitude by inverting the amplitude of the anomalous magnetic vector or the total gradient of the magnetic anomaly (Shearer & Li 2004) or by using 3-D joint inversion of the magnetic anomaly due to a vertical right circular cylinder with arbitrary polarization and its analytic signal (Chen et al. 2009).

Agarwal & Shaw (1996) calculate the analytic signal of a single magnetic pole anomaly and conclude in general, that the analytic signal is not symmetrical for arbitrary values of inclinations and declinations. A circular symmetry is observed for a field inclination of 90°, and for an inclination of 0° the anomaly is symmetric along the axis of the declination. In general, the anomaly is nearly circular and slightly elongated along the geomagnetic field declination. Haney et al. (2003) have substantiated the above observation through a mathematical formulation.

Though a number of techniques exist in the literature to invert 3-D analytic signal over assumed source geometry (Keating & Saillhac 2004 and Chen et al. 2009), none is available over generalized models because of the complexities involved in mathematical formulation. However, such complexities do not exist in the 2-D analytic signal of the magnetic field, which is independent of both geomagnetic field direction and the direction of polarization of the causative source (Nabighian 1972). However, it is worth mentioning here that no such inversion scheme has been developed using 2-D analytic signal. Therefore, we visualize the feasibility of a generalized inversion process from 2-D analytic signals or the total gradient of the magnetic anomaly to obtain an equation which is independent of the causative source geometries, namely, the contact, the pole and the dipole, involving unknown parameters, namely, horizontal location and depth only. By knowing the horizontal location and depth parameters, they further determined the inverse scattering (Donelli et al. 2006) and geophysics (Wilken et al. 2009).

PSO is based on the social behaviours of birds, fishes and insects such as ant, termites, etc., especially their knowledge sharing while food foraging. This has drawn attention in understanding the artificial form of intelligence and in developing efficient optimization algorithm (Kennedy & Eberhart 1995). The synchrony observed in flocking of birds—the V-shaped flight pattern—is the result of an effort of the flock(s) to optimize the interbird distance and maximize the energy saving in flight formation (Weimerskirch et al. 2001). Kennedy & Eberhart (1995) view PSO as a middle level form of the biological evolution spanning over thousands of years and the neural processing working in milliseconds range. PSO—a simple algorithm—has been successfully used in finding the minima for a wide range of functions commonly used as benchmark functions, namely, (i) Sphere, (ii) Rosenbrock, (iii) Rastrigrin, (iv) Griewank and (v) Schaffer F6 functions to test the efficiency of an optimization algorithm (Carlisle & Dozier 2001). It deals away with mimicking the complex processes of gene transfer and mutation from one generation to another required in GA or finding analogues of thermodynamic processes as in SA (Sen & Stoffa 1995). On the other hand, PSO has a memory component for each particle in the swarm so that both the cognitive and social knowledge of the particles are used simultaneously in deciding the excursion of the solution in the model space. The individual knowledge of a bird drives it towards the best location already known and its social knowledge allows flying towards a better location searched by other birds in the flock. Shaw & Srivastava (2007) applied PSO for the first time to invert various types of geophysical data. They compared their results with the ridge regression and GA. Sanyi et al. (2009) applied PSO for the inversion of various geophysical data and Wilken et al. (2009) for high-resolution reflection seismic data. In view of the above-mentioned advantages, we prefer PSO over other optimization techniques for the inversion of 2-D analytic signal of magnetic anomaly over generalized source geometry.

Applicability of the proposed technique has been evaluated through simulated magnetic anomalies over a contact and a thin dyke by selecting relatively large parametric space. We have further selected three magnetic profiles taken from published literatures and inverted composite AAS of measured magnetic anomaly (AASM) by PSO to determine horizontal location and depth, amplitude and structural index of causative sources. We have further compared the results obtained from PSO with those from the current state-of-the-art techniques in magnetic anomaly interpretation, namely, enhanced local wavenumber (ELW) and Euler deconvolution technique (EUL) methods. This comparison reveals that PSO results are, in general, comparable to those obtained by ELW and EUL in terms of stability of solutions and resolvability.

In view of the comparison of results obtained from PSO with ELW and EUL, a brief account of these techniques is given below.

### 1.1 Enhanced local wavenumber (ELW)

Thurston & Smith (1997) has developed an elegant interpretation technique based on the concept of the local phase of the 2-D analytic signal of any component of the Earth’s magnetic field measurements. They have used second-order derivatives (horizontal and vertical) of magnetic anomaly to obtain an equation which is independent of the causative source geometries, namely, the contact, the thin sheet, the pole and the dipole, involving unknown parameters, namely, horizontal location and depth only. By knowing the horizontal location and depth parameters, they further determined the
source geometry. The basic disadvantage of this technique is associated with the use of second-order derivatives, which are considerably sensitive to the random noise present in the observed magnetic field. However, the recent techniques of data acquisition are capable of measuring both fields and its various gradients. These gradients can also be utilized directly in determining the model parameters. Lyrio et al. (2004), While et al. (2006), Mikhailov et al. (2007) and Pajot et al. (2008) have developed noise reduction techniques for gradient measurements. Further, the estimated model parameters are dependent on the selection of the window length. Salem et al. (2005) computed the source parameters ‘... using the strategy of rejecting false peaks and using a single window of 21 data points ...’. We have used a FORTRAN code published by Agarwal & Srivastava (2008) to analyse field anomalies.

1.2 Euler deconvolution technique (EUL)

Thompson (1982) developed an automated technique, based on Euler Homogeneity equation, to compute horizontal location and depth from the analyses of magnetic anomaly by assuming causative source geometry. This technique makes use of first-order horizontal and vertical derivatives of the magnetic anomaly. A set of normal equations are then solved to determine horizontal location and depth by assuming source geometry. Euler deconvolution is similar to ELW technique and requires several solutions to be obtained with a moving window of pre-assumed length. All the solutions thus obtained are plotted and the interpreter looks for the clustering in a moving window of pre-assumed length. All the solutions thus obtained are plotted and the interpreter looks for the clustering in a moving window of pre-assumed length. All the solutions thus obtained are plotted and the interpreter looks for the clustering in a moving window of pre-assumed length. 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where $K$ is the amplitude factor related to the physical properties of the source, $x_0$ and $z_0$ are the horizontal location and depth of the source, respectively, and $q$ is known as shape factor. The eq. (7) attains a maximum at $x = x_0$. Salem et al. (2004) related the shape factor, $q$ and the structural index, $N$ (Thompson 1982) for different source geometries by the equation $N = 2q − 1$. Thus, the structural indices of 0, 1 and 2 for magnetic anomalies over a contact, a thin dyke and a horizontal cylinder, respectively, correspond to shape factors as 0.5, 1.0 and 1.5, respectively. In general, the magnetic anomaly is asymmetrical in nature because of the source magnetization and the Earth’s magnetic field directions. Nabighian (1972) has shown that the AAS over an individual corner of a magnetic source will be symmetrical irrespective of the direction of the Earth’s magnetic field and source polarization vector. The AAS of a 2-D magnetic source model approximated by an $m$-sided polygon will be the sum of the $m$ symmetric bell-shaped functions (eq. 7) approximately centred over corners of a polygon (Nabighian 1974; Huang & Guan 1998). In general, this sum of AAS will contain maximum influence from the upper corners of the source and the contribution from the bottom (lower) corners may be, in general, negligible. To substantiate this point, let us evaluate eq. (7) by assuming, $q = 1$, $K = 100$, $x = x_0$ and depth, $z_0$ assumes 3 and 10 units of data spacing. The AAS for depth 10 units drops down to approximately 90 per cent to that of for depth three units. Thus, if top and bottom of a source (forming vertically separated two corners) are at depths of three and 10 units, respectively, then it is possible to detect only the top of the source from AAS. Similarly, if the two corners of the same magnitude $K$ are separated horizontally, the corner at a large depth would be difficult to resolve by AAS because of its relatively small contribution. In general, the constants, $K$s are different in actual field situation. It is worth mentioning here that Salem et al. (2002) have also used similar bell shaped function (eq. 7) to estimate depth and shape factor.

We shall consider the composite AASM, to consist of several bells (eq. 7). PSO is used to invert AASM to determine horizontal location and depth, amplitude factor ($K$) and shape factor ($q$) for each corner of the assumed model. To determine these parameters, we have to specify search spaces for the constant $K$, the shape factor $q$, and the horizontal location and depth (in terms of data spacing).

4 STRATEGIES ON FIXING PARAMETER SEARCH SPACE

Proper selection of initial guess values for model parameters is an important part in any inversion strategy. These guess values depend upon the type of inversion strategy adopted. Recent optimization techniques, namely, SA, GA and PSO, works on a large search parameter space. Further, in every interpretation sampling interval (data spacing) plays a considerable role towards proper selection of model parameters. As a thumb rule, a continuous anomaly profile is better represented provided the ratio of depth to sampling interval is between 4 and 15 (Shaw et al. 2007; Srivastava & Agarwal 2009).

From the qualitative interpretation of AASM, we can estimate the horizontal position of the source corner roughly coinciding with the peak magnitude: the depth from half width distance and magnitude of constant from peak amplitude and depth. We have observed that the computation of the horizontal location is a substantially stable parameter and varies in a small range only. The maximum variation in shape factor can be chosen, say from 0.2 to 2.5 to incorporate all types of source geometries and a considerable large depth range varying from 1 to 25 units of data spacing. To have an idea of the variation in the magnitude of the constant $K$ (eq. 7), we compute probable range from the appropriate product of depth and shape factor (i.e. peak amplitude of AASM multiplied by the depth squared term by assuming shape factor as 1). With these factors in mind, the model parameter search space can be selected with good accuracy.

5 SIMULATED EXAMPLES

To evaluate the effectiveness of the proposed inversion technique based on PSO, we have selected two models, namely, a thin dyke and a contact. The total magnetic field anomaly due to these models is computed from equations derived by Nabighian (1972) for the parameters shown in Tables 1a and (b). In each case, the length of profile is 101 data points at an equi-spaced interval of 1 m. The simulated anomaly for a particular source model is also corrupted with random errors using RAN2 (.) subroutine (Press et al. 1984). The magnitude of the random noise is varied from 5 to 20 percent of the maximum amplitude of magnetic anomaly in steps of 5 percent.

2-D analytic signal has been computed from this noise corrupted simulated anomaly based on frequency domain filtering operations as described by Agarwal & Srivastava (2008). The search parameter space is chosen as per procedure outlined in Section 4. To study the influence of starting model (or the limits of the parameter space), a thin dyke model is interpreted with four different model spaces (Table 1a) showing substantially large variations. For shape factor, a wide range from 0.2 to 2.5 has been fixed as it varies only from 0.5 to 1.5 for all assumed source geometries, namely, a contact, a thin and a thick dyke. Further, in view of the thumb rule proposed earlier, a range for selection of depth parameter is assigned between 1 and 15 and/or 1 and 25 in units of data spacing. The range in the horizontal location can be assigned easily on the basis of the peak position in AASM. With such a large search parameter space (Table 1a), we have computed source parameters of all the assumed thin dyke models from PSO (Table 1a). A study of the Table 1a reveals that the rms error increases with increase in noise percentage in the simulated anomaly. Further, the parameter corresponding to the horizontal location of the source is very stable and accurate. The other source parameters, namely, the amplitude, depth and shape factor are interrelated with each other. Generally, any increase in the value of shape factor is associated with decrease in the amplitude and depth parameters (Table 1a). The maximum errors in depth and shape factor are found to be 16 and 20 per cent, respectively. The maximum variation from 40 to 87 per cent is observed for amplitude parameter. Similar studies are also conducted to analyse simulated anomaly due to a contact model (Table 1b). With the outcome of the thin dyke models, the variations in horizontal location have been reduced and depth parameter is varied in two sets, namely, 1 to 15 and 1 to 25, for inversion. PSO results, presented in Table 1b, reveal that all unknown parameters for contact models are very stable as compared to thin dyke model (Table 1a) analysed earlier.

Efficacy of the PSO technique has been further evaluated by analysing two peaks in AASM over two thin dykes of opposite/similar polarity with horizontal separation larger than the depths of the two sources. The model parameters of two thin dykes of opposite polarities are given in Table 1c. The total field magnetic anomaly (AM) and its AASM are shown in Fig. 1. It is obvious from Fig. 1 that the total field magnetic anomaly over two sources looks similar.
to an anomaly due to a single source which corresponds to a loss of resolution in the anomaly. However, AASM for this model exhibits two well-defined peaks situated close to their assumed horizontal positions. This profile is interpreted in two ways: (1) each peak individually and (2) two peaks collectively. For analyses of individual peaks, large variations in the model search spaces (Table 1c) are considered as per strategy outlined in Section 4. The inversion results from PSO are given in Table 1c which clearly reveal that both sources are thin dykes and computed model parameters are close to assumed ones. In the second approach, both the peaks are fitted together. As per earlier strategy, the model parameter search space (Table 1c) is varied considerably. The model parameters obtained from composite data are given in Table 1c. Fig. 1 displays a graph between AASM and AASC (computed from derived model parameters) from both the strategies and reveals an excellent match to confirm the correctness of both the interpretations.

### 6 FIELD EXAMPLES

#### 6.1 Matheson area, Northern Ontario, Canada

The total field aeromagnetic anomaly (Keating, private communication, 2005) over a magnetic body in the Matheson area of northern Ontario, Canada, is digitally acquired at an interval of about 12 m (Fig. 2). The flight height of the survey is 70 m above the ground level and the station spacing approximately 12 m (Ontario Geological Survey 2000; Salem et al. 2005). The survey was flown along N30°W, which is roughly perpendicular to the strike of the major geological features of the area. The anomaly is associated with a mapped bedrock diabase dyke and a borehole intersects the bedrock at 41 m (Vallee et al. 2004).

The amplitude of 2-D analytic signal, AAS (using eq. 6), from the measured magnetic field anomaly requiring the computation...
Table 1c. Model parameters computed from PSO for two thin dykes with various model space along with rms error.

<table>
<thead>
<tr>
<th>Model space</th>
<th>Computed model parameters</th>
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<tr>
<td></td>
<td>True model</td>
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<td>Left bell</td>
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<td>Amplitude</td>
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<td>( x_0^a )</td>
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<td>rms Error</td>
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<td>Right bell</td>
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<td>Amplitude</td>
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<tr>
<td>( z_0^a )</td>
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<td>( q )</td>
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<td>rms Error</td>
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<td>Two bells together</td>
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<td>Left bell</td>
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<td>Amplitude</td>
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<td>rms Error</td>
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\( x_0^a \) and \( z_0^a \) are the coordinate of the source location in unit of data spacing = 1 m.

Table 2a. Search space for Matheson area, Northern Ontario, Canada.

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<th>Model space</th>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude</td>
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<td>5000</td>
</tr>
<tr>
<td></td>
<td>( x_0^a )</td>
<td>35° (420)</td>
<td>60° (720)</td>
</tr>
<tr>
<td></td>
<td>( z_0^a )</td>
<td>5° (60)</td>
<td>15° (180)</td>
</tr>
<tr>
<td></td>
<td>( q )</td>
<td>0.5</td>
<td>1.5</td>
</tr>
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</table>

\( x_0^a \) and \( z_0^a \) are the coordinate of the source location in unit of data spacing = 12 m. Values in the parenthesis are in metres.

Figure 1. Simulated total field magnetic anomaly profile over two thin dykes. AM, measured total field magnetic anomaly; AASM, amplitude of 2-D analytic signal computed from measured anomaly; AASC, amplitude of 2-D analytic signal calculated using interpreted model parameters from PSO. The ‘+’ sign indicates the horizontal location of the source.

Figure 2. Total field magnetic anomaly profile over Matheson area, Northern Ontario, Canada (after Salem et al. 2005 and Keating 2005). AM, measured total field magnetic anomaly; AASM, amplitude of 2-D analytic signal from measured anomaly; AASC, amplitude of 2-D analytic signal computed using interpreted model parameters from PSO. The ‘+’ sign indicates the horizontal location of the source.

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6.2 Boston Township, Ontario, Canada

The amplitude of the vertical magnetic anomaly (Fig. 3) of Boston Township, Ontario, Canada (Grant & West 1965) is more than four times the intensity of the Earth’s magnetic field. This is due to a very shallow body that contains magnetite up to 30 per cent by volume. The magnetic inclination is 78° and the body strikes almost in the magnetic meridian. The maximum positive and negative amplitudes are +217 000 and −75 000 nT. On the basis of the result from a drill hole, Grant & West (1965) interpreted this anomaly using two ribbon models with opposing magnetic moments and separation ~9 m, and depths 2.5 and 4.5 m, respectively. This anomaly has been digitized at 0.61 m interval. The AASM exhibits two peaks of considerable different amplitudes (Fig. 3). Though, the measured anomaly in absence of any drill hole information, will be, in general, interpreted as a single source, however, the AASM clearly reveals the presence of two sources. We have inverted AASM by simultaneously fitting two bells corresponding to two peaks. The model parameter search spaces for both the bells are given in Table 3a. The rms error in this case is 1.54 nT. Table 3b gives the interpreted model parameters computed by PSO, EL W and EUL techniques. It is obvious from Table 3b that ELW and EUL techniques could not delineate Bell #1 and gave stable solutions for Bell #2 only. Both the peaks analysed by PSO indicate the nature of the causative sources to be horizontal cylinders separated by 8.0 m (= 27.8 – 19.8). The structural indices for the second peak by ELW and EUL indicate that the causative source may be approximated between a thin dyke and a horizontal cylinder (Roy 2001).

The analysis of the AASM by PSO technique reveal that the structural indices for two sources associated with corresponding peaks are 1.60 and 1.96, respectively. The structural index, approximately equals 2 (i.e. 1.96) for the second source can be unambiguously interpreted as a horizontal cylinder. Whereas, SI of 1.6 for the first source, at a first glance, could be inaccurate because of some interference effect between the two sources. However, from an equivalent simulated model analysis (Table 1c, and Fig. 1), as discussed in Section 5, the possibility of interference between two individual sources can be ruled out. We can, therefore, conclude that the geometry of the second source is closer to a horizontal cylinder than a thin dyke.

According to Grant & West (1965) ... subsequent drilling confirmed that the iron formation which consists of coarsely banded magnetite is inversely remanently magnetized at its western edge (the first bell). A strong normal remanent magnetization in the direction of the Earth’s field was also discovered in the eastern part (the second bell) of the formation. An overburden thickness of ~3.0 m above the second bell of the formation was confirmed by drilling . . . .

6.3 Barraute, North–West Quebec, Canada

We consider a vertical component magnetic profile (Line 69 + 00SE) taken along a perpendicular to the strike of the pyrite mineralization in Barraute, NW Quebec (Telford et al. 1976). The detailed magnetic survey in the area reveals two parallel pyrite zones in acidic flows, near a contact between the latter and rhyolite...
porphyry having a strike length of more than 305 m. Further, the zone nearer to the contact pinches out. The pyrite mineralization, in general, is associated with the magnetic trend of the area. The profile considered here exhibits reasonably large anomaly, which could be attributed to magnetite or possibly pyrrhotite. However, there is no specific indication of these minerals in the drill logs (Telford et al. 1976).

The magnetic anomaly profile (Fig. 4) digitized from Telford et al. (1976) exhibits two peaks of different magnitudes (approximately 700 and 900 nT). For the sake of clarity in write up, these anomaly peaks (AP) are numbered as AP #1 and AP #2 from left to right of the plot. The AP #2 is much broader as compared to AP #1 indicating that the source corresponding to AP #2 is at a considerable large depth as compared to peak #1. For interpretation, we have digitized this anomaly at an equal interval of 1.5 m. A careful observation of Fig. 4 indicates that AASM for AP #1 exhibits high amplitude, sharp and relatively symmetrical peak. The AASM for AP #2 together. The model parameter search spaces for both the

peak #1, #3 and #4) and two peaks (AASM #1 and #3), respectively. The horizontal location, depth and structural index computed from EUL and ELW are also tabulated in Table 4b for comparison purposes. The computed structural indices for AASM peak #1 and peak #3 are 0.39 and 0.71 from EUL indicating contact and dyke-type sources. Whereas, the interchange of source geometries, namely, a dyke and a contact is indicated by ELW for AASM peak #1 and peak #3, respectively. Thus, ELW and EUL do not provide identical results whereas PSO identifies all AASM peaks associated with thin dykes. Further, a source position corresponding to AASM peak #4 interpreted from EUL provides negative structural index, i.e. −0.45 that cannot be interpreted in terms of any type of source geometry.

In view of the differences in structural indices determined from EUL and ELW, the probable geological interpretation is visualized from PSO results. We interpret AASM peak #1 and peak #2 as two isolated thin dykes with a separation of about 43.3 m, whereas,
AASM peak #3 and peak #4 may be combined together to correspond to a thick dyke of width 16.8 m and top surface sloping towards right side from 10.7 to 13.7 m. Telford et al. (1976) analysed this anomaly by assuming two vertical dipoles of identical cross-section area separated by 48.8 m. The depths to the top of these ore bodies are 9.1 and 22.9 m, respectively. The drill hole adjacent to this line indicates two pyrite zones at depths of 4.6 (39 per cent pyrite) and 6.1 m (35 per cent pyrite), respectively. Thus, geological model visualized with PSO result matches fairly well with the drill hole data.

7 CONCLUSIONS
We have presented a technique using PSO to invert amplitude of the 2-D analytic signal determined from the first-order vertical and horizontal derivatives of the measured magnetic anomaly (AASM) over 2-D structures. These derivatives, if measured directly, can also be used. The AASM exhibits a peak (a bell function) directly situated over a corner formed by an inclined horizontal slab of infinite thickness and length. Apart from the depth, the decay rate (structural index) of AASM is related to the nature of the source geometry described in terms of shape factor. These properties of AASM are utilized in inversion process. For example, a thin dyke exhibits a single peak whereas a thick dyke will show two peaks corresponding to two boundaries. Therefore, AASM, in general, may exhibit more than one peak depending upon the depths of the upper surface of the source geometry approximated by several faulted slabs. A method based on AASM involving all data points is less susceptible to noise present in the observed anomaly than ELW and EUL where the solutions are obtained from a data set within a window of finite length. PSO determines the horizontal location and depth, amplitude and shape factor of the causative source from a very large search space to achieve the most probable global minimum. The efficacy of the proposed technique is evaluated through analysis of simulated anomaly (both noise-free and noise-corrupted) over a thin dyke and a contact with different model parameter search spaces. The inversion studies reveals that PSO can tolerate reasonably large variations in initial search model parameter space and up to 20 per cent of the random noise in the simulated anomaly. Further, PSO results are least sensitive to horizontal position. The applicability of PSO is evaluated through three measured magnetic anomalies taken from published literatures. We have also compared PSO results with well-established techniques, namely, ELW and EUL and the results are comparable.

ACKNOWLEDGMENTS
Thanks are due to the Department of Science and Technology, Government of India, New Delhi, for providing funds under different projects to develop infrastructural facility used in this work. The authors are grateful to Professors M. Diament, M. Everett, D. Gibert and two anonymous reviewers for their constructive criticism leading to substantial improvement in the revised manuscript. Authors are grateful to Drs. Mujibur Rahman and A. K. Behura of Department of English, Indian School of Mines, Dhanbad and Mrs Chaitali Sarkar for improving English of the manuscript.

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