A stacking approach to estimate $V_P/V_S$ from receiver functions

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SUMMARY

This note documents an approach to process a suite of receiver function (RF) traces in order to identify discontinuity-generated $P$-to-$S$ ($Ps$) conversions based on the timing of their associated reverberations. It is inspired by Kumar and Bostock’s method to estimate absolute $P$ wave speeds. From RF estimates recorded in the typical source range for direct $P$ arrivals, the method yields $P$ to $S$ wave speed ratios ($V_P/V_S$) without knowledge of crustal $V_P$ or thickness.

Key words: Time series analysis; Composition of the continental crust; Composition of the oceanic crust; Body waves; Crustal structure.

1 INTRODUCTION

Kumar & Bostock (2008) proposed a method to determine crustal $V_P$ and $V_P/V_S$ beneath a station from a suite of receiver functions (RFs) derived from teleseismic $P$ arrivals at a range of distances. The method relies on the slowness ($p$) dependence of the timing of the reverberations following the direct $P$-to-$S$ conversion ($Ps$) between the surface and the Moho, called $Pps$ and $Pss$ in their terminology. They combined expressions for the timing of $Ps$ and $Pss$ to define a quantity $X$, formally defined as

$$X = \frac{R^2 - p^2 V_P^2}{1 - p^2 V_P^2} = \frac{(t_{Pps} + t_{Ps})^2}{(t_{Pps} - t_{Ps})^2},$$

(1)

where $R = V_P/V_S$ and $t_i$ is the lag time of $x$ in the RF. By transforming (1) into the relation

$$R^2 + V_P^2 [pV_i(X_i - 1)] = X_i,$$

(2)

they demonstrated a way to separately estimate $R$ and $V_P$ from a suite of RFs, with $p_i$ and $X_i$ indicating the slowness and $X$ for the $i$th RF. An earth model provides $p$ from the source and receiver locations, and $X$ is determined from time picks in the RF from (1). With a collection of RFs it is possible to estimate, by least squares, $R$ and $V_P$ and their uncertainties from the uncertainty in time picks $t_i$ propagated into $X$.

They noted the difficulty in reliably estimating $R$ and $V_P$ if RFs at small $p$ were not available. Here we show that a data stacking method that does not rely on individual arrival time picks and can deliver good estimates for $R$ even if RFs recorded at small $p$ are not available.

2 METHOD

The key insight that constitutes the method's basis is the dependence of $X$ on slowness $p$ shown in Fig. 1. $X$ does not strongly depend on $V_P$, but primarily on $R$. This means that even an approximate stacking scheme that sums data across the slowness range will yield good constraints on $X_i$ and thus $R$.

How to achieve this stacking is not obvious because $X$ is not a directly observed quantity in the data, though $p$ is from the source-receiver geometry. Consider eq. (1). This gives a relation between $t_{Pps}$, $t_{Ps}$ and $t_{Pss}$ involving $X^{1/2}$:

$$t_{Pps}(X) = t_{Ps}(X^{1/2} + 1)/(X^{1/2} - 1)$$

$$t_{Pss}(X) = t_{Ps} 2X^{1/2}/(X^{1/2} - 1).$$

(3)

If one views $X^{1/2}$ as a stacking control parameter analogous to the role that slowness plays in a slant stack or vespagram derived from a time-distance diagram, then as $X^{1/2}$ is swept through the plausible ranges of $2 \leq X \leq 6$, the control parameter selects different times after the direct $Ps$ arrival. This mapping connects the time lag in an RF with $X$. Consequently, one may create a stack of a collection of RFs in a $t$-$X$ diagram by summing over $p$. The display will show, for each time lag in the RF collection (each potential $t_{ps}$ arrival), which values of $X$ have $Pps$ and $Pss$ reverberation energy associated with them by combining the signal amplitudes at $t_{ps}$, $t_{Pps}$ and $t_{Pss}$. Fig. 2 shows this kind of display.

The stacked image is referenced to the average slowness of the RF collection (typically around 0.065 s km$^{-1}$ due to the Earth’s source regions), and arrival moveouts are calculated from the RF lag and slowness. From the defining relations for $t_{ps}$, $t_{Pps}$ and $t_{Pss}$ (Zhu & Kanamori 2000; Kumar & Bostock 2008), one obtains

$$\frac{dt_{Ps}}{dp} = 2pV_P^2 \frac{t_{Ps}^2}{(1 - p^2 V_P^2)(X^{1/2} - 1)} \left( \frac{1}{t_{Pps} - t_{Ps}} - \frac{1}{t_{Pps} + t_{Ps}} \right)$$

$$\frac{dt_{Pps}}{dp} = -2pV_P^2 \frac{t_{Ps}^2}{(1 - p^2 V_P^2)(X^{1/2} - 1)} \left( \frac{1}{t_{Pps} - t_{Ps}} + \frac{1}{t_{Pps} + t_{Ps}} \right);$$

$$\frac{dt_{Pss}}{dp} = -2pV_P^2 \frac{t_{Ps}^2}{(1 - p^2 V_P^2)(X^{1/2} - 1)} \frac{2}{t_{Pss}}$$

(4)
Figure 1. Plots of $X(p)$ and their dependence on $R$, $V_p$ and $p$. $R$ primarily controls vertical position, whereas $V_p$ controls the $X$ trend at large $p$ (dash length indicates $V_p$, shortest 5.5 km s$^{-1}$ to longest 7 km s$^{-1}$ in 0.5 km s$^{-1}$ increments). Range of direct $P$ slowness shown, indicating that low slowness range is usually not recorded or analysed.

Figure 2. (a) Record section of receiver functions recorded at GIFN, aligned on the zero lag peak, and displayed in ascending slowness. Reference slowness is 0.0615 s km$^{-1}$. The Moho $t_{p_M}$ arrival is $\sim$5 s, and the $t_{p_S}$ arrival is $\sim$17 s. (b) $t$-$X$ plot of processed RFs. The contour indicates 1σ down from the peak value. $t$ uncertainty is ±0.25 s and $X$ uncertainty is ±0.18.
for brevity, the derivation details are omitted. These expressions are only weakly dependent on $V_p$ because, due to $p < V_p^{-1}$, the time shifts $d_t$ they entail are $O(p \times dp \times V_p^2) O(10^{-2}) \times$ stack related factors. Thus only a very rough guess is required to evaluate them; typically 6.1 km s$^{-1}$ is a good choice.

Fig. 2(a) shows a record section of RF estimates organized by slowness for the POLARIS network station GIFN [Gifford Fjord, Baffin Island, Canada (Eaton et al. 2005)] and the $t$-$X$ stack obtained from summing over slowness with plausible values of $X$. The $t$-$X$ stack (Fig. 2b) is derived at a reference slowness of 0.0615 s km$^{-1}$ and explores the range $2 \leq X \leq 6$ for possible $t_p$ between 2.5 and 15 s. The stack is formed by phase-weighted summation ($N = 2$) (Schimmel & Paulssen 1997) of the $t_{ps}$ and $t_{ps}(X)$ for all slownesses contained in the RF suite; other stacking methods are possible. The envelope of the time series for each $X$ forms each time streak in the grid. The peak is outlined with the 1σ uncertainty in $t$ and $X$. From the peak at 4.93 ± 0.25 s and 3.40 ± 0.18, $V_p/V_S$ is 1.750 ± 0.044 (assuming that $t$ and $X$ are uncorrelated yields smaller estimate uncertainties: ±0.04 s, ±0.03 and ±0.007, respectively). Changes in the guess for $V_p$ between 5.8 and 6.4 km s$^{-1}$ lead to changes in $V_p/V_S$ within 1σ. Compared to a standard $H$–$k$ analysis (Zhu & Kanamori 2000), $V_p/V_S$ for guessed $V_p$ values of 5.8, 6.1 and 6.3 km s$^{-1}$ is 1.742, 1.738 and 1.724, in agreement, within uncertainty, with our method.

The ridges in Fig. 2(b) are related to the mapping from $t$–$p$ into $t$–$X$. Fig. 3 illustrates the mapping and explains the general structure. At the stack reference slowness, the reverberation lags $t_{ps}$ and $t_{ps}(X)$ depend on $X$. At the appropriate $X$, the reverberation lags will be in phase with $P_s$. Thus direct $P_s$ arrivals trace horizontal lines across the $t$–$X$ stack, and reverberations diagonal lines. Stack peaks correspond to the times and $X$ [and $V_p/V_S$ through (1)] of $P_s$ arrivals and their reverberations, in clear analogy to Zhu & Kanamori’s (2000) method. The clear arrival in the record section (Fig. 2a) at ~8.5 s does not evidently generate multiples. A gradational layer is probably the cause of this because they suppress multiples due to repeated paths through the gradient. Inappropriate values of $X$ will alias $P_{3s}$ into $P_{ps}$ and vice versa, leading to subparallel ridges in the $t$–$X$ diagram (Fig. 3b; compare with 2b). These can create subsidiary peaks, but they are usually well separated from the main peak.
3 DISCUSSION AND CONCLUSIONS
What is the justification for yet another method to estimate crustal parameters from RF analysis? The novel feature of this approach is that it yields a time series for every value of $X$, and is closer to the data space of the inversion. Staying in the time domain retains the idea of a waveform, and thus benefits from the many methods of enhancing waveform signal-to-noise ratio through stacking or other signal processing methods. It also eliminates the explicit dependence on the unknown Moho depth, $H$, which is implicit in (4) through the factors $t_p/[\left(1 - p^2V_p^2\right)^{1/2}(X^{1/2} - 1)] = H/V_f$.

Another benefit of the method is that it does not require time picks of peaks in individual RFs, and thus is more amenable to mechanized analyses of stations with large RF data sets. By avoiding the need to estimate peak time pick uncertainty, it is less sensitive to outliers than least squares estimation using (2), as Kumar & Bostock (2008) advocate. Consequently, a RF collection over a broad range of $p$ is not required to stably estimate $V_f/V_s$.

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NOTE ADDED IN PROOF
Many of the ideas here were presaged in Zandt et al. (1995).

REFERENCES