

## Discussion

T. F. IRVINE, JR.<sup>6</sup> It would appear that the information contained in Figs. 2 to 5 of the paper would have an added significance if the abscissas had been chosen as the ratio of boundary-layer thickness to cylinder radius. This representation, which was mentioned by the authors, would enable the reader to visualize the results of the calculations better in terms of a physical flow picture. Such a method of data presentation would also permit a boundary-layer comparison of this free-convection solution with the similar forced-convection calculation by Seban and Bond cited in the paper.<sup>7</sup>

Since there is a great deal of interest at present in free convection at very low Prandtl numbers, it will be helpful for the authors to comment on the extension of their work into this region.

S. I. PAI.<sup>8</sup> In the authors' analysis, the solution is expressed in power series of  $x$ , the axial distance along the cylinder. Since one does not know the radius of convergence of such a series, it is usually assumed that the solution holds only for small value of  $x$ . On the other hand, the fundamental equations used in this analysis are the boundary-layer equations which hold true only for large value of  $x$ . This directly contradicts the method of solution. Have the authors any idea in what range of  $x$  their solution holds so that the two limitations mentioned, one due to

the method of solution and the other to the nature of the fundamental equations, are both satisfied?

### AUTHORS' CLOSURE

The authors wish to thank the discussers for their interest and for their comments.

It is felt that the abscissa variable used in Figs. 2 to 5 more readily facilitates application of the results than would the use of the ratio of the boundary-layer thickness to the cylinder radius. It is agreed that the interpretation of the findings in terms of the ratio of the boundary-layer thickness to the radius is worth while, and such a discussion is given in the paper. For low Prandtl number fluids (i.e., liquid metals), the numerical calculations are considerably more time-consuming than are those for gases. Calculations for low Prandtl numbers are now being contemplated.

Professor Pai's remarks cogently describe the usual uncertainties encountered whenever one uses series-expansion methods such as those of this paper. No positive information is available about convergence of such series, and one must be content with some remarks about the behavior of the terms which have been calculated. With reference to the limitations imposed by boundary-layer theory, one can make more definite statements. In the region near the leading edge (small  $x$ ), the vertical cylinder behaves very much like the vertical flat plate. So, it is quite reasonable to suppose that the value of the Grashof number at which the boundary-layer analysis first begins to give correct heat-transfer results for the flat plate also applies to the vertical cylinder. For air, it is found that this value of the Grashof number is  $10^4$ . (For Grashof numbers less than  $10^4$ , the heat-transfer results predicted by the boundary-layer analysis are lower than those determined experimentally.)

<sup>6</sup> Instructor, Department of Mechanical Engineering, University of Minnesota, Minneapolis, Minn.

<sup>7</sup> See reference (4) of the paper.

<sup>8</sup> Associate Research Professor, University of Maryland, College Park, Md.