A statistical approach to downscaling of sub-daily extreme rainfall processes for climate-related impact studies in urban areas

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Abstract This paper presents a spatial-temporal downscaling approach to describe the linkage between large-scale climate variables for daily scale to annual maximum (AM) precipitations for daily and sub-daily scales at a local site. More specifically, the proposed approach is based on a combination of a spatial downscaling method to link large-scale climate variables as provided by General Circulation Model (GCM) simulations with daily extreme precipitations at a local site and a temporal downscaling procedure to describe the relationships between daily extreme precipitations with sub-daily extreme precipitations using the scaling General Extreme Value (GEV) distribution. The feasibility of the proposed downscaling method has been tested based on climate simulation outputs from two GCMs under the A2 scenario (HadCM3A2 and CGCM2A2) and using available AM precipitation data for durations ranging from 5 minutes to 1 day at 15 raingage stations in Quebec (Canada) for the 1961 – 1990 period. Results of this numerical application has indicated that it is feasible to link large-scale climate predictors for daily scale given by GCM simulation outputs with daily and sub-daily AM precipitations at a local site. Furthermore, it was found that AM precipitations at a local site downscaled from the HadCM3A2 displayed a small change in the future, while those values estimated from the CGCM2A2 indicated a large increasing trend for future periods.

Keywords Climate change; downscaling methods; extreme rainfalls; frequency analyses; General Extreme Value distribution; statistical modelling

Introduction
The maximum rainfall amount for a given duration and for a selected return period is often required for the planning and design of urban drainage systems. For a site for which sufficient rainfall data are available, a frequency analysis of annual maximum rainfalls can be performed. Results of this analysis are often summarized by ‘intensity-duration-frequency’ (IDF) relationships for a given site, or are usually presented in the form of a ‘precipitation frequency atlas’, which provides rainfall accumulation depths for various durations and return periods over the region of interest (Hershfield, 1961; Hogg and Carr, 1985; Institute of Hydrology, 1999). Several probability models have been developed to describe the distribution of extreme rainfalls at a single site (Buishand, 1989; Wilks, 1993; Zalina et al., 2002). Unfortunately, these models are accurate only for the specific time frame associated with the data used. It has necessitated the need for formulating models that could statistically and simultaneously matches various properties of the rainfall process at different levels of aggregations. The most important practical implication of such models is that, from a higher aggregation model we could infer the statistical properties of the process at the finer resolutions that may not have been observed. Another major advantage of such procedure involves the parsimonious parameterisation since these models would normally require a much smaller number of parameters, while
traditional models need different sets of parameters for each particular time scale of the rainfall series considered (Burlando and Rosso, 1996; Kottekoda and Rosso, 1997).

Nguyen et al. (2002) has proposed a scaling General Extreme Value (GEV)-based estimation method that can be used to estimate extreme rainfalls for a given return period at a local site for sub-daily time scales (hourly, 30 minutes, etc.) from statistical properties of extreme rainfalls at a daily scale. Results of a numerical application of this scaling GEV approach using annual maximum rainfall time series from a network of 88 rain-gages in Quebec (Canada) has demonstrated the feasibility and accuracy of the proposed method (Nguyen, 2004).

Climate variability and change will have important impacts on the hydrologic cycle at different temporal and spatial scales. The temporal scales could vary from a very short time interval of 5 minutes (for urban water cycle) to a yearly time scale (for annual water balance computation). The spatial resolutions could be from a few square kilometers (for urban watersheds) to several thousand square kilometers (for large river basins). General Circulation Models (GCMs) have been recognized to be able to represent reasonably well the main features of the global distribution of basic climate parameters, but outputs from these models are usually at resolution that is too coarse (generally greater than 200 km) for many impact studies. Hence, there is a great need to develop tools for downscaling GCM predictions of climate variability and change to regional and local or station scales. In recent years, different downscaling methods have been proposed. Of particular importance for hydrological impact studies are those procedures dealing with the linkage of the large-scale climate variability to the historical observations of the precipitation process at a local site. If this linkage could be established, then the projected change of climate conditions given by a GCM could be used to predict the resulting change of the local precipitation characteristics. The required linkage can be developed using a wide range of (statistical and dynamic) downscaling methods (Nguyen et al., 2006).

In view of the above-mentioned issues, the present study proposes therefore a statistical downscaling approach that can be used to link the climate change scenarios given by GCMs to extreme rainfall characteristics at a local site. More specifically, the proposed approach is based on a combination of a spatial downscaling method to link large-scale climate variables as provided by GCM simulations with daily extreme precipitations at a local site and a temporal downscaling procedure to describe the relationships between daily extreme precipitations with sub-daily extreme precipitations using the scaling General Extreme Value (GEV) distribution. The proposed spatial-temporal downscaling method was tested using annual maximum (AM) precipitation data at 15 raingages in Quebec (Canada) and based on A2 climate change scenario simulation results (denoted by CGCM2A2 and HadCM3A2, respectively) available for the study region provided by the Canadian and UK GCMs for the current 1961–1990 period as well as for future 2020s, 2050s, and 2080s periods. Results of this numerical application have indicated that, after a bias-correction adjustment, it is feasible to develop an accurate linkage between the AM daily precipitations spatially downscaled from GCM simulations with the observed AM daily precipitations at local stations for the 1961–1990 period. These results suggest that it is possible to use the climate predictors given by GCM simulations under the A2 scenario for projecting the variability of AM daily precipitations for future periods. On the basis of these results for AM daily precipitations, the IDF curves for the current 1961–1990 period and for future periods (2020s, 2050s, and 2080s) were constructed using the proposed temporal GEV-scaling method for sub-daily AM precipitations. In general, it was found that the IDF curves based on HadCM3A2 simulations for future periods are quite similar to those for the current period while those using CGCM2A2 indicated a large increasing trend in the future.
A statistical downscaling approach

As mentioned above, the proposed downscaling approach consists of two basic steps: (1) a spatial downscaling method to link large-scale climate variables as provided by GCM simulations with daily extreme precipitations at a local site using the popular Statistical Downscaling Model (SDSM) (Wilby et al., 2002); and (2) a temporal downscaling procedure to describe the relationships between daily extreme precipitations with sub-daily extreme precipitations using the scaling GEV distribution (Nguyen et al., 2002) for the construction of the IDF curves at the site of interest.

A spatial downscaling method using SDSM headings

In general, a spatial (statistical) downscaling technique is based on the view that the regional climate is conditioned by two factors: large scale (global) climatic state and local physiographic features (von Storch et al., 1993). From this perspective, local information is derived by first determining a statistical model which relates global atmospheric variables (called predictors) to any of local weather variables (called predictands). Predictors given by GCM simulations are then fed into this statistical model to estimate the corresponding predictands. In particular, the linear regression-based spatial downscaling technique, called SDSM, as proposed by Wilby et al. (2002) have been commonly used in practice for constructing climate scenarios for various climate-related impact studies. The SDSM could provide a linkage between surface climate variables at individual sites for daily time scale (e.g., precipitation and temperature extremes) with grid-resolution daily GCM climate simulation outputs. Detailed description of the SDSM can be found in Wilby et al. (2002).

As expected, the daily AM precipitations that are extracted from daily precipitation series given by the spatial downscaling of GCM outputs using the SDSM method are often not comparable to the observed daily AM precipitations at a local site. Therefore, an adjustment procedure is needed in order to improve the accuracy of the spatial downscaling SDSM technique in the estimation of local daily AM precipitations. The proposed adjustment can be described in more detail as follows:

Let

\[ y_\tau = \hat{y}_\tau + e_\tau \]  

in which \( y_\tau \) is the adjusted daily AM precipitation at a probability level \( \tau \), \( \hat{y}_\tau \) is the corresponding GCM–SDSM estimated daily AM precipitation, and \( e_\tau \) is the residual associated with \( \hat{y}_\tau \). The estimated residual \( e_\tau \) can be computed using the following equation:

\[ e_\tau = m_0 + m_1 \hat{y}_\tau + m_2 \hat{y}_\tau^2 + \varepsilon \]  

in which \( m_0 \), \( m_1 \), and \( m_2 \) are parameters of the regression function, and \( \varepsilon \) is the resulting error term.

A temporal downscaling method using the scaling GEV distribution

The proposed temporal downscaling method is based on the concept of scale-invariance (or scaling). By definition, a function \( f(x) \) is scaling if \( f(x) \) is proportional to the scaled function \( f(\lambda x) \) for all positive values of the scale factor \( \lambda \) (see, e.g. Fedder, 1988). That is, if \( f(x) \) is scaling then there exists a function \( C(\lambda) \) such that

\[ f(x) = C(\lambda)f(\lambda x) \]  

It can be readily shown that

\[ C(\lambda) = \lambda^{-\theta} \]
in which \( \beta \) is a constant, and that
\[
f(x) = x^\beta f(1)
\]

Hence, the relationship between the non-central moment (NCM) of order \( k \), \( \mu_k \), and the variable \( x \) can be written in a general form as follows:
\[
\mu_k = E\{f^k(x)\} = \alpha(k) x^{\beta(k)}
\]
in which \( \alpha(k) = E\{f^k(1)\} \) and \( \beta(k) = \beta k \). Notice that if the exponent \( \beta(k) \) is not a linear function of \( k \), in such cases the process is said to be ‘multiscaling’ (Gupta and Waymire, 1990).

Application of the GEV distribution to model the annual series of extreme rainfalls has been advocated by several researchers (Natural Environment Research Council, 1975; Schaefer, 1990). The cumulative distribution function, \( F(x) \), for the GEV distribution is given as
\[
F(x) = \exp\left[-\left(1 - \frac{\xi - \frac{\alpha}{\kappa}}{\frac{\alpha}{\kappa}}\right)^{1/\kappa}\right] \quad \kappa \neq 0
\]
where \( \xi, \alpha \) and \( \kappa \) are respectively the location, scale and shape parameters. It can be readily shown that the \( k \)-th order NCM, \( \mu_k \), of the GEV distribution (for \( k \neq 0 \)) can be expressed as
\[
\begin{align*}
\mu_k &= \left(\xi + \frac{\alpha}{\kappa}\right)^k + (-1)^k \Gamma(1+k\kappa) \left(\frac{\alpha}{\kappa}\right)^k \Gamma(1+k\kappa) + \sum_{i=1}^{k-1} (-1)^i \left(\frac{\alpha}{\kappa}\right)^i \left(\xi + \frac{\alpha}{\kappa}\right)^{k-1-i} \Gamma(1+i\kappa) \\
&= \sum_{i=0}^{k-1} (-1)^i \binom{k}{i} \left(\frac{\alpha}{\kappa}\right)^i \left(\xi + \frac{\alpha}{\kappa}\right)^{k-1-i} \Gamma(1+i\kappa)
\end{align*}
\]
where \( \Gamma(\cdot) \) is the gamma function. Hence, on the basis of Equation (8), it is possible to estimate the three parameters of the GEV distribution using the first three NCMs. Consequently, the quantiles \( X_T \) can be computed using the following relation:
\[
X_T = \xi + \frac{\alpha}{\kappa} \left[1 - \left(1 - \ln(p)\right)^{\kappa}\right]
\]
in which \( p = 1/T \) is the exceedance probability of interest.

Further, for a simple scaling process, it can be shown that the statistical properties of the GEV distribution for two different time scales \( t \) and \( \lambda t \) are related as follows:
\[
\begin{align*}
\kappa(\lambda t) &= \kappa(t) \\
\alpha(\lambda t) &= \lambda^\beta \alpha(t) \\
\xi(\lambda t) &= \lambda^\beta \xi(t) \\
X_T(\lambda t) &= \lambda^\beta X_T(t)
\end{align*}
\]
Hence, based on these relationships it is possible to derive the statistical properties of short-duration (e.g. \( \lambda t = 1 \) less than 1 day) extreme rainfalls using the properties of daily \( (t = 1 \text{ day}) \) extreme rainfalls. The exponent \( \beta \) is computed based on the scaling properties of the NCMs of extreme rainfalls for various durations. Hence, the proposed scaling GEV can be used to derive the IDF relationships for AM precipitations for different durations.

**Numerical application**

To illustrate the application of the proposed spatial-temporal downscaling approach, a case study is carried out using both global GCM climate simulation outputs and at-site
AM precipitation data available at 15 raingage stations in Quebec (Canada). The selected global GCM predictors are given by the CGCM2A2 and HadCM3A2 simulations for the 1961–1990 period as well as for some future periods 2020s, 2050s, and 2080s, while the at-site AM rainfall series for durations ranging from 5 minutes to 1 day used in this study are available only for the 1961–1990 period. Furthermore, data for the 1961–1975 period were used for model calibration and data for the remaining 1976–1990 period were for validation purposes. The computational procedure for the proposed downscaling method can be summarized as follows:

(a) calibrate and validate the (spatial) downscaling SDSM model using the at-site daily precipitation as predictand and global GCM atmospheric variables as predictors (Nguyen et al., 2006);

(b) generate 100 samples of 30-year daily precipitation series at a given site using the calibrated SDSM and the corresponding GCM predictors, and extract daily AM precipitation series from these generated samples;

(c) perform necessary bias correction of the GCM-downscaled daily AM precipitation series using Equations (1) and (2) in order to obtain a good agreement between the mean series of GCM-downscaled daily AM precipitations with the at-site observed daily AM precipitations;

(d) investigate and establish the scaling relations between the NCMs of observed at-site AM precipitations for various durations;

(e) calibrate the scaling GEV model using observed at-site AM precipitations in order to establish the linkage between daily and sub-daily AM precipitations;

(f) construct IDF curves using the adjusted GCM-downscaled AM daily precipitations and the estimated sub-daily AM rainfall amounts given by the calibrated scaling GEV model.

Repeat steps b) to f) to construct IDF curves for future periods (2020s, 2050s, and 2080s) based on the corresponding GCM predictors for these periods.

For purposes of illustration, Figure 1 presents the probability plots of AM precipitations downscaled from CGCM2A2 and HadCM3A2 as compared to those of observed at-site AM precipitations for the 1961–1975 calibration period for Dorval station. It can be seen that the GCM-downscaled AM precipitations do not agree well with the observed at-site amounts. Figure 2 shows the good fit of the second-order correction function (Equation 2) to these differences (or residuals) for both GCMs. Hence, as indicated in Figure 1, after making the bias-correction adjustment of the downscaled AM precipitations.

![Figure 1](https://iwaponline.com/ws/article-pdf/7/2/183/418510/183.pdf)
precipitations using the fitted correction function, a very good agreement can be achieved between the adjusted mean GCM-downscaled amounts and the observed at-site values.

The bias-correction functions developed based on data for the 1961–1975 calibration period were then applied to the downscaled AM precipitations for the 1976–1990 period to assess their validity. Figure 3 shows the highly improved closeness between the adjusted downscaled AM precipitations and the observed values as compared to the unadjusted downscaled AM amounts. Hence, it is feasible to use the bias-correction function derived from data for the 1961–1975 calibration period for other time periods in the future. Similar results were found for other stations.

To assess the scaling behaviour of the at-site AM precipitation series, the log–log plots of the first three rainfall NCMs against duration are prepared for all 15 stations. For purposes of illustration, Figure 4 shows the plot for Dorval station. The log-linearity exhibited in the plot indicates the power law dependency (i.e., scaling) of the rainfall statistical moments with duration (Equation 6) for two time intervals: from 5 minutes to 1 hour, and from 1 hour to 1 day. Further, the linearity of the scaling exponent \( \beta(k) \) with the moment order \( k \) as shown in Figure 5 supports the assumption that the extreme rainfall series considered can be described by a simple scaling model. Hence, for a given location, it is possible to determine the NCMs and the distribution of rainfall extremes for short durations (e.g. 1 hour) using available rainfall data for longer time scales (e.g. 1 day) within the same scaling regime (\( \beta \) is known).

Figure 2 Error-adjustment functions for daily AM precipitations downscaled from CGCM2A2 and HadCM3A2 for Dorval station for the calibration 1961–1975 period

Figure 3 Probability plots of daily AM precipitations downscaled from CGCM2A2 and HadCM3A2 before and after adjustment for Dorval station for the validation 1976–1990 period
For illustrative purposes, Figure 6 shows the comparison between the empirical (observed), traditional fitted GEV and estimated scaling GEV distributions of 1-hour and 30-minute rainfall extremes at Dorval station. It can be seen that the estimated scaling GEV distribution is in very good agreement with the observations as indicated by the closeness of the estimated values with the perfect-fit 45° line. In addition, Figure 7 shows...
the accuracy in the estimation of 5-minute and 30-minute AM precipitations from 1-day amounts using both adjusted HadCM3A2-downscaling and scaling GEV procedures as compared to the traditional GEV fitting method. Similar results were found for the CGCM2A2 and for other durations as well as for other stations. These results have thus indicated that it is feasible to estimate accurately the sub-daily AM precipitations from adjusted GCM-downscaled daily AM precipitations.

Figure 8 shows the plots of daily and 5-minute AM precipitations at Dorval station for the 1961–1990 period and future periods (2020s, 2050s, and 2080s) using the proposed spatial-temporal downscaling method. It can be seen that the HadCM3A2 scenario
suggested a small change of AM precipitations in the future, while the CGCM2A2 model indicated a large increasing trend for future periods.

**Conclusions**

A spatial-temporal downscaling approach was proposed in the present study to describe the linkage between large-scale climate variables for daily scale to AM precipitations for daily and sub-daily scales at a local site. The feasibility of the proposed downscaling method has been tested based on climate simulation outputs from two GCMs under the A2 scenario (HadCM3A2 and CGCM2A2) and using available AM precipitation data for durations ranging from 5 minutes to 1 day at 15 raingage stations in Quebec (Canada) for the 1961–1990 period. Results of this numerical application has indicated that it is feasible to link daily large-scale climate variables to daily AM precipitations at a given location using a second-order bias-correction function. Furthermore, it was found that the AM precipitation series in Quebec displayed a simple scaling behaviour within two different time intervals: from 5 minutes to 1 hour, and from 1 hour to 1 day. Based on this scaling property, the scaling GEV distribution has been shown to be able to provide accurate estimates of sub-daily AM precipitations from GCM-downscaled daily AM amounts. Therefore, it can be concluded that it is feasible to use the proposed spatial-temporal downscaling method to describe the relationship between large-scale climate predictors for daily scale given by GCM simulation outputs and the daily and sub-daily AM precipitations at a local site. This relationship would be useful for various climate-related impact assessment studies for a given region.

Finally, the proposed downscaling approach was used to construct the IDF relations for a given site for the 1961–1990 period and for future periods (2020s, 2050s, and 2080s) using climate predictors given by the HadCM3A2 and CGCM2A2 simulations. It was found that AM precipitations at a local site downscaled from the HadCM3A2 displayed a small change in the future, while those values estimated from the CGCM2A2 indicated a large increasing trend for future periods. This result has demonstrated the presence of high uncertainty in climate simulations provided by different GCMs. Further studies are planned to assess the feasibility and reliability of the suggested downscaling approach using other GCMs and data from regions with different climatic conditions.

**References**


