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Discussion

A Method for Dynamic Fracture Initiation Testing of Ceramics[1]

F. I. Baratta[2] and P. J. Perrone.[3]

The authors of this paper, Duffy et al. (1988), are commended for the innovative approach used to precrack ceramic materials by compressive fatigue loading and their general approach used to dynamically test these materials. Yet, there are confusing and contradictory statements in this paper regarding the influence of notch tip field stresses on the crack tip. The results of the following described analyses, which consider only the static mode I stress intensity (K_I) for the cracked-notched round bar stressed by a far field stress σ (see Fig. 1), are presented to resolve this dilemma. Also shown in Fig. 1 is the nomenclature as used in the referenced paper; the appropriate dimensions are: do = 19mm, di = 9mm, $\rho = 0.127$mm, $t = 1.8$mm, and $\theta = 60$ deg.

This problem can be solved by adopting the method proposed by Lefort (1978), wherein a semi-emperical approach (SEA) was used to determine K_I for short cracks at the root of a notch of a round tensile bar. Since this method is readily available in the literature little detail is given here except to note that the notch geometry, shown in Fig. 1, closely approximates a deep hyperbolic shape. Thus, the formulation by Neuber (1946) was employed to obtain the stress distribution at the notch tip and utilized in subsequent stress intensity calculations.

Because the SEA was considered approximate, a more accurate finite element analysis (FEA) of the specimen was performed to also obtain *static* stress intensity factors. Calculations were based on linear elastic, small strain theory simplified for an axisymmetric geometry and loading. The axisymmetric finite element model, shown in Fig. 2(*a*), was constructed with six-noded, quarter point singularity elements at the crack tip (Henschell and Shaw, 1975, Barsoum, 1976; and Freese and Tracey, 1976) and biquadratic isoparametric elements elsewhere. Figure 2(*b*) shows the enlargement of the notch area.

Static stress intensities were calculated from resulting displacements of singularity elements, where for any ray emanating from the crack tip (radial direction) the singular elements provide the $\sqrt{r}$ displacement variation. As detailed by Tracey (1977) the numerical $\sqrt{r}$ term from the resulting displacement variation along a radial edge of a singularity element was equated to the analytic term of the asymptotic solution given by Williams (1957) which includes K_I and K_{II}, the mode II stress intensity factor. In this fashion K_I and K_{II} were determined along each radial edge of the singularity elements. For a pure mode I case, such as the one considered here, K_{II} served as a measure of the accuracy of the calculations.

From finite element calculations, the averages of the stress intensities were calculated along 13 equally spaced, radial singularity element boundaries, shown in Fig. 3. Stress intensity factors for crack lengths of 0.0065mm, 0.054mm,

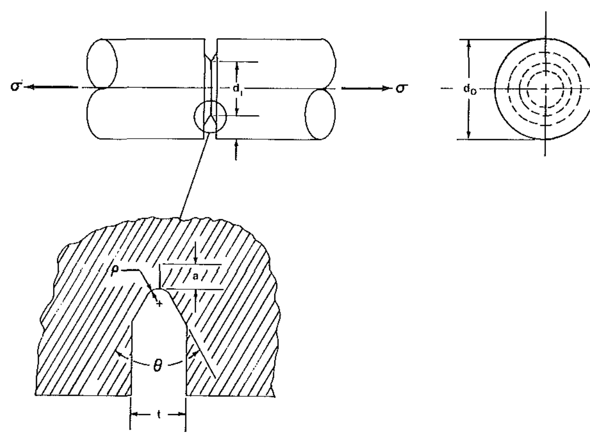

Fig. 1 Cracked-notched round tensile bar

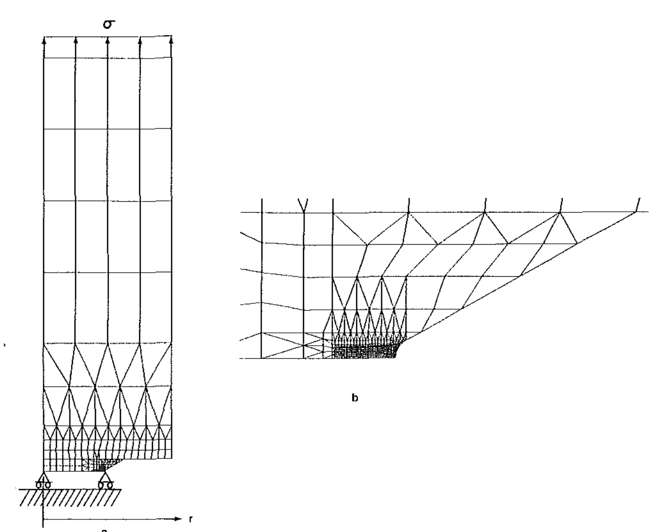

Fig. 2(*a*) Axisymmetric finite element mode; (*b*) enlargement of notch area

[1] By J. Duffy, S. Suresh, R. Cho, and E. R. Boff published in the JOURNAL OF ENGINEERING, MATERIALS AND TECHNOLOGY, Vol. 111, October 1988, pp. 326–331 and Addendum, published in the JOURNAL OF ENGINEERING MATERIALS AND TECHNOLOGY, Vol. 111, p. 86.

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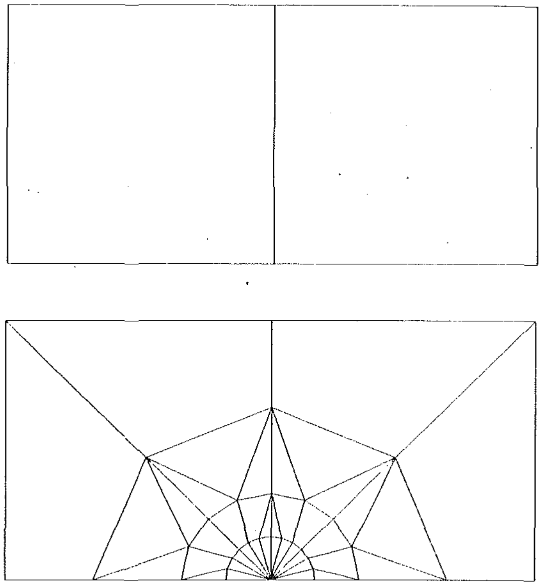

Fig. 3 Lower sketch: Crack tip mesh inserted into base mesh at the crack tip; Upper sketch: Eight noded quadrilaterals inserted at alternate crack tip locations.

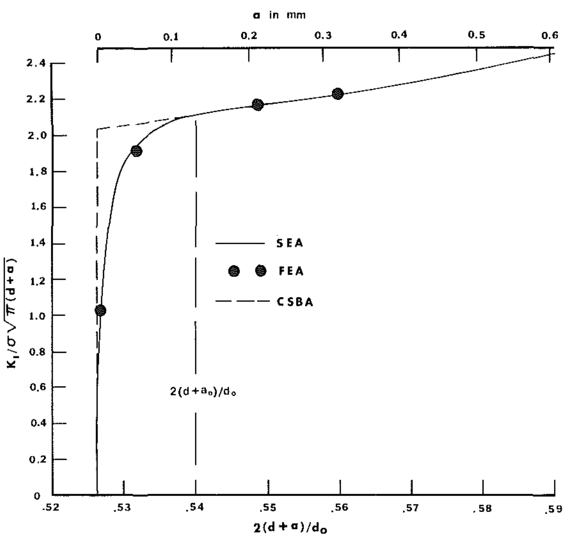

Fig. 4 Stress intensity factor for a circumferential crack emanating from a notch in a round tensile bar

0.2155mm, and 0.3200mm were then determined. For each crack length considered the difference between K_I values on any 2 radial element boundaries is less than 1 percent and K_{II} values is less than their corresponding K_I values by 3 orders of magnitude.

The nondimensional static mode I stress intensity factor $K_I/\sigma\sqrt{\pi(a+d)}$ as a function of both the cracked-notch depth ratio $2(d+a)/d_0$ and the fatigue precrack "a" in millimeters are shown in Fig. 4. The solid lined curve represents the results using the SEA and the circles represent the peviously described FEA data; the results of both analyses agree quite well. Also the stress concentration factor (without a crack present) obtained from the SEA and from the FEA closely agree, i.e., 27.5 and 28.3, respectively. The stress concentration factor was based on the maximum stress at the notch tip compared to the far field tensile stress σ.

Also shown in Fig. 4 is a dashed curve, which describes the result of the cracked-smooth bar assumption (CSBA), wherein the crack length is considered equivalent to the starter notch depth plus the fatigue crack $(d+a)$, after Tada et al. (1973), and adopted by Duffy et al. Notice that the two curves and the finite element results superimpose each other at $2(d+a_0)/d_0=0.54$, below which the validity of the cracked-smooth bar assumption becomes questionable. If the appropriate dimensions of the specimen given in the discussed paper are substituted in the above relationship, we find that $a_0=0.130$mm. This limiting CSBA value is approximately equal to ρ, not the recommended range of $\rho/20$ to $\rho/4$, as suggested in the paper. The limiting value is smaller than the maximum depth of the annular fatigue precrack of 170 microns used in the subject paper to determine fracture toughness (K_{IC}). Nevertheless, the largest error will occur at the shortest crack length. These errors, which are a function of the notch geometry and the fatigue precrack, are shown in Fig. 5. They are based on using the cracked-smooth bar assumption rather than considering the actual precracked-notch geometry when determining K_I. It is stated by the authors of the subject paper that "....a variation in the fatigue precrack length, from 25 microns to 550 microns, does not lead to any significant difference in the measured toughness value of the alumina ceramic (this study and Suresh et al. 1987)." As seen from Fig. 5 a crack length of 25 microns will yield an overestimate of K_I by 20 percent. However, the smallest fatigue precrack used by Duffy et al. to determine quasi-static K_{IC} was 35 microns; this will result in an error of approximately +11%.

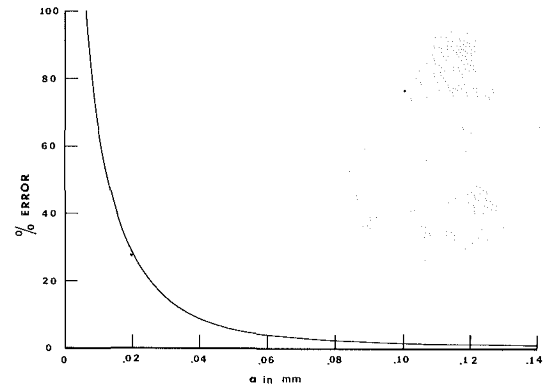

Fig. 5 Stress intensity factor error based on using the cracked-smooth bar assumption

It is hoped that the results of the analyses presented here, applicable to the specimen shown in Fig. 1, will be helpful to the authors of the subject paper. Nevertheless, it is not expected that one can assume that the static mode I analysis presented here will result in similar errors as those applicable to the mode III specimen when the effect of the notch geometry is ignored. Also, the dynamic mode III specimen, shown in Fig. 2 of the subject paper and used in the experiments by the authors appears to have had a hexagonal cross-section. It was also gripped at relatively short distances from the stress intensifier giving rise to end effects which may not have been accounted for in the K_{III} determination. These factors will result in markedly different K_{III} values compared to those of the static mode III specimen which was of circular cross-section and relatively long.

Additionally, the authors state in a later published addenum (Duffy et al. (1989)), that they have improved the accuracy in measuring the peak load of the transmitted pulse to the mode I specimen and thus showing a reduction in dynamic mode I fracture toughness (K_{Id}) compared to their original data, 3.68 MPa $\sqrt{\text{m}}$ and 5.63 MPa $\sqrt{\text{m}}$, respectively. The mean value of the static fracture initiation toughness was somehow changed from 3.35 MPa $\sqrt{\text{m}}$ to 2.9 MPa $\sqrt{\text{m}}$; yet static K_{IC} previously determined by Suresh and Tschegg (1987), would not have been affected by a "cleaner" improved dynamic pulse. Recalculating the ratio K_{Id}/K_{Ic} gives a ratio of 1.10, i.e., 3.68

MPa $\sqrt{m}$/3.35 MPa $\sqrt{m}$ rather than 1.27 as indicated by Duffy et al. (1989). Because this ratio is still within the experimental error, the conclusion that K_{Id} is greater than K_{IC} for the particular alumina examined is questionable.

It is suggested that the authors of the subject paper redetermine mode I fracture toughness using Fig. 4 in this communication, and derive appropriate mode III stress intensity factors for both static and dynamic cracked-notched torsion specimens such that they are comparable.

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Authors' Closure

The authors thank F. I. Baratta and P. J. Perrone for their interest in our paper (Duffy et al., 1988) and our addendum to the paper (Duffy et al., 1989a). Their numerical analysis of the notch-tip field effects on subsequent fracture toughness values is of considerable use in applying the compression fatigue precracking procedure for fracture toughness testing of brittle materials. We are pleased that they have presented their calculations in conjunction with the results published in our papers. At the same time, we are somewhat disappointed that they have failed to note some of the key points mentioned in the subject papers. This seems to have resulted in some misconceptions on their part, which they interpret as confusing and contradictory statements in the subject papers. Many of the points raised here have also been addressed in even greater detail in the sequels to subject papers (some of which are already published (Duffy et al., 1989b) and others are in print (Suresh et al., 1990). In this response, we pinpoint these issues raised by Baratta and Perrone (1990) and present a clarification.

(1) All the data presented in Duffy et al., (1989a) were for a precrack depth of greater than 88 μm. This, according to Fig. 5 in the comment by Baratta and Perrone (1990), causes an error of less than 4 percent. In view of this result, the precracking method, suggested by Suresh and Tschegg (1987) and used in Duffy et al. (1988, 1989a), is expected to give very accurate estimates of fracture toughness, compared to the other techniques that are available in the literature. The quoted value of critical precrack length in Duffy et al. (1988) was based on an estimate made from the results of Dowling (*Fat. Eng. Mater. Struct.*, Vol. 2, 1979, pp. 129–138). It is an experimental fact that for about 10 different quasi-static tension tests conducted on this material with precrack depths of 35-550 μm, we do not see any noticeable differences over and above the normal experimental scatter.

We wish to add that, subsequent to the publication of Duffy et al. (1988, 1989a), we have improved our experimental procedure for both precracking and dynamic tensile testing using an accurate, dynamic finite element analysis. These results have recently appeared in a paper (Suresh et al., 1990). In this latest work, we have precrack lengths that are typically greater than 100 μm for all the tests reported (i.e., less than 3 percent error as shown in Fig. 5 of Baratta and Perrone). The experimentally observed dynamic to static fracture toughness ratios are essentially the same as those reported by Duffy et al. (1989a).

(2) It is clearly stated in the addendum by Duffy et al. (1989a) that.. "We have repeated the Mode I dynamic and static fracture initiation experiments with this modified specimen and a *new batch* of AD-998 alumina.." Baratta and Perrone appear to have missed this point. Different batches of materials with the same composition, when processed in large quantities, often exhibit slightly different mechanical behavior. Therefore, the differences between the values of K_{Id}/K_{Ic} reported in the original paper (Duffy et al., 1988) and the addendum (Duffy et al., 1989a) are not because of an error in calculation (as incorrectly interpreted by Baratta and Perrone). Futhermore, as noted earlier, the minimum precrack depth was 88 μm, which is sufficient to provide an accuracy of better than 96 percent. The reported data for both dynamic toughness and static toughness were the actual experimental results obtained for a new batch of the AD-998 alumina. The conclusion in the addendum that the dynamic fracture toughness is greater than the static fracture toughness is *correct*. This conclusion has also been substantiated for a wide range of ceramics in our subsequent work (Suresh et al, 1990) and by other authors who have used very different methods to study dynamic fracture toughness in ceramics (e.g., Yang et al., 1989; Aoki et al., 1989).

It is also clear that differences in material properties arising from different processing conditions of the same ceramic material lead to scatter in fracture toughness values which is comparable to or even in excess of that attributable to the test technique. For this reason, we feel that it may not be worthwhile to spend much effort to debate the origin of scatter values typically smaller than 10%, especially considering the low fracture toughness values of ceramics.

(3) In their comment, Baratta and Perrone state that "... the dynamic Mode III specimen, shown in Fig. 2 of the subject paper and used in the experiments by the authors, appears to have had a hexagonal cross-section." It is clearly illustrated in Fig. 2 of the subject paper that the ligament that was subjected to dynamic fracture in Mode III was of circular cross-section; only the flanges used to apply the load were of hexagonal cross-section.

The experimental results reported in the subject paper constitute the first attempts to measure the Mode III fracture toughness of a brittle solid using a circumferentially notched rod geometry and a fatigue pre-crack. It is well known from the available literature on ceramics (e.g., Suresh and Tschegg, 1987) and on metals (e.g., Tschegg and Suresh, 1988) that considerable frictional sliding occurs between the mating faces of the fatigue precrack during Mode III loading. Furthermore, firmly gripping the specimen surface in the presence of such frictional forces is an experimental challenge, especially for dynamic tests lasting only a very short period of time. In view of these problems, we used a cylindrical specimen with a circular cross-sectional ligament and hexagonal cross-sectional flanges (to enable an efficient application of the torsional load). It is conceivable that some errors arise as a result of this design for frictional loading of the specimen. At the same