for certain values of $F(<1)$ and Rayleigh numbers possibly may be used to advantage in reducing pumping requirements.

Range of Validity of Results. This analysis rests upon assumptions such as laminar fully developed flow and heat transfer, uniform internal heat generation, uniform wall heat flux, and constant fluid properties (except for a density variation). In any application of the results of this analysis these assumptions must be good ones in order to expect the results to be valid. The effect of deviations from these assumptions cannot be predicted without a more refined analysis or an experiment. Qualitatively, it can be stated that a variation in an axial direction of such things as fluid properties and the heat-source term will have less effect than a radial variation.

The effect of entrances is not known at present nor is it known how far from an entrance these results may be safely applied. In an entrance, which always is both a hydrodynamic and thermal entrance when free-convection effects are present, radial-velocity components exist which make the momentum equation nonlinear. In addition, the axial temperature gradient is no longer independent of radius.

Conclusions

The general effect of large free-convection effects (large Rayleigh numbers) in combined forced and free-convection flows of the type considered is to decrease radial temperature differences and increase Nusselt numbers over those for pure forced-convection heat transfer. An increased free-convection effect also changes the axial pressure gradient from that for pure forced convection but the change may be in either direction depending upon the value of the heat-source parameter.

In all cases the comparison made with experimental data agreed more closely with the present analysis than it did with the pure forced-convection analyses although the agreement was not as good as may be desired. It is suggested that further experiments be designed with the particular purpose in mind of confirming this analysis. Special care should be taken to use very large length-to-diameter ratios for the tubes and the Reynolds numbers should be kept very low to insure no turbulence. Temperature differences should be kept small in order to minimize variable fluid-property effects.

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Discussion

Simon Ostrach.[7] From the introduction of the paper, it appears that the only part of the considered problem, which was not previously solved for tube flows, is that with a net through-flow including heat sources. However, the analysis is made for the entire problem and although it is pointed out that some of the presented solutions coincide with those previously obtained, the results in no way indicate distinctly the new contribution and the relationship of the duplicated work with that done earlier. The duplication is perhaps justified because some of the earlier work appears in Russian literature which is not readily available. However, the author's personal contribution should be clearly set apart from that which has already been done. Further, it is indicated (and not clearly so) that all aspects of the problem have been treated for the same types of flows between two parallel plates and it would be interesting to see what the relationship is between the flow in tubes and in the

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channel formed by the plates, but no such comparison is presented. The value of the paper would be enhanced by such a discussion.

The case where $N_{Ra} = 0$ is always referred to in the paper as the pure-forced-convection case. However, $N_{Ra} = 0$ could also be a natural-convection flow with no axial temperature gradient.

Myron Tribus.[8] It should be pointed out that the solutions quoted in this paper are all in the nature of asymptotic solutions and apply only far from the entrance to the tubes. Whether the boundary conditions associated with these solutions are physically realizable is not at present clear. This writer surmises, based on a study of the differential equations, that the entrance length in the absence of free-convection effects should be about the same for flow with internal heat sources as without. In the presence of free convection, no similar calculations were made. Can the author comment on the entry length for a fluid-flow system in the presence of free convection?

Author's Closure

Dr. Ostrach's comments concerning the part of the considered problem which was previously solved are incorrect. The statements made in the introduction are correct and are repeated here for emphasis. In references (2) and (3) Ostroumov gives the solution for the fully developed velocity and temperature *profiles* with constant-heat-flux boundary condition without internal heat sources. The mixed-mean-to-wall temperature differences are incorrectly given in reference (3) and the Nusselt numbers were never calculated. These results are given in the present paper. The fact that the velocity and temperature profiles, with constant-heat-flux boundary condition and no internal heat sources, were first obtained by Ostroumov is clearly indicated immediately after the derivations of Equations [13] and [15] of the text. Other results which are not new, but are pointed out because they represent limiting cases of the present analysis, are the pure-forced-convection temperature solution without internal heat sources (13) and the pure-forced-convection temperature solution with internal heat sources (12). This is also clearly indicated in the text. With these exceptions, to the best knowledge of the author, the remainder of the analysis is original.

A comparison with the corresponding solutions given by Dr. Ostrach for the same type of flows between parallel plates would be interesting. Mixed-mean-to-wall temperature differences were not given by Dr. Ostrach in reference (9), nor were the corresponding Nusselt-number relationships indicated, so a direct comparison of results of practical interest is not possible without further computations.

It is true that the case $N_{Ra} = 0$ could also be a natural-convection flow with no axial temperature gradient and this is discussed when presenting Equations [16], [17], and [18]. It should be emphasized, however, that the curves marked $N_{Ra} = 0$ in Figs. 2 and 3 are for $\beta = 0$ and A finite.

It is well that Professor Tribus has pointed out that the solutions given here are only for fully developed flows which satisfy the assumed boundary conditions because this should be reemphasized. Whether or not these boundary conditions are physically realizable could be shown by experiment but to the author's knowledge no such experiments have been run. There is no reason to expect that the situation will be any different from the case of pure forced-convection, where the same boundary conditions are physically realizable (within the accuracy of the experiments and the variability of the fluid properties).

The author has made no calculations of the entry lengths required to obtain fully developed flows of the types considered in the present paper. Such calculations, or an experiment, would be of definite value.

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