product of the voltage drop and cylinder current and a suitable conversion factor.

The Grashof-Prandtl modulus used for correlating free-convection heat-transfer data was calculated using the properties noted in Appendix 1 evaluated at the gas temperature.

Appendix 3

Test-Cylinder Design

The selection of the diameter and length of the test cylinder is dictated by the temperature distribution within the cylinder which results under the boundary conditions present at the time of test, and by the diameter of the test vessel itself. The test-vessel diameter sets the length of the test cylinder at about 6 in. The diameter of the test cylinder was determined such that the difference between the mid-point temperature of the cylinder and the average temperature, based upon the measured average resistance, will be less than 4 per cent of the total temperature rise of the test cylinder mid-point above the gas temperature. The end temperature will be assumed to be equal to the gas temperature, for the purposes of design; anything greater tends to minimize the departure between the mid-point temperature and the mean temperature. The uniformity of the cylinder temperature is important because the mean cylinder resistance and hence cylinder temperature is that quantity which is measured when the current through and the voltage drop across the test cylinder are noted.

The differential equation governing the longitudinal temperature distribution is

$$\frac{\partial^2 v}{\partial x^2} - m^2 v + b = 0$$

$$x = 0, l \qquad v = 0$$

The solution of this differential equation subject to the given boundary conditions is

$$v = \frac{b}{m^2}\left(1 - \frac{\sinh mx + \sinh m(l - x)}{\sinh ml}\right)$$

The mean temperature of the cylinder may be calculated from the integral

$$v_{\text{avg}} = \frac{1}{l}\int_0^l v\, dx$$

Performing the indicated integration yields

$$v_{\text{avg}} = \frac{b}{m^2}\left(1 - \frac{2(\cosh ml - 1)}{ml \sinh ml}\right)$$

The temperature at the mid-point of the cylinder is

$$v_{l/2} = \frac{b}{m^2}\left(1 - \frac{2\sinh \frac{ml}{2}}{\sinh ml}\right)$$

The departure of the mid-point temperature from the mean temperature may be determined by forming the ratio of the mid-point temperature to the average temperature

$$\frac{v_{l/2}}{v_{\text{avg}}} = \frac{\sinh ml - 2\sinh \frac{ml}{2}}{\sinh ml - \frac{2}{ml}(\cosh ml - 1)}$$

The departure of the mid-point temperature from that of an ideal cylinder, that is, one in which no heat is lost through the ends, is given by

$$\psi = \frac{v_{\text{ideal}} - v_{l/2}}{v_{\text{ideal}}} = \frac{2\sinh \frac{ml}{2}}{\sinh ml}$$

A plot of ψ and $v_{l/2}/v_{\text{avg}}$ as a function of the cylinder parameter ml shows that the departure of the mid-point temperature from that of the ideal cylinder is negligible for ml greater than 10 and that the average temperature is within 4 per cent of the mid-point temperature for ml greater than 60.

For a 10-mil platinum wire operating in air, a nominal value of the heat-transfer coefficient is 10 Btu/hr sq ft deg F, and the other constants necessary to calculate m are

$p = 2.62 \times 10^{-3}$ ft $\qquad k = 40.2$ Btu/hr ft deg F

$A = 54.5 \times 10^{-8}$ sq ft $\qquad i = 0.532$ amp

$\alpha = 2.2 \times 10^{-3}$/deg F $\qquad \rho_G = 0.4 \times 10^{-6}$ ohm ft

$$m = \sqrt{\frac{hp}{kA} - \frac{3.413 i^2 \rho_G \alpha}{kA^2}} = 36 \qquad ml = 19.1$$

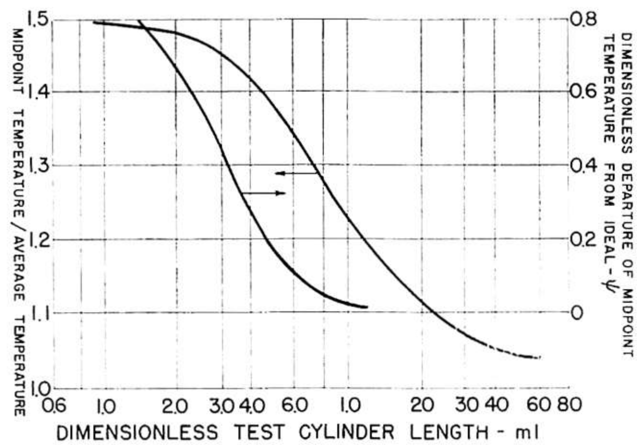

Fig. 8 Relationship of Cylinder Mid-Point Temperature to Average Temperatures

Since ml is below that value said to be the lowest acceptable, guard-heating of the cylinder ends is indicated and this was done for all runs. A similar calculation for the case of free convection to Freon 12 shows that ml is well above the required minimum value so that guard-heating of the cylinder ends was not required. Fig. 8 is a plot of the design parameters mentioned in the foregoing.

A calculation which assumes that all of the heat generated in the test cylinder is liberated at the center line indicates that the radial temperature gradient is negligible.

Discussion

T. W. Jackson.[5] For some time the writer has wondered whether or not free-convection Nusselt numbers can be correlated precisely by a Grashof-Prandtl number product. However, because of past experience this appears to be the convenient method of approach, and the authors are justified in its use.

Obtaining high free-convection heat-transfer coefficients, as stated by the authors, would offer possibilities of sealed heat-transfer systems and, consequently, eliminate pumps and

[5] Chief, Mechanical Sciences Division, Georgia Institute of Technology, Atlanta, Ga. Mem. ASME.

associated problems. The paucity of data on free convection at or near the critical state, however, indicates the difficulty of doing this and of obtaining experimental results in this region. The writer regrets that the data were not for water which would be a more interesting fluid from an engineering standpoint. Perhaps the present paper will inspire someone to consider a water system with its high pressure (P_{cr} = 3206 psi) and comparatively high temperatures (T_{cr} = 705 F).

The writer concurs with the authors that, since the heat-transfer coefficient h in free convection becomes a function of the temperature difference, the real merit of defining a coefficient may not be as justifiable as for the case of forced convection. In addition, at or near the critical point the dependency of the physical properties of the heat-transfer medium on temperature makes a correlation difficult. Therefore the authors' procedure of plotting heat rate and heat-transfer coefficient versus temperature differences and reduced pressure seems practical. The practical use of, say, a tenfold increase in heat-transfer coefficient may not be possible because of the temperature and pressure dependencies. However, the final decisions on this must await further experimental data.

The authors' comment that it was possible to obtain a twofold increase in the value of the heat-transfer coefficient for a given heat rate, depending on whether the data were taken with increasing or decreasing steps in the heat rate, is interesting. In determining the viscosity of steam using the transpiration method the writer experienced this type of elusive phenomenon. Perhaps it is the result of distortions in the normal steady-state distribution of molecular energies of the fluid. The distributions may be affected by a temperature gradient—one way for a positive gradient and another for a negative gradient.

The writer wishes to congratulate the authors for tackling a difficult problem. It is hoped that they can continue or encourage the continuance of this work so that data on water will become available.

Authors' Closure

The authors thank Mr. Jackson for his discussion of the paper and agree with him that it would be of greater engineering interest to have used water vapor as the test gas. Experimental difficulties with pressures and temperatures near the critical for water are most formidable however.

Since the preparation of this paper Mr. R. B. Ramsey, Jr., of the E. I. du Pont de Nemours Co., has called to our attention new data on the thermodynamic properties of Freon 12. Time has not permitted a re-evaluation of the experimental data in the light of this new thermodynamic information, but it is hoped that this will be done in the near future.

Since this work was intended only as a preliminary effort, it is the authors' hope, of course, that it will encourage others to investigate, both analytically and experimentally, this interesting field of heat transfer.