Finite element modelling of marine controlled-source electromagnetic responses in two-dimensional dipping anisotropic conductivity structures

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SUMMARY

We present an adaptive finite element (FE) solution for the marine controlled-source electromagnetic (CSEM) forward problem in 2-D dipping anisotropic conductivity structures. Our code is implemented on an unstructured triangular mesh, which allows for arbitrary model geometries including bathymetry and dipping layers. We have verified the FE code using a layered 1-D model with anisotropy. For this model, the FE algorithm provides accurate results. The relative amplitude error between the analytical solution and numerical results for all electromagnetic components except $H_z$ for large offsets is less than 1 per cent and the error in phase is less than $0.5^\circ$. We simulate the marine CSEM responses of a 2-D anisotropic model, and the numerical results show that the CSEM fields are affected considerably by anisotropy in both the reservoir target and the surrounding sediment, but to different extent.

Key words: Numerical solutions; Numerical approximations and analysis; Electrical properties; Marine electromagnetics.

INTRODUCTION

In the past decade, the marine controlled-source electromagnetic (CSEM) method has emerged as a useful exploration technique for mapping offshore hydrocarbon reservoirs (e.g. Eidesmo et al. 2002) and characterizing gas hydrates bearing shallow sediments (e.g. Weitemeyer et al. 2006). The CSEM method uses a horizontal electric dipole (HED) source, which is towed a few tens of metres above the seafloor. The HED source transmits low-frequency (0.1–10 Hz) electromagnetic signals to an array of receivers positioned on the seabed. The receivers detect and record the electromagnetic signals from the source. The resistivity distribution below the seabed may be determined by interpreting the recorded electromagnetic fields as a function of source–receiver offsets. When interpreting the marine CSEM data, the seabed is usually assumed to be electrically isotropic. However, it is commonly known that the layered sedimentary sequences are capable of producing anisotropy on a macroscopic scale (Tompkins 2005). Thus, ignoring anisotropy in interpreting marine CSEM data may lead to a distorted image of seabed conductivity structures, even misinterpretation. The effects of anisotropy on the marine CSEM responses have been rarely discussed in geophysical literatures.

Tompkins (2005) investigated the effects of vertical anisotropy on marine CSEM responses of 1-D layered models. Loseth & Ursin (2007) presented a method for calculating the electromagnetic field from a dipole source in stratified media with general anisotropy. Lu & Xia (2007) studied the EM field distribution in a double half-space with a vertical anisotropy and the impact of electrical anisotropy on marine CSEM data. Kong et al. (2008) considered the marine CSEM forward problem in 2-D earth media with diagonal anisotropy. Li & Key (2007) reported an adaptive finite element (FE) algorithm for forward modelling of the marine CSEM fields in 2-D isotropic conductivity structures. In this paper, the FE treatment is extended to dipping anisotropic models.

The outline of the paper is as follows. First, we describe the FE approximation of the marine CSEM forward problem in 2-D dipping anisotropic conductivity structures. Next, we validate the FE code using a layered model with anisotropy. We then investigate the effects of anisotropy upon the marine CSEM responses.

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THEORY

Governing equations

We consider a 2-D conductivity model with dipping anisotropy. Let \( x \) be the structural strike direction. The coordinate system is right handed with the \( z \)-axis pointing positive downwards. An HED source is located above the seafloor. Assuming a time variation \( e^{-i\omega t} \), the governing equations for the electric and magnetic fields (\( E \) and \( H \)) in the quasi-stationary approximation are

\[
\nabla \times E = i\mu_0 H, \tag{1}
\]

\[
\nabla \times H - \sigma E = J, \tag{2}
\]

where \( \mu_0 \) is the magnetic permeability of free space, \( \omega \) is the angular frequency and \( J \) is the source current distribution, and

\[
\sigma = \begin{pmatrix}
\sigma_{xx} & 0 & 0 \\
0 & \sigma_{yy} & \sigma_{yz} \\
0 & \sigma_{zy} & \sigma_{zz}
\end{pmatrix}
\tag{3}
\]

is the electric conductivity tensor in dipping anisotropy media. The conductivity tensor is symmetric, that is, \( \sigma_{12} = \sigma_{21} \) and hence it is characterized by four components. This tensor can be rotated into its principal axes \( (x', y', z') \), and can then be described by its three principal values \( (\sigma_{1}, \sigma_{2}, \sigma_{3}) \) and the dipping angle \( \alpha \). To remove the source - point singularities, the electromagnetic fields are split into a primary part and a secondary part. The primary fields (\( E^p \) and \( H^p \)) induced by an HED in a 1-D layered structure with primary conductivity \( \sigma^p(z) \) can be calculated analytically. The primary conductivity structure consists of the first layer and the underlying half-space or layered Earth. The first layer is assumed to be isotropic and has the same resistivity as sea water in which the electric dipole source is located. The underlying half-space (or layers) may be isotropic or anisotropic. In our numerical examples presented in the later sections, we assume that the underlying medium in the reference model is isotropic, and the singularity can be well removed. The secondary fields (\( E^s \) and \( H^s \)) caused by inhomogeneities with anomalous conductivity tensor \( \sigma^s = \sigma - \sigma^p(z) \) are computed by the FE method. The secondary fields satisfy the following equations:

\[
\nabla \times E^s = i\mu_0 H^s, \tag{4}
\]

\[
\nabla \times H^s - \sigma E^s = \sigma^s E^p. \tag{5}
\]

Although the conductivity distribution is 2-D, eqs (4) and (5) are 3-D differential equations. They can be transferred into 2-D equations by Fourier transformation with respect to the strike direction \( x \), resulting in the following partial differential equations:

\[
\frac{\partial \hat{E}_x^s}{\partial y} - i k_x \sigma_{xx} \hat{E}_x^s = i\mu_0 \hat{H}_y^s, \tag{6}
\]

\[
\frac{\partial \hat{E}_y^s}{\partial z} - \frac{\partial \hat{E}_z^s}{\partial y} = i\mu_0 \hat{H}_y^s, \tag{7}
\]

\[
\frac{\partial \hat{E}_z^s}{\partial y} - i k_x \hat{E}_x^s = i\mu_0 \hat{H}_x^s, \tag{8}
\]

\[
\frac{\partial \hat{H}_y^s}{\partial y} - \frac{\partial \hat{H}_z^s}{\partial z} - \sigma_{yy} \hat{E}_y^s = \sigma_{yy} \hat{E}_y^s + \sigma_{yz} \hat{E}_z^s, \tag{9}
\]

\[
\frac{\partial \hat{H}_z^s}{\partial y} - i k_x \hat{E}_y^s - (\sigma_{yy} \hat{E}_y^s + \sigma_{yz} \hat{E}_z^s) = \sigma_{yy} \hat{E}_y^s + \sigma_{yz} \hat{E}_z^s, \tag{10}
\]

\[
\frac{\partial \hat{H}_x^s}{\partial y} - i \sigma_{yy} \hat{E}_y^s - (\sigma_{yz} \hat{E}_z^s \hat{E}_y^s + \sigma_{zz} \hat{E}_y^s) = \sigma_{yz} \hat{E}_z^s + \sigma_{zz} \hat{E}_z^s, \tag{11}
\]

where \( k_x \) is the wavenumber along the strike direction \( x \). Note that a hat (\( \hat{\cdot} \)) denotes the quantity in the wavenumber domain \( (k_x, y, z) \). Eqs (6)–(11) can be combined to yield two partial differential equations for the strike-parallel transformed electromagnetic components \( \hat{E}_x^s(k_x, y, z) \) and \( \hat{H}_y^s(k_x, y, z) \).

\[
\nabla \times \left( \tau_y \nabla \hat{E}_y^s \right) - \sigma_{yy} \hat{E}_y^s + \nabla \cdot \left( \tau_y \nabla \hat{H}_y^s \right) = p_\times - i k_x \left( \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right), \tag{12}
\]

\[
\nabla \times \left( \tau_y \nabla \hat{H}_y^s \right) - \sigma_{yy} \hat{H}_y^s + \nabla \cdot \left( \tau_y \nabla \hat{E}_y^s \right) = -i\mu_0 \left( \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right), \tag{13}
\]
with
\[ \tau_1 = \frac{i \mu \omega_0}{\gamma^2 z_y} \begin{pmatrix} \gamma^2_y & -i \mu \omega \sigma_{yz} \\ -i \mu \omega \sigma_{yz} & \gamma^2_z \end{pmatrix}, \]
\[ \tau_2 = \frac{1}{\gamma^2 z_y} \begin{pmatrix} \sigma_{yy} \gamma^2_z + i \mu \omega \sigma_{yz} & k^2 \sigma_{yz} \\ k^2 \sigma_{yz} & \sigma_{zz} \gamma^2_y + i \mu \omega \sigma_{yz} \end{pmatrix}, \]
\[ \tau_3 = \frac{i k}{\gamma^2 z_y} \begin{pmatrix} -i \mu \omega \sigma_{yz} & -\gamma^2_y \\ \gamma^2_z & i \mu \omega \sigma_{yz} \end{pmatrix}, \]
\[ \tau_4 = \frac{i k}{\gamma^2 z_y} \begin{pmatrix} i \mu \omega \sigma_{yz} & -\gamma^2_z \\ -\gamma^2_y & i \mu \omega \sigma_{yz} \end{pmatrix}. \]

\[ Q = \frac{1}{\gamma^2 z_y} \left( \gamma^2_y p_y + i \mu \omega \sigma_{yz} p_z \right), \quad R = \frac{1}{\gamma^2 z_y} \left( \gamma^2_z p_z + i \mu \omega \sigma_{yz} p_y \right). \]

\[ p_y = \sigma_{yy} \hat{E}_y, \quad p_z = \sigma_{zz} \hat{E}_z + \sigma_{yz} \hat{E}_y, \quad p_z = \sigma_{zz} \hat{E}_z + \sigma_{yz} \hat{E}_y, \]
\[ \gamma^2_y = k^2 - i \mu \omega \sigma_{yy}, \quad \gamma^2_z = k^2 - i \mu \omega \sigma_{zz}, \quad \gamma^2_{yz} = \gamma^2_y \gamma^2_z + \omega^2 \mu^2_0 \sigma_{yz}, \]
\[ \nabla_z = \left( \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right). \]

**Figure 1.** 1-D canonical reservoir model with vertical anisotropy. An HED (Tx) is located 50 m above the seafloor. A 100-m-thick, isotropic resistive layer of 50 \( \Omega \) overlies an isotropic half-space of 1 \( \Omega \). The 1-km-thick overburden layer is vertically anisotropic. The overburden resistivity in the horizontal plane is \( \rho_x = \rho_y = \rho_h = 1 \Omega \), and in vertical direction \( \rho_v = 4 \Omega \).

**Figure 2.** Amplitude (a) and phase (c) of the inline and broadside geometry electromagnetic field components on the seafloor for the 1-D model shown in Fig. 1. The anisotropy coefficient is \( \lambda = 2 \) (i.e. \( \rho_h = 1 \Omega \), \( \rho_v = 4 \Omega \)) and the transmission frequency is 0.25 Hz. The solid lines indicate the analytical solutions and the asterisks the numerical results obtained using the adaptive FE technique. The relative error in amplitude (b) and the absolute error in phase (d).
To solve eqs (12) and (13) for the unknown fields $\hat{E}_s^x$ and $\hat{H}_s^x$, the homogeneous Dirichlet boundary conditions are applied on the outer boundary of the model.

**FE approximation**

The weak formulation of the boundary value problem can be obtained by the multiplication of eqs (12) and (13) with a test function $\delta \hat{E}_s^x$ and $\delta \hat{H}_s^x$, respectively, and the integration over the model area $\Omega$. Integration by parts using Green’s formula and substitution of the boundary

![Figure 3. A 2-D canonical reservoir model. Both the reservoir and the surrounding sediment are assumed to be electrically anisotropic. The CSEM responses of the 2-D model are calculated using the finite element algorithm presented in this paper.](Image)

![Figure 4. The inline (left-hand side) and broadside (right-hand side) geometry horizontal electric field along the seafloor for four different vertical resistivities ($\rho_z = 1, 2, 4$ and $10 \ \Omega m$) in the anisotropic sediment from Fig. 3. The horizontal resistivities in the $x$ and $y$ direction in the sediment are the same and equal to $1 \ \Omega m$. The reservoir is isotropic with resistivity $50 \ \Omega m$. Amplitude is shown in the top row and phase in the middle row. The amplitude in (a) and (d) is normalized by the corresponding values of the 1-D and 2-D reference model, respectively, and illustrated in the bottom row.](Image)
conditions leads to
\[
\iint_{\Omega_1} \left[ \tau_1 \nabla_2 \delta \hat{E}_s \cdot \nabla_2 \delta \hat{E}_s + \sigma_{xx} \delta \hat{E}_s \right] \, d\Omega + \iint_{\Omega_1} \left[ \tau_2 \nabla_2 \delta \hat{H}_s \cdot \nabla_2 \delta \hat{H}_s \right] \, d\Omega
\]
\[\quad = - \iint_{\Omega_1} p_\delta \hat{E}_s \, d\Omega - \iint_{\Omega_1} ik_i \left( Q \frac{\delta \hat{E}_s}{\partial y} + R \frac{\delta \hat{E}_s}{\partial z} \right) \, d\Omega \quad (14)
\]
and
\[
- \iint_{\Omega_1} \tau_4 \nabla_2 \delta \hat{H}_s \cdot \nabla_2 \delta \hat{H}_s \, d\Omega - \iint_{\Omega_1} i \omega \mu_0 \delta \hat{H}_s \delta \hat{H}_s \, d\Omega - \iint_{\Omega_1} \tau_2 \nabla_2 \delta \hat{E}_s \cdot \nabla_2 \delta \hat{H}_s \, d\Omega
\]
\[\quad = \iint_{\Omega_1} i \omega \mu_0 \left( R \frac{\delta \hat{H}_s}{\partial y} - Q \frac{\delta \hat{H}_s}{\partial z} \right) \, d\Omega. \quad (15)
\]

The FE discretization of eqs (14) and (15) results in a linear system of equations in the matrix form (e.g. Li 2002; Li & Key 2007; Li & Pek 2008).

\[
\mathbf{K} \mathbf{u} = \mathbf{p},
\]
where \( \mathbf{u} \) is the column vector, consisting of the unknown transformed secondary fields (\( \hat{E}_s \) and ˆ\( \hat{H}_s \)) at all vertices, and \( \mathbf{p} \) is the known vector resulting from the terms of the right-hand side of eqs 14 and 15. The global matrix \( \mathbf{K} \) is complex, sparse and symmetric. Substituting the electric and magnetic values on the outer boundary into eq. (16), the equations can be solved for \( \hat{E}_s \) and \( \hat{H}_s \) at the internal vertices, for example, using the direct solver SuperLU (Demmel et al. 1999) or the quasi-minimal residual (QMR) method (Freund & Nachtigal 1994; Weiss 2001).

Electric and magnetic fields

Once the strike-parallel components of secondary fields are obtained in the wavenumber domain (\( k_x, y, z \)), the other two electric components \( \hat{E}_s \) and \( \hat{E}_z \) as well as two magnetic components \( \hat{H}_y \) and \( \hat{H}_z \) can be determined by spatially differentiating

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**Figure 5.** The real part of inline geometry electric field in the \( \text{yz} \)-plane for the model of Fig. 3 with vertical anisotropy in the anisotropic sediment: (a) \( \rho_x = \rho_y = \rho_z = 1 \Omega \text{m} \); (b) \( \rho_x = \rho_y = 1, \rho_z = 4 \Omega \text{m} \); (c) \( \rho_x = \rho_y = 1, \rho_z = 10 \Omega \text{m} \). The reservoir is isotropic with resistivity 50 \( \Omega \text{m} \). Arrows show the direction of the electric fields and the shaded colours correspond to the magnitude of the electric fields in a logarithmic scale. The blue dashed lines indicate the 2-D target’s outline.
\[ \hat{E}_y = \frac{i}{w} \mu_0 \gamma_{yz} [\gamma_y^2 F_2 + i \omega \mu_0 \sigma_{yz} F_1], \]
\[ \hat{E}_z = \frac{i}{w} \mu_0 \gamma_{yz} [\gamma_y^2 F_1 + i \omega \mu_0 \sigma_{yz} F_2], \]
\[ \hat{H}_y = -\frac{i k_x}{\gamma_{yz}} [\gamma_y^2 F_1 + i \omega \mu_0 \sigma_{yz} F_2] + \frac{1}{i \omega \mu_0} \frac{\partial \hat{E}_z}{\partial z}, \]
\[ \hat{H}_z = \frac{i k_x}{\gamma_{yz}} [\gamma_y^2 F_2 + i \omega \mu_0 \sigma_{yz} F_1] - \frac{1}{i \omega \mu_0} \frac{\partial \hat{E}_y}{\partial y}, \]

with
\[ F_1 = \frac{\partial \hat{H}_z}{\partial y} - \frac{k_x}{\omega \mu_0} \frac{\partial \hat{E}_z}{\partial z} + p_z, \quad F_2 = -\frac{\partial \hat{H}_y}{\partial z} - \frac{k_x}{\omega \mu_0} \frac{\partial \hat{E}_y}{\partial y} + p_y. \]

The electric and magnetic fields in the wavenumber domain can be transformed into the space domain \((x, y, z)\) by inverse Fourier transformation and are then added to the primary fields to obtain the total electromagnetic fields.

**A posteriori error estimator and adaptive grid refinement**

Adaptive FE methods provide a powerful approach for numerical modelling of complex problems. They can automatically improve mesh design and offer reliable solution at a reasonable computational cost. This algorithm starts with a coarse mesh and successively refined meshes are generated according to a posteriori error estimator which estimates the error between the current FE solution and the true solution to the partial differential equation on each element. In this paper, we use the dual error estimate weighting (DEW) method (Ovall 2006), in which the global influence on the local error is taken into consideration. This method has been shown to be effective for the 2-D MT (Key & Weiss 2006; Li & Pek 2008) and the 2.5-D marine CSEM forward problem (Li & Key 2007). In a manner similar to our preceding papers (Li &...
Key 2007; Li & Pek 2008), we can derive the coupled DEW error indicator for element \( e \).

\[
\hat{\eta}_e = \eta^E_e \bar{\eta}^E_e + \eta^H_e \bar{\eta}^H_e + \eta^{EH}_e + \eta^{HE}_e
\]  

(21)

with

\[
\eta^E_e = \frac{\left\| (R - I) \nabla \hat{E}_s^e \right\|_{L^2(e)}}{\left\| (R - I) \nabla \hat{E}_s^e \right\|_{L^2(e)}},
\]

\[
\eta^H_e = \frac{\left\| (R - I) \nabla \hat{H}_s^e \right\|_{L^2(e)}}{\left\| (R - I) \nabla \hat{H}_s^e \right\|_{L^2(e)}},
\]

(22)

\[
\bar{\eta}^E_e = \frac{\left\| \tau_3 (R - I) \nabla \hat{E}_s^e \right\|_{L^2(e)}}{\left\| \tau_3 (R - I) \nabla \hat{E}_s^e \right\|_{L^2(e)}},
\]

\[
\bar{\eta}^H_e = \frac{\left\| \tau_1 (R - I) \nabla \hat{H}_s^e \right\|_{L^2(e)}}{\left\| \tau_1 (R - I) \nabla \hat{H}_s^e \right\|_{L^2(e)}},
\]

(23)

\[
\eta^{EH}_e = \frac{\left\| \tau_4 (R - I) \nabla \hat{E}_s^e \cdot (R - I) \nabla \hat{H}_s^e \right\|_{L^2(e)}}{\left\| \tau_4 (R - I) \nabla \hat{E}_s^e \cdot (R - I) \nabla \hat{H}_s^e \right\|_{L^2(e)}},
\]

(24)

\[
\eta^{HE}_e = \frac{\left\| \tau_2 (R - I) \nabla \hat{H}_s^e \cdot (R - I) \nabla \hat{E}_s^e \right\|_{L^2(e)}}{\left\| \tau_2 (R - I) \nabla \hat{H}_s^e \cdot (R - I) \nabla \hat{E}_s^e \right\|_{L^2(e)}},
\]

(25)

where \( I \) is the identity matrix, and \( \nabla \hat{E}_s^e \) and \( \nabla \hat{H}_s^e \) is the gradient of a FE solution \( \hat{E}_s^e \) and \( \hat{H}_s^e \), respectively. \( R \) is a gradient recovery operator.

The details about the derivation of eqs (21)–(25) are given in the Appendix.

For our implementation, we refine 5–10 per cent of elements with the largest \( \hat{\eta}_e \). The grid refinement is carried out by calling the grid generation code ‘Triangle’ (Shewchuk 1997, 2002). After an improved mesh is generated, the problem is solved again on the new mesh and the error indicator \( \hat{\eta}_e \) is updated. This process is repeated until the required level of accuracy is achieved. For details about the implementation of the adaptive FE method please refer to Li & Key (2007).

**Validation of the Adaptive FE Code**

To validate the FE code, we simulated a 1-D model and compared the computed results with the quasi-analytic solution. The test model is a 1-D reservoir model shown in Fig. 1. The sea water is isotropic with a resistivity of 0.3 \( \Omega m \) and a depth of 1 km. An HED is located at a height of 50 m above the seafloor. A 100-m-thick, isotropic resistive layer of 50 \( \Omega m \) overlies an isotropic half-space of 1 \( \Omega m \). The 1-km-thick...
overburden layer is vertically anisotropic. This means that all three principal axes of the resistivity tensor of this layer are coincident with the axes of (x,y,z) coordinates and the resistivity $\rho_h$ for any horizontal direction differs solely from the resistivity in the vertical direction $\rho_v$ (i.e. $\rho_x = \rho_y = \rho_h$ and $\rho_z = \rho_v$). The degree of vertical anisotropy is defined by the anisotropy coefficient $\lambda = \sqrt{\rho_v/\rho_h}$. The overburden resistivity in the horizontal plane (x,y) is assumed to be $\rho_h = 1 \, \Omega\,m$ and in vertical direction $\rho_v = 4 \, \Omega\,m$ (i.e. $\lambda = 2$). For the FE modelling, the two-layer earth consisting of 1-km-thick, 0.3 $\Omega\,m$ sea water and an isotropic half-space of 1 $\Omega\,m$ is regarded as the reference model. Numerical modelling was done for both broadside and inline geometry at a frequency of 0.25 Hz. A total of 80 receivers was positioned along the seafloor at ranges between $y = 0.1$ km and 8 km. Note that in broadside geometry (the transmitter pointing towards x-axis and the receivers positioned along the y-axis) there exist an electric component ($E_x$) and two magnetic components ($H_y$ and $H_z$), and in inline geometry (the transmitter pointing towards y-axis and the receivers positioned along the y-axis), there are two electric field components ($E_x$ and $E_z$) and one magnetic field component ($H_y$). For all numerical examples in this section, we used the same model domain of 40 km width and 40 km height (including 20 km in the air), which was initially discretized into 3352 triangular elements with 1701 vertices. Adaptive refinement iterations were set to refine 5 per cent of elements with largest error indicator calculated at wavenumbers $k_x = 0.00001, 0.0001$ and 0.001 m$^{-1}$. Refinement iterations were stopped when $\tilde{E}_s$ and $\tilde{H}_s$ at the receivers varied by no more than 0.2 per cent between subsequent iterations. The final refined grid, consisting of 21 958 vertices and 58 266 triangular elements was used to compute the spatial domain fields using 56 logarithmically spaced wavenumbers from $10^{-7}$ to 0.9 m$^{-1}$.

Figs 2(a) and (c) show the computed amplitude and phase (asterisks) of all six electromagnetic components for the 1-D model (Fig. 1). The analytical solutions (solid line) are also shown for comparison. One can see that the FE algorithm provides accurate results. The relative amplitude error between the analytical and the computed results for all EM field components except $H_z$ for large offsets is less than 1 per cent (Fig. 2b) and the error in phase is less than 0.5° (Fig. 2d). Note that the amplitude of electric and magnetic field components plotted are normalized by the source dipole moment.

**EFFECTS OF ANISOTROPY**

In this section, a simple 2-D model shown in Fig. 3 is used to demonstrate effects of anisotropy upon marine CSEM responses. A 6-km-wide, 100-m-thick reservoir is embedded into a homogeneous background sediment at a depth of 1 km. The sea water is isotropic with resistivity 0.3 $\Omega\,m$ and the water depth is 1000 m. The horizontal electrical dipole is positioned 50 m above the seafloor over the centre of the reservoir. A total of 200 receivers are positioned along the seafloor at ranges between $y = -10$ and 10 km. Anisotropy in the reservoir and the background media is modeled by assigning different principal resistivities to the box. The principal resistivities of the anisotropic sediment are $\rho'_x = \rho'_y = 1 \, \Omega\,m$ and $\rho'_z = 10 \, \Omega\,m$. The reservoir is isotropic with resistivity 50 $\Omega\,m$. Amplitude is shown in the top row and phase in the middle row. The amplitude in (a) and (d) is normalized to the 1-D and 2-D reference model response, respectively, and illustrated in the bottom row.

![Figure 8](https://example.com/fig8.png)

**Figure 8.** The inline horizontal electric (left-hand side) and magnetic (right-hand side) field curves for five different dip angles ($\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and $90^\circ$) in the anisotropic sediment from Fig. 3. The principal resistivities of the anisotropic sediment are $\rho'_x = \rho'_y = 1 \, \Omega\,m$ and $\rho'_z = 10 \, \Omega\,m$. The reservoir is isotropic with resistivity 50 $\Omega\,m$. Amplitude is shown in the top row and phase in the middle row. The amplitude in (a) and (d) is normalized to the 1-D and 2-D reference model response, respectively, and illustrated in the bottom row.

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sediment is considered separately. It is assumed that the principal axis $x'$ is horizontal and in strike-direction $x$ (i.e. $x' = x$) and the remaining two principal axes $y'$ and $z'$ are in the vertical plane ($y,z$) with a dip angle $\alpha$ with respect to the $y$-axis. The anisotropic resistivity is given by

$$
\rho = \begin{pmatrix}
\rho_x' & 0 & 0 \\
0 & \rho_y' \cos^2 \alpha + \rho_z' \sin^2 \alpha & (\rho_y' - \rho_z') \sin \alpha \cos \alpha \\
0 & (\rho_y' - \rho_z') \sin \alpha \cos \alpha & \rho_y' \sin^2 \alpha + \rho_z' \cos^2 \alpha
\end{pmatrix},
$$

(26)

where $\rho_x'$, $\rho_y'$ and $\rho_z'$ are principal resistivities in the principal axes $x'$, $y'$ and $z'$, respectively. Although our code allows for models in which all three principal resistivities are different from each other, we assume in the numerical examples that the resistivities in the bed parallel directions are the same and differ from the resistivity in the perpendicular to the bed plane. All calculations are made at a frequency of 0.25 Hz, unless stated otherwise.

Anisotropy in the sediment

The surrounding sediment is assumed to be anisotropic and the reservoir is isotropic with resistivity 50 $\Omega m$. Three types of anisotropy are considered: transverse isotropy with a vertical axis of symmetry (TIV, also called vertical anisotropy), transverse isotropy with a horizontal axis of symmetry (TIH) and transverse isotropy with a tilted axis of symmetry (TTI).

First, we study the effect of the vertical resistivity in the sediment ($\rho_z$) upon marine CSEM responses. All three principal axes of the resistivity tensor of the background sediment are coincident with the axes of the $(x, y, z)$ coordinates. The horizontal resistivities of the anisotropic sediment ($\rho_x$ and $\rho_y$) remain constant and are set to be 1 $\Omega m$, while the vertical resistivity $\rho_z$ varies from 1 to 10 $\Omega m$. Fig. 4 shows the inline and broadside geometry horizontal electric field responses along the seafloor for four different vertical resistivities $\rho_z$ ($=1, 2, 4, 10 \Omega m$). From Fig. 4, one can see that both magnitude and phase of the horizontal electric field are dependent on the resistivity in vertical direction $\rho_z$. The magnitude of the electric field increases with increasing vertical resistivity $\rho_z$, while the phase decreases. We can explain this observation by looking at the distribution of the electrical fields. Fig. 5 shows the real part of the inline geometry electrical field in the $yz$-plane for three vertical resistivities $\rho_z$ ($=1, 4, 10 \Omega m$). Arrows show the direction of the electric field and the shaded colours correspond to the magnitude of the electric field in a logarithmic scale. From Fig. 5 one can see that (1) the electric field decays exponentially away from

Figure 9. The real part of inline geometry electric field in the $yz$-plane for the model of Fig. 3 with dipping anisotropy in the anisotropic sediment: (a) $\alpha = 30^\circ$; (b) $\alpha = 45^\circ$; (c) $\alpha = 60^\circ$. The principal resistivities of the anisotropic sediment are $\rho_x' = \rho_y' = 1 \Omega m$ and $\rho_z' = 10 \Omega m$. The reservoir is isotropic with resistivity 50 $\Omega m$. Arrows show the direction of the electric fields and the shaded colours correspond to the magnitude of the electric fields in a logarithmic scale. The blue dashed line indicates the 2-D target’s outline and the black arrow the dip direction in the sediment.
the source and it decays faster in sea water than in the seafloor sediment; (2) the electrical field decay becomes slower with increasing the vertical resistivity of the seafloor sediment.

To highlight the effect of a target, the electrical field amplitude is often normalized by 1-D background model responses. The amplitude of the electric field response shown in Figs 4(a) and (d) is normalized by the corresponding value for the 1-D reference model which is consisted of the air layer, 1000 m depth water layer and 1 Ωm isotropic half-space and then illustrated by the dashed line in Figs 4(c) and (f).

In an anisotropic case, the normalized field by the 1-D isotropic reference model contains both the effect of the target and the effect of the anisotropy. To highlight the effect of the anisotropy alone, the electrical field response is divided by the response of the 2-D isotropic reference model which consists of the 1-D isotropic background model and the 50 Ωm isotropic target and is shown by the asterisks in Figs 4(c) and (f). The effect of the vertical resistivity in the sediment for inline geometry is much larger than that for broadside geometry. Note that the horizontal electric field response curves are symmetric with respect to \( y = 0 \) (the HED location and the centre of the model).

Next, we investigate the effect of horizontal resistivity in the anisotropic sediment upon marine CSEM responses. Both horizontal resistivities \( \rho_x \) and \( \rho_y \) affect the marine CSEM responses, but to different extent. Here we consider two cases of transversely isotropic sediment: one with a horizontal symmetry \( y \)-axis and another with horizontal symmetry \( x \)-axis. In the case with the symmetry \( y \)-axis, resistivities in the vertical plane \((x, z)\) are assumed to be the same and equal to 1 Ωm, and the resistivity \( \rho_y \) varies from 1 to 10 Ωm. Fig. 6 shows the inline and the broadside geometry horizontal electric field response for four different horizontal resistivities \( \rho_y = 1, 2, 4, 10 \) Ωm. From Fig. 6, one can see that while both the inline and the broadside geometry horizontal electric fields are dependent on the resistivity in the \( y \)-direction \( \rho_y \), the effect on the inline geometry fields is greater than on the broadside geometry one. In the case with horizontal symmetry \( x \)-axis, resistivities in the vertical plane \((y, z)\) are the same and equal to 1 Ωm, and resistivity \( \rho_x \) varies from 1 to 10 Ωm. Fig. 7 shows the inline and the broadside geometry horizontal electric field response for four different horizontal resistivities \( \rho_x = 1, 2, 4, 10 \) Ωm. From Fig. 7, one can see that while both broadside and inline geometry horizontal electric fields are affected by \( \rho_x \), the effect on the broadside geometry field is much greater than on the inline geometry field. From Figs 4, 6 and 7, we can see that while the inline geometry is sensitive to the vertical resistivity \( \rho_z \) in the sediment, the broadside geometry is sensitive to the horizontal resistivity \( \rho_x \) and it is not sensitive to \( \rho_y \).

Then, we consider a TTI case in which non-zero anisotropic dipping angle is incorporated into the above anisotropic sediment model. The principal resistivities of the anisotropic sediment are \( \rho_x'/\rho_y'/\rho_z' = 1/1/10 \) Ωm. Fig. 8 shows the inline horizontal electric field \( (E_x) \) and the inline horizontal magnetic field \( (H_x) \) for five different dip angles \((\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ \text{ and } 90^\circ) \) at 0.25 Hz. The CSEM responses of a 1 Ωm (black dotted line) and 10 Ωm (black dashed line) isotropic half-space are also shown for comparison. From Fig. 8, one can see that

**Figure 10.** The inline horizontal electric field curves for four different vertical resistivities \((\rho_z = 50, 100, 200 \text{ and } 500 \) Ωm \() in the anisotropic reservoir from Fig. 3. The horizontal resistivities are \( \rho_x = \rho_y = 50 \) Ωm. The sediment is isotropic with resistivity 1 Ωm. The amplitude is normalized to the response of the 2-D reference model with the isotropic reservoir of 50 Ωm (dashed lines) and with that of geometric mean resistivity of the anisotropic reservoir (asterisks), respectively.
(1) The electromagnetic responses are affected considerably by the anisotropy dip angle $\alpha$. The amplitude of the EM field decreases with increasing dip angle, while the phase increases except for far offsets at which the air wave starts to dominate.

(2) The amplitude and phase are asymmetric with respect to both the centre of the model and the HED location in all but two cases: $\alpha = 0^\circ$ and $\alpha = 90^\circ$. The asymmetry of the normalized response clearly shows the effect of the anisotropic dip angle $\alpha$ upon marine CSEM fields. The survey line on the left side yields larger normalized responses than those on the right side. Fig. 9 shows the real part of the inline geometry electrical field in the $yz$-plane for three dipping angles $\alpha (=30^\circ, 45^\circ, 60^\circ)$. Again, arrows show the direction of the electric field and the shaded colour corresponds to the magnitude of the electric field in a logarithmic scale. From Fig. 9 one can see that the distribution of the electrical field in the subseaflor is asymmetric with respect to both the HED location and the centre of the model (i.e. $y = 0$) and it is dependent of the dipping angle. The magnitude of the electric field decays much more slower along the direction of low resistivity (dipping direction) than in the direction of high resistivity in the anisotropic seafloor sediment.

(3) If $\alpha = 0^\circ$, the CSEM responses correspond to those produced by a model with resistivities of $1 \Omega m$ along the $x$ and $y$ directions and $10 \Omega m$ along the $z$ direction, namely, the previous TIV model for $\rho_x = \rho_y = 1 \Omega m$ and $\rho_z = 10 \Omega m$. If $\alpha = 90^\circ$, the CSEM responses correspond to those produced by a model with resistivities of $1 \Omega m$ along the $x$ and $z$ directions and $10 \Omega m$ along the $y$ direction, that is, the previous TIH model for $\rho_x = \rho_z = 1 \Omega m$ and $\rho_y = 10 \Omega m$.

For the above FE modelling, a model domain of 40 km width and 40 km (including 20 km in the air) height was initially discretized into 689 triangular elements with 360 vertices. Adaptive refinement iteration was set to refine 5 per cent of elements with largest error indicator. For most models, the maximum relative difference of the electric field $\hat{E}_x$ and the magnetic field $\hat{H}_x$ between subsequent iterations at receiver locations converged to less than 0.5 per cent after 25 grid refinements. For the vertical anisotropy model with $\rho_x = \rho_y = 1$ and $\rho_z = 10 \Omega m$, for instance, the maximum relative difference of the EM fields $\hat{E}_x$ and $\hat{H}_x$ converged to 0.2 percent after 25 grid iterations, resulting in 21450 triangles and 10743 vertices. The grid refinement iteration took 120 s and the refined grid required 365 s to compute the spatial domain EM fields (Apple 2.4 GHz Intel Core 2 Duo).

Figure 11. The real part of inline geometry electric field in the $yz$-plane for the model of Fig. 3 with vertical anisotropy in the reservoir body: (a) $\rho_z = 100 \Omega m$; (b) $\rho_z = 200 \Omega m$; (c) $\rho_z = 500 \Omega m$. The horizontal resistivities in the $x$ and $y$ direction in the reservoir are $50 \Omega m$. The sediment is isotropic with resistivity $1 \Omega m$. Arrows show the direction of the electric fields and the shaded colours correspond to the magnitude of the electric fields in a logarithmic scale. The blue dashed lines indicate the 2-D target’s outline.
Anisotropy in the reservoir

Now, we consider the anisotropic reservoir case in which the reservoir is assumed to be anisotropic and the surrounding sediment is isotropic with a resistivity of 1 \( \Omega \text{m} \).

Fig. 10 shows the inline geometry horizontal electric field response along the seafloor for four different vertical resistivities \( \rho_z \) (50, 100, 200, 500 \( \Omega \text{m} \)). The horizontal resistivities of the anisotropic reservoir \( (\rho_x \text{ and } \rho_y) \) remain constant and are set to be 50 \( \Omega \text{m} \). From Fig. 10, we can see that the horizontal electric field is dependent on the vertical resistivity in the reservoir. The magnitude of the electric field increases with increasing \( \rho_z \), while the phase decreases. To highlight the effect of the vertical anisotropy in the reservoir, the amplitude of electric fields shown in Fig. 10(a) is divided by the corresponding value for the 2-D isotropic reference model which consists of the 1-D isotropic background model and the 50 \( \Omega \text{m} \) isotropic reservoir and is shown in Fig. 10(c) (dotted line). The normalized field clearly indicates the dependence of electric field responses on a vertical resistivity.

The normalized field by the 2-D reference model contains both the effect of vertical anisotropy and the effect of the changing bulk resistivity in the reservoir. To highlight the effect of anisotropy alone, the electric field response is normalized by that of a 2-D isotropic reference model with the geometric mean resistivity of the anisotropic reservoir and is shown by asterisks in Fig. 10(c). Note that the effect of the vertical resistivity in the reservoir on the CSEM responses is much less than that in the surrounding sediment. Fig. 11 shows the inline geometry electrical field in the \( yz \)-plane for three different vertical resistivities \( (\rho_z = 100, 200, 500 \Omega \text{m}) \) in the reservoir body. The distribution of the electrical field for three different vertical resistivities in the reservoir is very similar to each other. This means that the vertical anisotropy in the reservoir has little effect.

Fig. 12 shows the inline geometry horizontal electric field response for four different horizontal resistivities \( \rho_y \) (50, 100, 200, 500 \( \Omega \text{m} \)). The reservoir’s resistivities in the vertical plane \( (x, z) \) are practically the same and equal to 50 \( \Omega \text{m} \). The influence of the horizontal resistivity \( \rho_y \) of the reservoir on the CSEM responses can be hardly seen in Figs 12(a) and (b). The electrical field amplitude is normalized, respectively, by the response of two reference models: one consisting of 1-D isotropic background and the 50 \( \Omega \text{m} \) isotropic target, and another consisting of 1-D isotropic background and the isotropic 2-D block with the geometric mean resistivity of the anisotropic reservoir, and is shown in Fig. 12(c).

Figure 12. The inline horizontal electric field curves for four different horizontal resistivities \( (\rho_y = 50, 100, 200 \text{ and } 500 \Omega \text{m}) \) in the anisotropic reservoir from Fig. 3. The principal resistivities \( \rho_x = \rho_z = 50 \Omega \text{m} \). The sediment is isotropic with resistivity 1 \( \Omega \text{m} \). The amplitude is normalized to the response of the 2-D reference model with the isotropic reservoir of 50 \( \Omega \text{m} \) (dashed lines) and with that of geometric mean resistivity of the anisotropic reservoir (asterisks), respectively.
**Figure 13.** (a) A 2-D model with a dipping anisotropic overburden. (b) A 2-D model obtained by trial-and-error forward model computations under the assumption of the isotropic overburden. The dotted lines indicate the correct left and right boundary of the target. Note that the underlying target is laterally stretched for 0.5 km and its right boundary is shifted to the left for 0.5 km.

**IMPACT OF DIPPING ANISOTROPIC OVERBURDEN ON AN UNDERLYING TARGET**

In fold and thrust belt areas, hydrocarbon reservoirs are frequently overlain by thick, dipping sedimentary sequences including shales and thinly interbedded sandstones (Yan et al. 2004). The thin bedded sand-shale sequences can exhibit macroscopic electrical anisotropy with the main symmetry axis perpendicular to bed plane. Folding and faulting can thrust these stratigraphic horizons to the surface with steep dip angles (Yan et al. 2004). In this environment, electrical anisotropy can be regarded as transversely isotropic with a tilted axis of symmetry (TTI).

Fig. 13(a) shows a 2-D model, which is similar to the previous 2-D model shown in Fig. 3, but with a TTI overburden layer of 1000 m thick. The overburden layer has principal resistivities \( \rho_x = \rho_y = 1 \), \( \rho_z = 4 \) \( \Omega \)m and a dipping angle \( \alpha = 30^\circ \). The underlying target has a width of 6 km (from \( y = -3 \) to 3 km) and thickness of 100 m and is isotropic with a resistivity of 50 \( \Omega \)m. The half-space basement is assumed to be isotropic with resistivity 1 \( \Omega \)m. A total of 151 receivers was located along the seafloor at ranges between \( y = -15 \) and 15 km. The horizontal electrical dipoles are located at \( y = -5, -2, 0, 2, 5 \) km and at a height of 50 m above the seafloor. Fig. 14 displays the amplitude of the inline geometry horizontal electric field component from five transmitters (Tx) at a frequency of 0.25 Hz. The blue line indicates the electric field amplitude of the true model shown in Fig. 13(a). For comparison, the field amplitude (green line) of the isotropic overburden with resistivity 2 \( \Omega \)m (i.e. the geometric mean resistivity of the anisotropic overburden) is also shown. The influence of the dipping anisotropy in the overburden is evident. After trial-and-error forward model computations, we have found an isotropic model (Fig. 13b) and its response fits the field amplitude of the anisotropic overburden model well. However, the underlying target is laterally stretched for 0.5 km (i.e. its width changes to 6.5 km at ranges between \( y = -4 \) and 2.5 km) and it is shifted to the left for 0.5 km, so its centre is moved to \( y = -0.75 \) km from \( y = 0 \). This example clearly shows that if anisotropic overburden were treated as isotropic in marine CSEM interpretation, the reservoir properties, such as lateral extent and location, could be erroneously estimated.

**CONCLUSIONS**

We have presented an adaptive FE algorithm for the marine CSEM forward problem in 2-D dipping anisotropic media. The use of unstructured triangular mesh enables us to model arbitrary 2-D structures. Our numerical modelling shows that an electrical anisotropy has a significant effect upon marine CSEM field responses. The marine CSEM responses are dependent on both vertical and horizontal resistivities of surrounding sediments, while the influence of horizontal resistivity of the reservoir on CSEM responses can hardly be observed. The CSEM responses are affected considerably by anisotropy dip angle. Both inline and broadside geometry CSEM responses are affected by anisotropy, but to different extent. Although the effect of vertical anisotropy in the sediment on inline geometry fields is much larger than on broadside geometry responses, the effect of horizontal anisotropy along strike direction on broadside geometry fields is much larger than on inline
geometry fields, and the effect of horizontal anisotropy across strike direction is minimal. The effect of sediment anisotropy on the CSEM responses is much larger than that of the reservoir anisotropy.

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REFERENCES

APPENDIX: DEW FOR 2.5-D CSEM ANISOTROPY MODELLING

The DEW uses the solution to a dual problem to bias refinement towards elements that affect the solution at the EM receiver locations and enables the computation of asymptotically exact solutions to the partial differential equations. Consider a functional $G$, which is some measure of the solution error $u - u_h$, where $u$ is the true solution of the partial differential equation and $u_h$ is the FE approximation.

Using inner product notation, eqs (14) and (15) can be expressed as

$$B(u, v) = F(v),$$ (A1)

where

$$B(u, v) = \int_{\Omega} \left[ \alpha \nabla^2 u \cdot \nabla^2 v + \eta u v \right] d\Omega + \int_{\Omega} \beta \nabla^2 p \cdot \nabla^2 v d\Omega$$ (A2)

with

$$u = \hat{E}_x, \quad v = \delta \hat{E}_x, \quad p = \hat{H}_x, \quad \alpha = \tau_1, \quad \beta = \tau_2, \quad \gamma = \sigma_{xx} \quad \text{for eq. (14)},$$ (A3)

$$u = \hat{H}_x, \quad v = \delta \hat{H}_x, \quad p = \hat{E}_x, \quad \alpha = \tau_1, \quad \beta = \tau_2, \quad \eta = i \omega \mu_0 \quad \text{for eq. (15)}.$$ (A4)

The dual problem of eq. (A1) reads

$$B^*(w, v) = G(v)$$ (A5)

for $w$, in which $B^*$ is a dual or adjoint operator and is defined as $B^*(w, v) = B(v, w)$. We then have

$$G(u - u_h) = B^*(w, u - u_h) = B(u - u_h, w) = B(u - u_h, w - w_h),$$ (A6)

where $w$ and $w_h$ are the true solution and the FE solution of the dual problem, respectively. In the derivation of the last term on the right-hand side of eq. (A6), we have used the orthogonality property $B(u - u_h, w_h) = F(w_h) - B(u_h, w_h) = 0$. Provided with FE solutions $u_h$ and $w_h$, the right-hand side of eq. (A6) can be used to compute the equivalent solution to the error functional $G$.

$$B(u - u_h, w - w_h) = \int_{\Omega} \left[ \alpha \nabla(u - u_h) \cdot \nabla(w - w_h) + \gamma (u - u_h)(w - w_h) \right] d\Omega$$

$$+ \int_{\Omega} \beta \nabla^2 p \nabla(w - w_h) d\Omega.$$ (A7)

Approximating the gradient terms in eq. (A7) by

$$\nabla(u - u_h) \approx (R - I) \nabla u_h, \quad \nabla(w - w_h) \approx (R - I) \nabla w_h,$$ (A8)

yields

$$B(u - u_h, w - w_h) \approx \int_{\Omega} \alpha (R - I) \nabla u_h \cdot (R - I) \nabla w_h d\Omega + \int_{\Omega} \beta [(R - I) \nabla w_h \cdot \nabla p] d\Omega.$$ (A9)

where the contribution of the second term in the first area integral in eq. (A7) has been neglected. Eq. (A9) is an approximation of the error functional $G$, which can be computed using $u_h$ and $w_h$, respectively. From (A9), we define the coupled DEW error indicator as eq. (21).