MPa $\sqrt{m}$/3.35 MPa $\sqrt{m}$ rather than 1.27 as indicated by Duffy et al. (1989). Because this ratio is still within the experimental error, the conclusion that K_{Id} is greater than K_{IC} for the particular alumina examined is questionable.

It is suggested that the authors of the subject paper redetermine mode I fracture toughness using Fig. 4 in this communication, and derive appropriate mode III stress intensity factors for both static and dynamic cracked-notched torsion specimens such that they are comparable.

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Authors' Closure

The authors thank F. I. Baratta and P. J. Perrone for their interest in our paper (Duffy et al., 1988) and our addendum to the paper (Duffy et al., 1989a). Their numerical analysis of the notch-tip field effects on subsequent fracture toughness values is of considerable use in applying the compression fatigue precracking procedure for fracture toughness testing of brittle materials. We are pleased that they have presented their calculations in conjunction with the results published in our papers. At the same time, we are somewhat disappointed that they have failed to note some of the key points mentioned in the subject papers. This seems to have resulted in some misconceptions on their part, which they interpret as confusing and contradictory statements in the subject papers. Many of the points raised here have also been addressed in even greater detail in the sequels to subject papers (some of which are already published (Duffy et al., 1989b) and others are in print (Suresh et al., 1990). In this response, we pinpoint these issues raised by Baratta and Perrone (1990) and present a clarification.

(1) All the data presented in Duffy et al., (1989a) were for a precrack depth of greater than 88 μm. This, according to Fig. 5 in the comment by Baratta and Perrone (1990), causes an error of less than 4 percent. In view of this result, the precracking method, suggested by Suresh and Tschegg (1987) and used in Duffy et al. (1988, 1989a), is expected to give very accurate estimates of fracture toughness, compared to the other techniques that are available in the literature. The quoted value of critical precrack length in Duffy et al. (1988) was based on an estimate made from the results of Dowling (*Fat. Eng. Mater. Struct.*, Vol. 2, 1979, pp. 129–138). It is an experimental fact that for about 10 different quasi-static tension tests conducted on this material with precrack depths of 35-550 μm, we do not see any noticeable differences over and above the normal experimental scatter.

We wish to add that, subsequent to the publication of Duffy et al. (1988, 1989a), we have improved our experimental procedure for both precracking and dynamic tensile testing using an accurate, dynamic finite element analysis. These results have recently appeared in a paper (Suresh et al., 1990). In this latest work, we have precrack lengths that are typically greater than 100 μm for all the tests reported (i.e., less than 3 percent error as shown in Fig. 5 of Baratta and Perrone). The experimentally observed dynamic to static fracture toughness ratios are essentially the same as those reported by Duffy et al. (1989a).

(2) It is clearly stated in the addendum by Duffy et al. (1989a) that.. "We have repeated the Mode I dynamic and static fracture initiation experiments with this modified specimen and a *new batch* of AD-998 alumina.." Baratta and Perrone appear to have missed this point. Different batches of materials with the same composition, when processed in large quantities, often exhibit slightly different mechanical behavior. Therefore, the differences between the values of K_{Id}/K_{1c} reported in the original paper (Duffy et al., 1988) and the addendum (Duffy et al., 1989a) are not because of an error in calculation (as incorrectly interpreted by Baratta and Perrone). Futhermore, as noted earlier, the minimum precrack depth was 88 μm, which is sufficient to provide an accuracy of better than 96 percent. The reported data for both dynamic toughness and static toughness were the actual experimental results obtained for a new batch of the AD-998 alumina. The conclusion in the addendum that the dynamic fracture toughness is greater than the static fracture toughness is *correct*. This conclusion has also been substantiated for a wide range of ceramics in our subsequent work (Suresh et al, 1990) and by other authors who have used very different methods to study dynamic fracture toughness in ceramics (e.g., Yang et al., 1989; Aoki et al., 1989).

It is also clear that differences in material properties arising from different processing conditions of the same ceramic material lead to scatter in fracture toughness values which is comparable to or even in excess of that attributable to the test technique. For this reason, we feel that it may not be worthwhile to spend much effort to debate the origin of scatter values typically smaller than 10%, especially considering the low fracture toughness values of ceramics.

(3) In their comment, Baratta and Perrone state that "... the dynamic Mode III specimen, shown in Fig. 2 of the subject paper and used in the experiments by the authors, appears to have had a hexagonal cross-section." It is clearly illustrated in Fig. 2 of the subject paper that the ligament that was subjected to dynamic fracture in Mode III was of circular cross-section; only the flanges used to apply the load were of hexagonal cross-section.

The experimental results reported in the subject paper constitute the first attempts to measure the Mode III fracture toughness of a brittle solid using a circumferentially notched rod geometry and a fatigue pre-crack. It is well known from the available literature on ceramics (e.g., Suresh and Tschegg, 1987) and on metals (e.g., Tschegg and Suresh, 1988) that considerable frictional sliding occurs between the mating faces of the fatigue precrack during Mode III loading. Furthermore, firmly gripping the specimen surface in the presence of such frictional forces is an experimental challenge, especially for dynamic tests lasting only a very short period of time. In view of these problems, we used a cylindrical specimen with a circular cross-sectional ligament and hexagonal cross-sectional flanges (to enable an efficient application of the torsional load). It is conceivable that some errors arise as a result of this design for frictional loading of the specimen. At the same time, it is also well known from the earlier-mentioned studies that, even for a fixed cross sectional geometry for the Mode III specimen, frictional sliding between the faces of the fatigue precrack can influence the apparent K_{III} by as much as a factor of two. With increasing precrack length, the effect of friction also increases, thereby resulting in apparently large fracture toughness values. The choice of a particular crack length for the test is just arbitrary. In view of these uncertainties, which cannot be quantified precisely, we feel that further refinement of our experimental technique is unwarranted at this time.

In summary, we thank Messers. Baratta and Perrone for presenting their calculations of the effects of notch-tip strain field on fracture toughness measurement. These calculations will be useful for future experimental work. Our most recent measurements of dynamic and static fracture toughness (Suresh et al., 1990) are already available for the open literature; these results, according to the calculations of Baratta and Perrone, have an accuracy of better than 97 percent and are consistent with the conclusions of our original papers (Duffy et al., 1988, 1989a). Our main conclusions are also consistent with those of other researchers (e.g., Yang et al., 1989; Aoki et al., 1989). Therefore, we do not feel that it is necessary to reinterpret any of the results published in our original papers.

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