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discussion

The Modeling of Flow Concentration in Two-Phase Materials[1]

S. M. COPLEY.[2] I have two comments for the authors of this paper. These are as follows:

1 It must be assumed in the calculations that dislocations can always be emitted by a source in the matrix at a stress less than that required for a dislocation (or in the case of an alloy with ordered precipitates, a pair of dislocations) to glide through precipitates in the alloy. Otherwise, there could be no static equilibrium, because dislocations once emitted would continue to glide. It is not clear, however, that this assumption can be justified in many real alloy systems. Consider, for example, the nickel-base superalloy Mar M200 [24].[3] Slip in this alloy is distributed inhomogeneously and involves massive shears on widely spaced slip planes. The constitution of Mar M200, which is representative of that of the cast nickel-base superalloys, consists of a random fcc solid solution hardened by about 60 vol pct of a fcc ordered precipitate phase. After solutionizing for 4 hr, air quenching and aging at 927°C for 64 hr, the microstructure of this alloy can be described as a close-packed arrangement of precipitate spheres 2.5×10^{-7} m in diameter, with a spacing between spheres of 1.9×10^{-8} m measured along a line connecting sphere centers. It is reasonable to take the sphere of maximum diameter that could be placed in an octahedral hole in a close-packed array of spheres 2.5×10^{-7} m in diameter as a measure of the matrix volume available in which a dislocation source could emit dislocations. The diameter of this sphere would be $0.414 \times 2.5 \times 10^{-7}$ m $= 1.04 \times 10^{-7}$ m. The stress required for a Frank-Read source to emit a single dislocation is

$$S = S_o + \frac{2Gb}{L} \qquad (1)$$

where S_o is the shear stress for dislocation glide in the matrix, G is the shear modulus of the matrix, b is the Burgers vector and L is the source length. Setting L equal to 1×10^{-7} m, G equal to 5.67×10^4 MPa, b equal to 2.53×10^{-10} m and S_o equal to 1.17×10^2 MPa (see reference [25], for a discussion of the values of these parameters), we obtain S equal to 4.09×10^2 MPa. This is greater than the stress calculated by Copley and Kear for a dislocation pair to glide through precipitates in Mar M200 at a velocity of 10^{-5} m sec^{-1}, which was 3.9×10^2 MPa [25]. It is slightly less than the experimentally determined critical resolved shear stress of this alloy, which in the heat treated condition is 4.27×10^2 MPa. On the basis of these values and considering that (i) a source would require very fortuitous placement in order to operate in the available matrix volume and (ii) a source would have to produce a pair of dislocations in the superalloy, which would require a greater stress than to produce a single dislocation, it appears unlikely that a matrix source would emit dislocations at a stress less than that required for dislocation pairs to glide through the precipitates as assumed by the authors in their calculations.

2 No effort is made to include the effect of dislocation debris accumulation in the slip band. This effect would increase the microhardeneing rate associated with a single dislocation source and should be included in a comprehensive treatment of flow concentration.

[1]By T. S. Cook, C. A. Rau, and E. Smith, published in the April, 1976, issue of the Journal of Engineering Materials and Technology, Trans. ASME, Series H, Vol. 98, pp. 180–189.

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[3]Numbers 24-25 in brackets designate Additional References at end of discussion.

Additional References

24 Copley, S. M., Kear, B. H., and Piearcey, B. J., *Proc. Intern. Congr. Electron Microscopy*, 6th, Kyoto, Japan, 1966.

25 Copley, S. M., and Kear, B. H., *Trans. Met. Soc. AIME*, Vol. 239, 1967, pp. 984–992.

Authors' Closure

Let me first thank Dr. Copley for his perceptive comments. We had in fact considered those points which he raises, and had incorporated them in the calculations we have performed. Specifically, we would agree that in a uniformly distributed, very high-volume fraction of ordered precipitates, the source operation stress may well exceed the propagation resistance of the matrix. Yielding of some materials like MAR M200 may be controlled by source activation. We have modeled a range of source activation stresses numerically by reducing the spacing between the hypothetical source and the initial barrier (S in Fig. 4 of our paper) and by varying barrier strength. In our two-dimensional model, we have parametrically varied the ratio of S/L_1 where L_1 is the average inter-particle spacing. As this ratio decreases, the source activation stress eventually exceeds the stress required to continue propagation through the matrix. We have carried out calculations to define this limit, above which dislocation glide continues unstable (no static equilibrium) at the stress required to activate the dislocation source (i.e., penetrate the first barrier).

It is important to note, however, that most precipitation hardened alloys have a much lower volume fraction of precipitate than that discussed by Dr. Copley, and more importantly, a wider statistical distribution in the size and spacing of hardening precipitates. It is reasonable to assume that since the stress to activate a given dislocation source is inversely proportional to the source length, which will in turn be related to the inter-particle spacing, that there will be some source sites which will have considerably lower stress for activation than average. It is, in fact, these statistically easy sites for dislocation nucleation which produce initial microyielding at stresses well below that required for glide through the array (controlled by average spacing). In these cases, the present model appropriately accounts for the influence of both source strength and average glide resistance on microstrain hardening and the development of macroscopic planar slip.

Dr. Copley's second comment addresses the need to incorporate glide-band hardening. We agree, and have already analyzed the cases of both glide-band hardening and softening at various rates with the passage of dislocations. These results will be reported separately; but in general, glide-band hardening increases the stability of slip and will reduce flow concentration. Perhaps the more interesting case is where particle strength degradation occurs with continued penetration. For example, in the alloy discussed by Dr. Copley, the particle strength results primarily from its long range order, which can be gradually destroyed by continued dislocation cutting. If the effective particle strength decreases with continuing slip, unstable planar glide may develop after a period of stable microstrain hardening. In fact, surprisingly gradual percentage reductions in particle strength are sufficient to induce unstable planar slip.

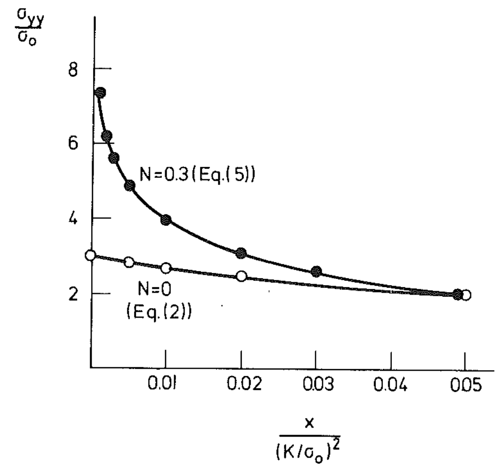

Fig. 1 Stress distribution ahead of crack

Finite Element Solutions of Crack-Tip Behavior in Small Scale Yielding[1]

K.-H. SCHWALBE.[2] In completion of D. M. Tracey's paper I would like to point out that some of his results coincide with some simple estimations.

The stress distribution in the plastic zone of a crack in a work hardening material can be derived as follows: consider the work hardening relation

$$\sigma = \sigma_0 \left(\frac{\epsilon}{\epsilon_0}\right)^N \qquad (1)$$

The stress is magnified through work hardening by the factor $(\epsilon/\epsilon_0)^N$ compared with ideal plastic behavior. In the plastic zone of a mode I crack in ideal plastic materials the "flow stress" is not a constant but given by a distribution function shown in Fig. 4 of reference [1]. An approximate formula for the stress distribution is

$$\sigma_{yy} = \frac{\sigma_0 \cdot 0.3}{\dfrac{x}{(K/\sigma_0)^2} + 0.1} \qquad (3)$$

For the strain distribution a mode III type solution is assumed [2]:

$$\epsilon = \epsilon_0 \left(\frac{\omega_0}{x}\right)^{\frac{1}{1+N}} \qquad (3)$$

where ω_0 denotes the length of the plastic zone on the x-axis [2]:

$$\omega_0 = (1 - 2\nu)^2 \frac{K^2}{(1 + N)\pi\sigma_0^2} \approx 0.05 \frac{K^2}{\pi\sigma_0^2(1 + N)} \qquad (4)$$

Inserting equations (2) and (3) into equation (1) we obtain

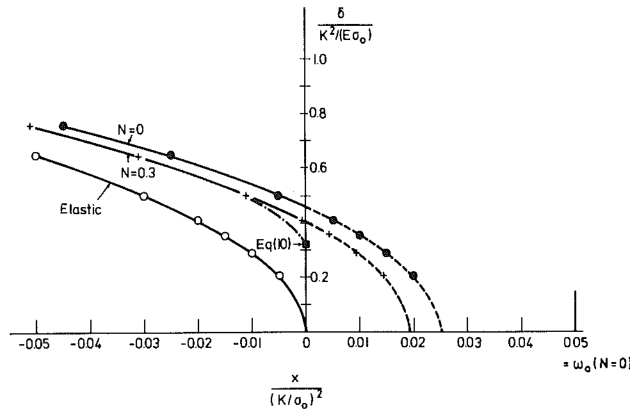

Fig. 2 Calculated crack tip profiles

$$\frac{\sigma_{yy}}{\sigma_0} = \frac{0.3}{\dfrac{x}{(K/\sigma_0)^2} + 0.1} \left(\frac{\omega_0}{x}\right)^{\frac{N}{1+N}} \qquad (5)$$

In Fig. 1 equation (5) is plotted for $N = 0.3$ along with equation (2). Comparing the result with Fig. 6 of Tracey's paper shows that the use of mode III analogs is not unrealistic. Hence, stress and strain distributions in mode I plastic zones can be described by simple closed-form solutions. This is also true for the crack tip opening displacement, δ_t. The solution for mode III is [3]

$$\frac{\delta_t}{2} = \gamma_0 \cdot \omega_0 \qquad (6)$$

Assuming this solution to be universal, a mode I analog would be [4] for one crack face

$$\frac{\delta_t}{2} = \gamma_0 \cdot \omega_{70} \qquad (7)$$

where ω_{70} is the maximum extent of the plane strain plastic zone inclined at 70° to the x-axis [3, 4]:

$$\omega_{70} = \frac{\sin^2 70 \cos^2 35}{(1 + N)\pi\sigma_0^2} K^2 \qquad (8)$$

For the ideal plastic case ($N = 0$) we obtain

[1]By D. M. Tracey, published in the April, 1976, issue of the Journal of Engineering and Technology, Trans. ASME, Series H, Vol. 98, pp. 146–151.

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