Rupture directivity of microearthquakes on the San Andreas Fault from spectral ratio inversion

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SUMMARY
Earthquakes often occur on faults separating materials with different elastic properties. On theoretical grounds, it is expected that earthquakes on such bimaterial interfaces might have a preferred rupture propagation direction, that being the direction of motion of the more compliant material. The goal of this paper is to determine whether a large sample of natural earthquakes on a bimaterial interface exhibits this tendency. Since the creeping section of San Andreas Fault has a large across-fault velocity contrast and has produced thousands of microearthquakes over the last few decades, the rupture directions of ∼3100 magnitude 0.5–3.0 earthquakes were studied using spectral ratio analysis. The spectral ratios of all earthquake pairs in spatially defined clusters were fitted with synthetic spectral ratios at qualified stations. The synthetics were computed from a simple moving point source model in which each modelled earthquake has four parameters: two rupture lengths (one to the SE and one to the NW) and their propagation velocities. The resolution of rupture directivity increases with event size, such that nearly 900 events, mostly those larger than ∼70 m, appear reasonably well resolved. The inversion results suggest that ∼40 per cent of the well-resolved events are roughly bilateral, although more than ∼80 per cent of the 144 events classified as "strongly unilateral" rupture to the SE, consistent with the theoretical prediction. For those rupture halves that were large enough for the propagation speed to be somewhat resolved, that speed was greater by roughly 10 per cent for those halves propagating to the SE, qualitatively consistent with numerical and laboratory experiments. We also found that events with nearby foreshocks within several hours tend to rupture away from those foreshocks, whether to the NW or to the SE, indicating that asymmetry of prior stressing history can exert a stronger influence on rupture directivity than the material contrast.

Key words: Earthquake dynamics; Earthquake source observations; Rheology and friction of fault zones.

1 INTRODUCTION
Natural earthquakes often occur on bimaterial interfaces—for example, plate boundaries separating different rock types or the interface between fault gouge and more rigid host rock. Analytical and numerical studies have shown that the mechanics of earthquakes on such an interface can be dramatically different from those in homogeneous media. Weertman (1980) showed that steady-state slip pulses propagating in the direction of motion of the more compliant (slower wave speed) side of the fault are associated with a reduction in fault-normal stress. This has led to the expectation that earthquakes might propagate preferentially in this direction, which in the context of the central San Andreas fault would be to the southeast (the more compliant Franciscan sediments on the North American Plate move to the southeast, relative to the stiffer Salinian granites on the Pacific plate moving to the northwest). Note that the coupling between shear slip and normal stress changes occurs only at the mode-II rupture front, where the slip vector is perpendicular to the rupture front. From symmetry it does not occur at the mode-III edge, where the slip vector is parallel to the front.

Due to the higher amplitude of ground motion ahead of a rupture front than behind, establishing whether a preferred earthquake rupture propagation direction exists on natural faults is important for hazards assessment. For this reason the bimaterial literature is extensive and not without controversy. To summarize the more recent models: When the sliding friction is too large to allow slip at the ambient shear and normal stress levels, rupture is possible only in the form of unilateral Weertman-style pulses (Andrews & Ben-Zion 1997; Ben-Zion & Andrews 1998; Cochard & Rice 2000). Models with slip-weakening friction, on the other hand, and a sliding friction smaller than the ratio of the ambient shear to normal stress, lead to slightly asymmetric bilateral ruptures, with larger slip and propagation speeds at the front moving in the Weertman direction (Harris & Day 1997; Rubin & Amuero 2007). Shi & Ben-Zion (2006) generated unilateral pulses using a slip-weakening model, but in their case the nucleation process, which was localized in time as well as...
space, ended before the sliding strength decreased to the level of the ambient shear stress, making this case similar to those in which the slipping friction is too large to allow slip at the ambient shear and normal stress. Using a more strongly velocity-weakening friction law, which can produce pulse-like propagation even on faults in homogeneous media, Ampuero & Ben-Zion (2008) generated unilateral pulses that propagated predominantly in the Weertman direction. Given a sufficiently heterogeneous pre-stress, however, a minority also propagated in the opposite direction. A sufficiently heterogeneous pre-stress can also give rise to a preferred propagation direction for slip-weakening friction (Brietzke et al. 2009). Clearly, and not surprisingly, the behaviour of simulated ruptures on a bimaterial interface depends upon (at least) the adopted friction law and the pre-stress.

Direct evidence of a preferential propagation direction in nature has been difficult to find, perhaps because of the relatively small statistical sample of well-instrumented earthquakes occurring on faults with a large and well-characterized velocity contrast. Harris & Day (2005) pointed out that five of the eight magnitude 4-6 earthquakes near Parkfield since 1934 propagated to the northwest (NW), while only three propagated to the southeast (SE). Originally this was thought to be contrary to the bimaterial expectation; however, Zhao et al. (2010) interpret this behaviour as consistent with the complex spatial distribution of the velocity contrast across the fault [see also Ben-Zion (2006) and Harris & Day (2006) for an earlier comment and reply]. Dor et al. (2006, 2008) reported that near-surface damage occurs predominantly on one side of major strike-slip faults in California and Turkey. They propose that this results from unidirectional ruptures that consistently propagate in the Weertman direction, with damage being concentrated on the tensile (higher-wavespeed) side of the rupture front. Seismic velocity models, where available, are consistent with this interpretation.

Clear evidence of a bimaterial effect can be seen in the asymmetric distribution of aftershocks in a catalogue of 5000 relocated microearthquakes along 60 km of the San Andreas Fault near San Juan Bautista (Rubin & Gillard 2000; Rubin 2002a). Of the 169 aftershocks occurring within 10 hr and 1 estimated mainshock radius of the mode-II edges of the mainshock (those where the vector connecting mainshock and aftershock centroids lies within 45° of horizontal), nearly three times more occurred to the NW than to the SE (125 vs. 44). In contrast, beyond the mode-III ends of (above and below) the mainshock, where no bimaterial effect is expected, 61 aftershocks were shallower and 64 deeper. Based on their numerical simulations, Rubin & Ampuero (2007) attributed this asymmetry to dying slip pulses generated by Weertman-style tensile stresses as the SE-propagating rupture fronts were slowed and stopped by stress barriers. Because these tensile stresses are a purely dynamic phenomenon, after rupture ceases the SE margins of the mainshocks are left far below the static failure threshold, whereas the NW margins are typically just slightly below that threshold. Note that in these simulations strongly asymmetric stress fields were produced by bilateral ruptures. Very likely this asymmetry could also result from unilateral ruptures, via the same ‘dying slip pulse’ mechanism (Ampuero & Ben-Zion 2008). Ampuero & Ben-Zion (2008) suggested that macroscopic rupture directivity is a more robust feature of numerical simulations than is the dying slip pulse, and that for this reason it is more likely to be responsible for the aftershock asymmetry. However, the mechanism by which macroscopic directivity would lead to more aftershocks in the opposite direction has not been specified (note that the relative earthquake locations used to identify the asymmetry refer to the event centroids, not hypocentres).

Recently Zaliapin & Ben-Zion (in press) summarized aftershock asymmetry on 25 faults in California. Faults with a strong velocity contrast (>5 per cent) showed increased activity of temporally and spatially close aftershocks in the direction opposite to the expected propagation direction of the mainshock, consistent with the asymmetry of immediate aftershocks found by Rubin & Gillard (2000) and Rubin (2002a). On the other hand, later and more distant aftershocks tended to occur in the expected mainshock propagation direction. By analogy with large earthquakes, which can produce distant aftershocks preferentially in the mainshock propagation direction, they interpret this as indicative of a preferential propagation direction for the microearthquakes in their catalogues. At best, however, observations of aftershock asymmetry provide indirect measures of microearthquake directivity that are difficult to interpret quantitatively.

In this study we attempt to measure directivity directly, and circumvent the ‘statistics of small numbers’ problem by searching in a large catalogue of microearthquakes from the same region in which aftershock asymmetry was observed. Due to the availability of precise relative locations and time-dependent station corrections, we use the ‘southern box’ of Rubin (2002a) (Fig. 1), at the northern end of the creeping section of the San Andreas fault. This consists of ~3100 magnitude 0.5–3.0 events occurring along a 30-km segment of the fault between 1984 and 1997. Using P-wave traveltime delays, we estimate a P-wave velocity contrast across this section of the San Andreas fault that varies from ~5–20 per cent. McGuire & Ben-Zion (2005) obtained an even higher 20–50 per cent contrast from differential traveltimes between head waves and P waves in an overlapping region. The extensive and significant material contrast and the large amount of seismicity in the region make this an ideal location to explore the bimaterial effect.

**Figure 1.** Earthquake and station locations used in this study. The 3142 relocated events are shown as blue dots in the red box and the black stars indicate the station locations.
2 METHOD

The data are vertical component 100-samples per seconds velocity seismograms from the Northern California Seismic Network (NCSN) catalogue. Using the multitaper algorithm of Thomason (1982), a 0.64 s signal window consisting of both the direct P wave and P coda was used to compute the power spectral density, which was then smoothed and truncated above 25 Hz due to low signal to noise ratios. Changing the proportion of coda to direct arrival did not alter the results significantly (Appendix A). To account for the changing magnitudes of low frequency energy with event size, the spectra were normalized at low frequency (~5 Hz) before taking ratios. Spectral ratios are plotted on a logarithmic scale, which makes the ratio and its inverse mirror images of each other.

We select qualified stations for earthquake pairs based on their cross-correlation coefficients, coherency and signal-to-noise ratios (See Appendix B for details). For two earthquakes within ~200 m, and with neither event so small as to have a low signal to noise ratio or so large as to be badly clipped, a typical number of qualifying stations is between 10 and 30. The spectral ratios of nearby earthquake pairs at acceptable stations often exhibit azimuthally consistent ratios of high-to-low frequency energy, with the amplitude of variation as a function of azimuth related to differences in size and directivity. As event separation grows, both the number of qualified stations and data quality decrease. Pairs of large earthquakes with strong signal of directivity tend to be more immune to the azimuthal inconsistency because the signal amplitudes are often larger. But in general, pairs separated by >400 m have fewer than 10 qualified stations (<5 are common), and their spectral ratios are azimuthally inconsistent, presumably because the propagation paths become significantly different. On the other hand, events separated by ~200–300 m often have more stations with smaller amplitudes of inconsistency, and we used distance-dependent weights (see eq. B2 in Appendix B) to further make sure that pairs separated by larger distances get lower weights.

Rubin (2002b) used repeating earthquake clusters to identify time-dependent changes in station delay times that resulted from changing electronic components at many stations in the NCSN network. Since these changes might also have affected the seismic spectra, we checked the spectra of repeating earthquakes across the identified changes. The spectra of repeating earthquakes are expected to be very similar, which is consistent with most of our observations. However, for the ~30 stations with previously identified changes of station delays, the spectra of most repeating clusters experienced frequency-dependent shifts at those times. The amplitudes of these spectral shifts are often larger than the small-amplitude spectral ratio signals observed (~0.1 at 25 Hz on a logarithmic scale). Empirical corrections based on the difference of the mean spectra of repeating earthquakes across each change were applied to earthquake pairs spanning the times of those changes (see Appendix C for details).

To quantify the extent of directivity, we fitted the spectral ratios with a forward model consisting of two point sources moving horizontally in the two opposite directions along the fault strike (Ben-Menahem 1961). The apparent duration \( \Delta t_{\text{app}} \) of the P-wave pulse from each earthquake half at a receiver can be written as a function of the rupture length \( L \), azimuth of the station with respect to the rupture direction \( \theta \), the rupture propagation velocity \( V_r \), the P-wave velocity \( c_p \) and the take-off angle \( \eta \) (with \( \theta \) being straight down from the event):

\[
\Delta t_{\text{app}} = L \left( \frac{1}{V_r} - \frac{\cos \theta}{c_p / \sin \eta} \right).
\]

For a given unilateral earthquake and P-wave velocity, the Doppler effect can be observed as the variation in apparent duration \( \Delta t_{\text{app}} \) by \( 2L/(c_p / \sin \eta) \) as \( \theta \) varies from 0 to \( \pi \). In eq. (1), the station azimuth \( \theta \) can be obtained from station and earthquake locations, the SE direction along the fault strike is referred to as 0, and symmetric stations with respect to the fault are assigned the same azimuths. The P-wave velocity \( c_p \) is estimated by interpolating the NCSN layered velocity model for the study region (different for each side of the fault); this model is also used to determine \( \eta \). In addition to the known \( \theta \), \( \eta \) and \( c_p \), each half of a bidirectional source model has two parameters: the rupture length \( L \) and propagation velocity \( V_r \). Since our model consists of two halves, a bidirectional earthquake has four parameters: two rupture lengths, \( L_{\text{SE}} \) and \( L_{\text{NW}} \), which are directly related to rupture directivity (a directivity coefficient can be defined as \( L_{\text{SE}}/(L_{\text{SE}} + L_{\text{NW}}) \)), and the corresponding rupture propagation velocities \( V_{\text{SE}} \) and \( V_{\text{NW}} \).

Of course real ruptures might have a significant or dominant component of vertical propagation (e.g. Fletcher & Spudich 1998), but with so few stations having steep take-off angles we found this difficult to constrain. Rather than attempting to completely determine the directivity properties of these events, we view this study more specifically as a test of the prediction of a preferred horizontal (SE) propagation direction. Vertically-propagating (or nearly so) ruptures should appear in the inversion as bilateral (or nearly so) ruptures, and both imply no (or nearly no) preference for either SE or NW propagation. Nonetheless, it is evident that by applying any crude model of this sort to real earthquakes, robust conclusions will require resolving parameters for hundreds of events.

The model parameters are constrained by some prior information. Using the moment-magnitude relation of Abercrombie (1996), we compute the seismic moment from the catalogue coda magnitude using \( \log (M_0) = 1.0M + 9.8 \). With the scaling of event size to seismic moment and a circular rupture assumption, the moment can be written as

\[
M_0 = \mu A = \frac{16}{7} \Delta \sigma r^3.
\]

(Eshelby 1957), where \( \mu \) is the shear modulus, \( A \) and \( \delta \) are the slipped area and mean displacement, \( \Delta \sigma \) is the stress drop of the earthquake and \( r \) is the radius of the circular rupture. The total rupture length of a bidirectional earthquake \( L_{\text{SE}} + L_{\text{NW}} \) expected be close to the diameter of the theoretical circular rupture (the catalogue length, \( L_{\text{cat}} = 2r \)) within uncertainties. A typical stress drop \( \Delta \sigma \) for earthquakes in this magnitude range is estimated as 3 MPa (Abercrombie 1995). Since \( r \) is proportional to the cube root of \( M_0/\Delta \sigma \), even a combined uncertainty of a factor of eight in stress drop and catalogue magnitude leads to only a factor of two uncertainty in rupture diameter. Thus we allow \( L_{\text{SE}} + L_{\text{NW}} \) to be within 0.5 to 2 times the catalogue length, with the probability distribution shown in Fig. 2(a). The empirical prior distribution of rupture propagation velocities peaks at 0.85 times the shear wave speed \( c_s \) and falls rapidly to zero at the shear wave speed and less rapidly to zero at low speed (Fig. 2b). The shear wave speed for each event is determined using the relation \( c_s = c_p / 1.73 \).

With the forward model and prior constraints, we performed inversions by sampling the posterior distributions of model parameters using a Markov chain Monte Carlo method (See Appendix B for details). In Fig. 3, the seismograms, spectral ratios, synthetic spectral ratios and posterior distributions of rupture lengths are plotted for two pairs of earthquakes with different signs of directivity. The posterior distributions are constrained by the spectral ratios shown as well as by those taken with other nearby earthquakes. In both
pairs, the seismograms of the two events are plotted in blue and red above the spectral ratios, which use blue curves to show the ratio of the spectrum of the blue event to the red event and red curves to show the reverse. The seismograms and spectral ratios from different stations are sorted from upper left to lower right based on the station azimuth from the SE direction along the fault strike (indicated above the spectral ratios). The grey scale curves in the bottom panels are the synthetic spectral ratios of the blue event to the red computed from the posterior samples. The latter are plotted as 2-D marginal distributions of rupture lengths $L_{SE}$ and $L_{NW}$ on the right.

The synthetics in the spectral ratio plots fit the data fairly well, and the directivity that one can infer directly from the spectral ratios is consistent with the inversion results. The upper pair consists of two earthquakes with catalogue diameters of 80 m separated by only 6 m; they are two members of a 7-event repeating earthquake cluster. Despite an average cross-correlation coefficient exceeding 0.96, there is a clear signal of directivity, with the blue event having relatively more high-frequency energy at stations to the SE and less at stations to the NW. There is also an asymmetry in signal amplitude, with the spectral ratios being largest at stations to the SE and close to 1 ($\log = 0$) at stations more than 90° from the SE. These observations are consistent with the posterior samples which show that the blue event is close to bilateral and the red event is unilateral to the NW (the NW rupture length of the red event is the largest of the four rupture halves, making the signal amplitude largest at stations in the opposite direction). These results also show that even repeating earthquakes can differ in their directivity (see Section 4).

Note that even at stations with clipped records, such as those at azimuths of 25.5° and 114.1°, the spectral ratios are consistent with those at their unclipped neighbours. The resolution of directivity was improved by including nearby events outside the repeating cluster in one single inversion, for example, a test inversion of the
clusters using this algorithm, with sizes ranging from 5 to 90. Ultimately we placed 3103 of the 3142 original earthquakes in 133 multiplets of two events; similarly, if one event has a much smaller size than the other, the smaller event will not be resolved. Improvements can be made by including more events in a single inversion to increase the variability of size and directivity. We built earthquake multiplets from the spatial distribution within the relocated catalogue. Ultimately we placed 3103 of the 3142 original earthquakes in 133 clusters using this algorithm, with sizes ranging from 5 to 90 events. The spectral ratios of all pairs of events in a multiplet are fitted by synthetics to invert for model parameters of all the events in the multiplet, which means that an N-event multiplet has up to N(N − 1)/2 pairs, and each pair may have a different number of qualified stations (see Appendix B).

In the course of this work we became aware of two independent studies that also used spectral ratios to estimate the directivity of microearthquakes. Kane et al. (2009) presented preliminary results for microearthquakes on the San Andreas fault near Parkfield, about 110 km SE of our study area. Also near Parkfield, Lengline & Got (2011) used spectral ratios to determine propagation direction, but restricted their study to clusters of repeating earthquakes and assumed that ruptures propagated unilaterally, either to the NW or to the SE. As was noted above, we found it advantageous to include as wide a range of earthquake size as possible in the inversion, provided the source separation was sufficiently small.

3 RESULTS

We use the two rupture lengths along the fault strike to quantify rupture directivity. Since the inversion was carried out by sampling the posterior distributions of model parameters using the Markov chain Monte Carlo method, the inversion results for rupture lengths of each event are a cloud of Monte Carlo samples in 2-D model space (e.g. Fig. 3). To view the results for the entire catalogue without having them obscured by the large number of samples, only one representative point of the posterior distribution for each event is shown in Fig. 4. In Fig. 4(a) the central position of the Monte-Carlo samples for each event is plotted, and Fig. 4(b) plots the maximum likelihood points for comparison. Since both numerical simulations of ruptures on a bimaterial–material interface (Rubin & Ampuero 2007) and the inverted rupture propagation velocities (to be shown later) suggest that earthquake ruptures may propagate faster in the preferred (SE) direction, in Fig. 4(c) we plot the central positions of events from inversions in which the prior distribution of rupture speeds is 10 per cent faster to the SE than to the NW. Since we were uncertain of the extent to which the spectral ratio data could constrain both rupture length and propagation speed, as opposed to rupture duration \( L/V \), we also explored model parameterizations in which \( V_{SE} \) and \( V_{NW} \) were fixed at 0.85\( c_s \), so that each earthquake had only the two parameters \( L_{SE} \) and \( L_{NW} \) (Fig. 4d). The colour scale in Fig. 4 indicates the ‘resolution’ of directivity, defined as the 90 per cent confidence interval of \( L_{SE}/(L_{SE} + L_{NW}) \); that is, the width of the central 90 per cent of the posterior probability density function (pdf) projected onto lines with a slope of \(-45^\circ\). Thus low values (browns and black) are very well resolved, whereas values >0.5 (yellows and whites) are very poorly resolved. The quantity \( L_{SE}/(L_{SE} + L_{NW}) \) is a measure of the extent of directivity with 0, 0.5 and 1 corresponding to NW unilateral, bilateral and SE unilateral events.

One notable feature in Fig. 4 is that as event size becomes larger, the resolution of directivity increases. This is because the ‘zero-crossing’ frequency of a boxcar function with duration \( \Delta t_{app} \) is \( 1/\Delta t_{app} \), which is typically higher than 25 Hz for events smaller than 80 m. Therefore, the smallest events are not expected to be well resolved and are important only as EGFs for the larger ones. Because of the low resolution of single rupture lengths smaller than 20 m, there is a lack of central points near the two axes except in Fig. 4(b). The resolution of the two-parameter inversion (Fig. 4d) is generally higher than that of the four parameter inversions, because of the fewer degrees of freedom.

The inversion results show that the directivity is fairly widely scattered; there are numerous approximately bilateral events as well as more nearly unilateral events propagating in each direction. However, several SE-propagating strongly unilateral events lying near the lower right in the panels of Fig. 4 have no NW-propagating counterparts on the upper left. A similar tendency for SE propagation can be seen from the asymmetric distribution near the diagonal line, with more events having slightly larger SE than NW rupture lengths. The SE-dominated propagation exists in all four inversion results, suggesting that this trend persists regardless of the choice of model parameterizations. The central points in Fig. 4(c) (10 per cent higher prior rupture speed to the SE) are approximately a clockwise-rotated version of those in Fig. 4(a), due to a correlation between rupture length and rupture speed (see Fig. B1 in Appendix B), making the SE dominance of propagation more pronounced in Fig. 4(c). Statistics for the different inversions are shown in Table 1.

In addition to the rupture lengths, the four-parameter inversions also return estimates of propagation velocities. Although the rupture speeds of individual events may vary with time, it would be interesting to see if the average rupture speeds for halves propagating in the different directions are statistically different. Since the fall-off of the seismic spectrum with increasing frequency is largely determined by the apparent rupture duration, for most of the resolved halves in our catalogue there is a significant trade-off between rupture length and propagation velocity (see Fig. B1). However, since the forward model is more sensitive to longer rupture durations,
it might be possible to resolve the rupture length and propagation speed independently for the larger events. In Fig. 5(a) we plot the summed posterior pdfs of rupture propagation velocities for individual halves larger than 60 m from an inversion with the same priors for $V_{SE}$ and $V_{NW}$. The blue and red bars show the summed rupture speeds to the SE and NW, respectively. The summed posterior pdfs of both $V_{SE}$ and $V_{NW}$ for halves smaller than 60 m are very close to the prior distribution, indicating very little constraint from the data on their rupture speeds. In Fig. 5(a), however, the posterior pdfs are skewed towards the upper limit of the prior distribution (thick black curve), indicating that the inversion preferably chose higher propagation velocities. Moreover, the posterior pdfs in the two directions are different, with the median velocity to the SE 4–5 per cent higher than to the NW. Because the inversion penalizes deviations from the prior distribution, it is likely that the actual difference is even greater.

To estimate the difference between $V_{SE}$ and $V_{NW}$, we compared the difference in the medians of the summed posteriors of $V_{SE}$ and

Table 1. Statistics for the well-resolved events from different inversions.

<table>
<thead>
<tr>
<th>Figure</th>
<th>#Events</th>
<th>Median</th>
<th>Mean</th>
<th>Var</th>
<th>#SE / #Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(a)</td>
<td>884</td>
<td>0.529</td>
<td>0.543</td>
<td>0.018</td>
<td>61.5 per cent</td>
</tr>
<tr>
<td>4(b)</td>
<td>884</td>
<td>0.538</td>
<td>0.544</td>
<td>0.020</td>
<td>62.1 per cent</td>
</tr>
<tr>
<td>4(c)</td>
<td>887</td>
<td>0.541</td>
<td>0.550</td>
<td>0.015</td>
<td>68.6 per cent</td>
</tr>
<tr>
<td>4(c)(&gt;100 m)</td>
<td>443</td>
<td>0.561</td>
<td>0.568</td>
<td>0.017</td>
<td>70.2 per cent</td>
</tr>
<tr>
<td>4(d)</td>
<td>928</td>
<td>0.519</td>
<td>0.539</td>
<td>0.017</td>
<td>60.0 per cent</td>
</tr>
<tr>
<td>A1(a)</td>
<td>892</td>
<td>0.527</td>
<td>0.538</td>
<td>0.015</td>
<td>62.4 per cent</td>
</tr>
<tr>
<td>A1(b)</td>
<td>877</td>
<td>0.521</td>
<td>0.529</td>
<td>0.014</td>
<td>58.5 per cent</td>
</tr>
<tr>
<td>9(SE)</td>
<td>20</td>
<td>0.433</td>
<td>0.432</td>
<td>0.010</td>
<td>30.0 per cent</td>
</tr>
<tr>
<td>9(NW)</td>
<td>27</td>
<td>0.594</td>
<td>0.605</td>
<td>0.010</td>
<td>81.5 per cent</td>
</tr>
<tr>
<td>10(SE)</td>
<td>15</td>
<td>0.574</td>
<td>0.588</td>
<td>0.013</td>
<td>80.0 per cent</td>
</tr>
<tr>
<td>10(NW)</td>
<td>42</td>
<td>0.586</td>
<td>0.576</td>
<td>0.010</td>
<td>76.2 per cent</td>
</tr>
</tbody>
</table>

The well-resolved events are those whose inverted total rupture lengths are larger than 70 m and whose widths of the central 90 per cent of $L_{SE}/(L_{SE} + L_{NW})$ are smaller than 0.3 (0.4 for the last four rows). Columns 2–4 are the medians, means and variances of $L_{SE}/(L_{SE} + L_{NW})$ for the well-resolved events. Figs A1(a) and (b) are for different data windows, Figs 9 and 10 are for mainshocks with foreshocks and aftershocks, respectively, located to the SE or NW.
From the inversion results it is clear that there is a preferential tendency for propagation to the SE among the well-resolved earthquakes, consistent with both numerical and laboratory experiments (Weertman 1980; Shi & Ben-Zion 2006; Bhat et al. 2010). These results also show that the tendency for SE propagation appears to increase with event size. There is theoretical justification for such behaviour, in that the normal stress change at the rupture front increases with propagation speed (Weertman 1980) and from fracture energy arguments one expects rupture speed to increase with the rupture dimensions (at least in the absence of off-fault damage). However, given the above-mentioned correlation between resolution and directivity near our resolution limit, we are reluctant to conclude that the apparent dependence upon rupture size is real. Obtaining directivity measurements for more earthquakes could help clarify this issue.

4 DISCUSSION

From the inversion results it is clear that there is a preferential tendency for propagation to the SE among the well-resolved earthquakes on this portion of the San Andreas fault, consistent with the theoretical expectation. This result is relatively insensitive to model parameterization (Table 1 and Fig. A2). However, it is also clear that a large number of events are nearly bilateral or propagate preferentially to the NW. This seems unlikely to be due to small-scale reversals of the across-fault velocity contrast. We routinely

\( V_{NW} \) to the difference in the priors. For inversions where the prior \( V_{SE} \) was 15 per cent or 10 per cent larger than \( V_{NW} \), the medians of the posterior were 13.3 per cent and 9.8 per cent larger, respectively.

Since the difference between the prior ratio and the posterior ratio is so small in the latter case, the actual velocity difference between \( V_{SE} \) and \( V_{NW} \) might be close to 10 per cent. Fig. 5(b) shows the summed pdfs for this case (note that \( V_{SE} \) and \( V_{NW} \) are scaled by their own prior \( c_s \) in (b)). Given a 10 per cent difference in priors, the resolved rupture speeds have almost identical distributions with respect to their own priors (Fig. 5b). For this reason, for the remainder of the paper we treat Fig. 4(c) (10 per cent higher prior rupture speeds to the SE) as our preferred parameterization.

The sense of the velocity asymmetry, with faster propagation speeds to the SE, is consistent with the slip-weakening numerical simulations of Rubin & Ampuero (2007). The source of the asymmetry is the reduction in normal stress behind the rupture propagating in the direction of motion of the more compliant side of the fault. The magnitude of asymmetry is also generally consistent with these simulations. This asymmetry in the simulations depended upon propagation distance but was typically about 10 per cent (although in one case close to 20 per cent) for velocity contrasts appropriate for this portion of the San Andreas fault. Xia et al. (2005) also found asymmetry in propagation velocities of laboratory earthquakes on a bimaterial interface, with velocities ~5 per cent faster in the preferred direction when both rupture fronts propagated at subshear velocities. Supershear ruptures have also been observed in both directions in laboratory and numerical experiments (Shi & Ben-Zion 2006; Bhat et al. 2010). We have run test inversions where the prior velocities extended up to 1.6\( c_s \), in both directions, but do not have the resolution to definitively say whether any particular rupture is supershear.

To quantify the extent of any preferred rupture propagation direction, we examine the statistics of the events in Fig. 4(c) that we consider to be ‘well-resolved’. First note that resolution in Fig. 4 appears to be correlated with directivity, in that the near limit of our resolution (event sizes of 60–70 m) the nearly bilateral events appear to be better-resolved (red colours) than the moderately unilateral events (yellow and orange). This is plausible because for a given rupture length the spectral ratio amplitudes depend upon directivity. To avoid biasing our statistics in this way, we impose a minimum length criterion as well as a resolution criterion on events classified as ‘well-resolved’ (\( |L_{NW} + L_{SE}| > 70 \text{ m} \); 90 per cent confidence interval of \( L_{SE}/(L_{NW} + L_{SE}) < 0.3 \)). In Fig. 6(a) we plot the cumulative number of well-resolved events in Fig. 4(c) as function of event size. The model space in Fig. 4(c) was divided into symmetric regions with respect to the 45° line using the coloured dashed lines to compare the number of events with the same magnitude of directivity. As indicated by the black curve in Fig. 6(a), 39 per cent of the 887 well-resolved events are close to bilateral, defined as \( L_{NW}/L_{SE} > 0.8 \) (recall that these may include events that rupture predominantly vertically). The comparison between the strongly unilateral events (defined as \( L_{NW}/L_{SE} < 0.5 \)) shows that ~4 times as many propagated to the SE than to the NW. The moderately unilateral events also show a preference for SE propagation, although with a lower SE to NW event number ratio (~2.4).

To avoid the arbitrary nature of the domain boundaries in Fig. 6(a), Fig. 6(b) plots histograms of \( L_{NW}/L_{SE} \) for the well-resolved events as a whole (black) as well as those in two size bins (blue and red for inverted lengths smaller and larger than 100 m, respectively). The statistics of directivity for all the well-resolved events and those larger than 100 m are summarized in Table 1, with ~70 per cent of the events propagating greater distances to the SE. Comparison of the blue and red curves in Fig. 6(b) shows that the tendency for SE propagation appears to increase with event size. There is theoretical justification for such behaviour, in that the normal stress change at the rupture front increases with propagation speed (Weertman 1980), and from fracture energy arguments one expects rupture speed to increase with the rupture dimensions (at least in the absence of off-fault damage). However, given the above-mentioned correlation between resolution and directivity near our resolution limit, we are reluctant to conclude that the apparent dependence upon rupture size is real. Obtaining directivity measurements for more earthquakes could help clarify this issue.
Figure 6. (a) Cumulative number of the 887 well-resolved events in five different regions of Fig. 4(c), indicated by the dashed lines in that figure. The cumulative events are plotted as function of total inverted rupture length $L_{SE} + L_{NW}$. (b) Histograms of $L_{SE}/(L_{SE} + L_{NW})$ for all the well-resolved events in Fig. 4(c) (black) and for those with total inverted length smaller than 100 m (blue, 444 events) and larger than 100 m (red, 443 events).

Figure 7. (a) Directivity of the well-resolved unilateral and moderately unilateral earthquakes (same definition as in Fig. 6) on a vertical along-strike cross section. Black dots denote events with inferred sizes smaller than 70 m that could act as Empirical Green’s functions for the well-resolved events. Plot on the lower left shows histograms of the ratio of $c_p$ on the Pacific (PA) and North American (NA) sides of the fault ($c_{p-PA}/c_{p-NA}$) for moderately or strongly unilateral events propagating to the SE (blue) and NW (red), with dashed lines (indistinguishable) indicating the median $P$-wave velocity contrast for the two populations. (b) $P$-wave velocity contrast. To have a clearer view of the spatial patterns, the event sizes in both plots are represented using circles whose diameters are 1.5 $L_{cat}$. © 2011 The Authors, *GJI*, 186, 852–866 Geophysical Journal International © 2011 RAS
made estimates of this contrast in our earthquake relocations, and at the scale of individual multiplets hundreds of metres to kilometres across the P-wave velocity was typically higher on the Pacific Plate by 5–20 per cent (Fig. 7). Other processes that could compete with the contrast in wave speed in influencing propagation direction include a contrast in poroelastic parameters (Rudnicki & Rice 2006), asymmetry in the pre-existing stress field (e.g. Stein et al. 1997; Ampuero & Ben-Zion 2008) and non-planar fault surfaces, which like the material contrast introduces a coupling between shear slip in the mode-II direction and normal stress changes.

Since both rupture directivity and across-fault material contrast may have local variations, in Fig. 7 we plot the distribution of rupture directivity and P-wave velocity contrast on a vertical cross section along the fault strike. Fig. 7(a) shows the directivity of well-resolved events from the inversion results in Fig. 4(c), with blue and red indicating unilateral SE and unilateral NW events, respectively (for clarity, ‘bilateral’ events with their shorter halves larger than 80 per cent of the longer halves are not shown). Fig. 7(b) plots the multiplet-based P-wave velocity ratios based on the travelt ime delay measurements (Rubin 2002a), with cyan and green indicating velocity differences of 0–10 per cent and 10–20 per cent, respectively. The section between 30 and 40 km in Fig. 7(a) show a dominance of SE unilateral events, whereas events to the NW and SE of this region show a greater mix of directivity. This spatial pattern is not obviously correlated with that of the P velocity contrast. This can also be seen in the inset in Fig. 7(a), which shows that the median velocity contrasts in the vicinity of those events propagating to the NW and those propagating to the SE are indistinguishable. This is in contrast to the results of Lengline & Got (in press), who found an apparent correlation between their directivity estimates near Parkfield and the P-wave velocity contrast from Zhao et al. (2010). However, the P-wave velocity contrast provides only a gross indication of the material contrast most relevant to dynamic rupture on a bimaterial interface. As illustrated by Rubin &ampuero (2007), the most important parameter (neglecting possible contrasts in poroelastic parameters) is the ratio of shear wave speeds. Unfortunately, we have found this to be difficult to measure due to the poor quality of S-wave arrivals on the North American side of the fault.

A possible clue as to why many events propagate to the NW comes from different signs of directivity within some repeating earthquake clusters. We found that more than 60 of the 78 repeating earthquake clusters identified by Rubin (2002a) have roughly flat spectral ratios as a function of frequency at all azimuths, which suggests that either the events have similar signs and extents of directivity or are too small to resolve. Some of these repeating earthquakes are resolved by comparison to one or more smaller companions and exhibit a particular pattern of directivity, that is, they repeatedly nucleate at the same location and rupture to the same direction (either NW or SE). However, there are also a few repeating earthquake clusters in which the events have different signs of directivity; this was also observed by Lengline & Got (in press). Examination of the relocated catalogue shows that some of these have adjacent clusters which might have influenced the propagation direction. The blue circles in Fig. 8(a) show a 7-event repeating cluster trend. This cluster had a smaller adjacent cluster, shown in red, and no other events within 300 m. The inverted directivity of the seven blue events is shown as a function of time in Fig. 8(b). Directivity of the red events cannot be well resolved due to their small sizes and the lack of smaller reference events, so only their origin times are shown (red dashed lines). The inversion results suggest that events 1, 2, 3, 5, and 6 propagated to the NW, while events 4 and 7 are close to bilateral (events 4 and 5 are the blue and red events in the upper pair of Fig. 3). Four of the five NW-propagating events (1, 3, 5, 6) have SE foreshocks (loosely defined) within the previous year; the time separation between events 1, 3 and their foreshocks are just 14 and 19 mins. Event 2 is the only one which propagated to the NW without a SE foreshock, although we cannot exclude the possibility of a small foreshock below the detection threshold. It seems reasonable to suppose that these NW-propagating events were affected by the SE foreshocks and that their SE ends, closer to the foreshocks, ruptured first.

To assess the influence of mode-II foreshocks on mainshock directivity on the scale of the entire catalogue, we plotted the inversion results of only those earthquakes with diameters being the estimated catalogue length $L_{\text{cat.}}$ (b) Inverted directivity of the blue cluster events, with 90 per cent error bars, plotted against their origin times. The origin times of the red events are indicated by the vertical red lines. When red and blue events are close in time, red arrows indicate if the red event occurred before or after the blue.

A possible clue as to why many events propagate to the NW comes from different signs of directivity within some repeating earthquake clusters. We found that more than 60 of the 78 repeating earthquake clusters identified by Rubin (2002a) have roughly flat spectral ratios as a function of frequency at all azimuths, which suggests that either the events have similar signs and extents of directivity or are too small to resolve. Some of these repeating earthquakes are resolved by comparison to one or more smaller companions and exhibit a particular pattern of directivity, that is, they repeatedly nucleate at the same location and rupture to the same direction (either NW or SE). However, there are also a few repeating earthquake clusters in which the events have different signs of directivity; this was also observed by Lengline & Got (in press). Examination of the relocated catalogue shows that some of these have adjacent clusters which might have influenced the propagation direction. The blue circles in Fig. 8(a) show a 7-event repeating cluster trend. This cluster had a smaller adjacent cluster, shown in red, and no other events within 300 m. The inverted directivity of the seven blue events is shown as a function of time in Fig. 8(b). Directivity of the red events cannot be well resolved due to their small sizes and the lack of smaller reference events, so only their origin times are shown (red dashed lines). The inversion results suggest that events 1, 2, 3, 5, and 6 propagated to the NW, while events 4 and 7 are close to bilateral (events 4 and 5 are the blue and red events in the upper pair of Fig. 3). Four of the five NW-propagating events (1, 3, 5, 6) have SE foreshocks (loosely defined) within the previous year; the time separation between events 1, 3 and their foreshocks are just 14 and 19 mins. Event 2 is the only one which propagated to the NW without a SE foreshock, although we cannot exclude the possibility of a small foreshock below the detection threshold. It seems reasonable to suppose that these NW-propagating events were affected by the SE foreshocks and that their SE ends, closer to the foreshocks, ruptured first.

To assess the influence of mode-II foreshocks on mainshock directivity on the scale of the entire catalogue, we plotted the inversion results of only those earthquakes with foreshocks within two mainshock radii and $10^{15}$ s in Fig. 9(a). These are the distance and timescales that maximized the aftershock asymmetry observed by Rubin (2002a), although the physics underlying the influence of the mainshock on aftershock asymmetry and the influence of foreshock on mainshock directivity could of course be different (the designation ‘foreshock’ or ‘mainshock’ here refers only to relative timing and not event size). In Fig. 9(a) events with SE foreshocks are contained in blue boxes while the ones with NW foreshocks are contained in red; the colour of the contained points still represents the resolution of $L_{\text{SE}}/(L_{\text{SE}} + L_{\text{NW}})$. To increase the number of ‘well-resolved’ events we relax the definition to $L > 70$ m and resolution <0.4 rather than <0.3 (the distributions resulting from the two criteria appear similar). The time separations between the foreshocks and the mainshocks are reflected by different box sizes, with smaller separations in larger boxes. In Fig. 9(a), it can be seen that blue and red boxes tend to lie above and below the diagonal.
Given the discussion in the literature, it is also of interest to examine the influence of mainshock directivity on aftershock distribution. We note first that the asymmetry of rupture directivity in Fig. 6(b) seems smaller than the asymmetry of aftershock production. That is, if the probability of producing an aftershock to the NW was \( L_{NW} / L \) times the probability of producing an aftershock to the SE, and if all well-resolved events were equally likely to produce aftershocks, then the ratio of NW to SE aftershocks would be far less than the 3:1 observed in the catalogue. A more quantitative assessment would be to compare the distribution of the directivities of events with NW aftershocks and those with SE aftershocks to the distribution of directivities of all well-resolved events. If these distributions appear very similar, for example, then one could conclude that mainshock directivity has no appreciable influence on aftershock productivity, in the sense that SE- and NW-propagating mainshocks are equally likely to produce aftershocks to the NW.

In Fig. 10(a) we plot the summary of inverted rupture lengths for the well-resolved events \((L > 70 \text{ m} \text{ and resolution } < 0.4)\) with mode-II aftershocks, with blue and red boxes indicating whether those aftershocks are located to the SE or NW, respectively. We define an aftershock as an event within two mainshock radii and \(10^4 \text{s} \) of the mainshock, the same definition used by Rubin (2002a). The number of well-resolved events with SE and NW aftershocks are 15 and 42, respectively, in the same 1:3 ratio as for the catalogue as a whole. The distributions of directivity for events with SE and NW aftershocks are shown in Fig. 10(b) and summarized in Table 1. Although the events with SE aftershocks might be affected by the statistics of small numbers, it appears that both those events with NW aftershocks and those with SE aftershocks have a greater tendency for SE propagation than does the catalogue as a whole.

**Figure 9.** (a) Inverted rupture lengths of well-resolved events [for better statistics, the width of the 90 per cent confidence interval of \( L_{SE} / (L_{SE} + L_{NW}) \) can be as large as 0.4] which have mode-II foreshocks within \(10^4 \text{s} \) and 2 times the mainshock radius. The inversion results are from Fig. 4(c) in which the prior distribution of \( V_{SE} \) is 10 per cent faster than \( V_{NW} \). Blue and red boxes enclose events which have foreshocks to the SE and to the NW, respectively. Different box sizes denote the temporal separation between the two events (indicated on the right). (b) Histograms of \( L_{SE} / (L_{SE} + L_{NW}) \) of events within blue and red boxes.

**Figure 10.** (a) Inverted rupture lengths of the well-resolved events which have mode-II aftershocks within \(10^4 \text{s} \) and 2 times the mainshock radius. The priors of rupture speeds are 10 per cent faster to the SE. Blue and red boxes enclose events which have aftershocks to the SE and to the NW, respectively. Different box sizes denote the temporal separation between the two events (indicated on the right). (b) Histograms of \( L_{SE} / (L_{SE} + L_{NW}) \) of events with aftershocks.
whole. We thought this might be due to the correlation between SE directivity and mainshock size suggested by Fig. 6(b), with larger earthquakes also tending to have more aftershocks, but in fact the earthquakes with aftershocks in Fig. 10 have a size distribution very similar to that of all the well-resolved events in Fig. 4(c). The two distributions for events with aftershocks in Fig. 10(b) (blue and red curves) appear quite similar and a Kolmogorov-Smirnov test yields a probability of 86.9 per cent for differences at least this large under the null hypothesis that the two sets of samples are drawn from the same distribution. In addition, mainshocks with $L_{SE} > L_{NW}$ have 32 aftershocks to the NW and 12 to the SE, while mainshocks with $L_{NW} > L_{SE}$ have 10 aftershocks to the NW and 3 to the SE, both in roughly the same ratio as for the catalogue as a whole. So it seems that mainshocks propagating in either direction produce more aftershocks to the NW, consistent with the mechanism of aftershock asymmetry proposed by Rubin & Ampuero (2007). Although a larger database is needed to answer this question definitively, we tentatively conclude that macroscopic mainshock directivity does not provide the dominant influence on the aftershock asymmetry described by Rubin & Gillard (2000) and Rubin (2002a).

5 CONCLUSIONS

We performed inversions of spectral ratios of microearthquakes on the northern creeping section of the San Andreas Fault to see if they propagated preferentially to the SE as predicted. For spatially close event pairs, the normalized spectral ratios at stations with high signal-to-noise ratios, cross-correlation coefficients and coherencies are azimuthally consistent. The moving point source model is able to fit the spectral ratio data reasonably well and gives estimates of the rupture lengths in both directions along with the level of resolution. The results of inversions using different model parameterizations and data selections all suggest that while ~40 per cent of the earthquakes in our catalogue are nearly bilateral ($L_{short} > 0.8 L_{long}$), nearly twice as many have $L_{SE} > L_{NW}$ consistent with the theoretical prediction, and of those classified as strongly unilateral ($L_{short} < 0.5 L_{long}$), more than three times as many propagate to the SE. For events with resolved rupture speeds, the propagation velocities to the SE are faster than to the NW by ~10 per cent, consistent with numerical and laboratory simulations of bilateral ruptures on a bimaterial interface. The percentage of SE-propagating events and the $P$-wave velocity contrast have different spatial patterns on a cross section along the fault strike; future studies of the $S$-wave velocity contrast may provide more information on this problem.

Our results also show that events with nearby foreshocks tend to propagate away from those foreshocks. The relationship between foreshock location and mainshock directivity holds only for pairs separated by rather small space and time windows, but in those cases, the effect of foreshock location on mainshock directivity is more significant than the material contrast. There is not a strong correlation between mainshock directivity and aftershock location, suggesting that the aftershock asymmetry observed by Rubin & Gillard (2000) and Rubin (2002a) is not controlled by mainshock directivity. Rather, we view both the aftershock asymmetry (more to the NW) and preferential directivity (to the SE) as largely independent manifestations of the across-fault material contrast.

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### APPENDIX A: DATA AND MODEL PARAMETERIZATION

A possible concern with our analysis procedure is that the 0.64-s (64-sample) data window is much longer than the earthquake duration (~0.03 s for a rupture length of 100 m). Thus most of our data consists of coda, rather than the direct arrival, and if energy leaving the source in multiple directions is scattered to the station this could reduce the signal of directivity. Nonetheless, including coda of some duration is inevitable if one is to obtain high signal-to-noise ratios and reliable spectral ratios. To address this issue we tested two data windows other than that used in the main text, which started 10 samples before and the P-wave arrival. One accentuated the direct arrival by starting 30 samples before and extending to only 34 samples after, and one excluded the direct arrival entirely by extending from 20 to 84 samples after. The results are shown in Fig. A1, both of which appear qualitatively similar to Fig. 4(a) but with a stronger asymmetry when less coda is used (Fig. A1a). To show the influence of windowing more clearly, Fig. A2 plots the histograms of \( L_{SE}/(L_{SE} + L_{NW}) \) for all the well-resolved events (same definition as in Table 1). As the length of coda decreases, the fraction of well-resolved events with longer SE halves increases from 58.5 per cent to 61.5 per cent to 62.4 per cent (Table 1). However, the means and medians of directivity for inversions with different windowings are quite close to each other and lack a monotonic trend, so the signal degradation that comes from including coda waves is not strong enough to greatly affect the inverted tendency for SE propagation.

The assumption of an infinitely short rise time implies that the seismic signal of one rupture ‘half’ at any station appears as a boxcar with duration \( \Delta t_{app} \) and corner frequency \( 1/(\pi \Delta t_{app}) \), above which the spectrum falls off with frequency \( f \) as \( f^{-1} \). Adding a finite rise time of duration \( \Delta t' \) adds a second corner frequency of \( 1/(\pi \Delta t') \), above which the spectrum decays as \( f^{-2} \) (Shearer 1999), which is more consistent with the observed spectral decay of natural earthquakes at high frequency (Abercrombie 1995; Ide et al. 2003). However, provided the rise time is short compared to the rupture duration, for most of the \( M < 3 \) events in this study the \( f^{-2} \) trend occurs well beyond the 25 Hz range examined, and the zero-rise time assumption seems appropriate.

Another factor which can greatly affect the inversion results is the choice of model parameters. Although the rupture process of natural earthquakes may not be in the manner of focused point

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Figure A1. Unlike the four plots in Fig. 4 where the 64-sample seismic data windows start 10 samples prior to the first P arrival, Figs A1(a) and (b) plot the central points of inversions comparable to Fig. 4(a) except that the windows start 30 samples before and 20 samples after the P arrival.
sources running horizontally in opposite directions, the limited information from the spectral ratios alone is not expected to be able to resolve a more complex source model. Candidate parameters for a single earthquake include two rupture lengths to the SE and NW along the fault strike and their rupture propagation velocities. We tested four different model parameterizations: (1) The two rupture lengths and rupture speeds are four independent free parameters (as in Fig. 4a). (2) Only the two rupture lengths are free parameters and the two rupture propagation velocities are fixed at 0.85cs (Fig. 4d). (3) Besides fixing the rupture propagation velocities at 0.85cs, force LSE +LNW to equal the catalogue length. (4) LSE = LNW, leaving only one degree of freedom (the total rupture length) for each event [inversion results of (3) and (4) not shown in Fig. 4].

Using the downhill-simplex method (Press et al. 1986), we performed inversions on many earthquake multiplets using these four parameterizations. As a straightforward way of showing what proportion of the directivity signal can be fitted by the various models, in Fig. A3 the binned minimum deviations are plotted against the signal amplitudes of the spectral ratio data at 25 Hz. The unit of both signal amplitude and deviation is the log10 of the spectral ratios. The four-parameter inversion fits the data only slightly better than the two-parameter inversion, which is consistent with the observed trade-off between rupture length and rupture propagation velocity. The increasing difference between the deviations of the two and four parameter inversions with signal amplitudes suggests that rupture lengths and speeds of larger halves can be independently resolved to some extent, which is also implied by the difference in posterior pdfs of VSE and VNW for halves larger than 60 m (Fig. 5). The advantage of having VSE and VNW as free parameters is that the uncertainties of rupture propagation velocities can be mapped into the posterior pdfs, avoiding underestimating the uncertainties of rupture lengths. Because of the extra degree of freedom, the two-parameter model is clearly better than the two one-parameter ones. The comparison between the bilateral inversion (LSE = LNW) and all the others suggest that a large proportion of the largest signals can be explained only by asymmetric lengths of rupture propagation.

APPENDIX B: INVERSION PROCEDURE

The first step in data processing is to select stations with high-quality data for each pair of earthquakes. A seismic station is deemed acceptable when the cross-correlation coefficient between seismograms, signal-to-noise ratio in each frequency band, and spectral coherence between the two events are higher than 0.5, 2 and 0.4, respectively. The waveforms of every two events at each station were aligned by cross-correlation. The inversion was then performed as follows: if we have Nf samples in the spectral domain and Nsta available stations for one earthquake pair, the problem is to solve a set of Nf · Nsta non-linear equations with eight unknowns (four for each earthquake). We seek the least absolute value solution of the problem to suppress the bias caused by outliers. The misfit function is defined as (i and f are indices for stations and frequency samples)

\[
\text{Misfit} = \sum_{i,f} |\text{Ratio}^{\text{syn}}(i, f) - \text{Ratio}^{\text{obs}}(i, f)|.
\]

We first used the downhill-simplex method (Press et al. 1986) to search for the minimum misfit model and from this estimated the mean deviations of the spectral ratio data νf. This optimization algorithm does not need the gradient of the misfit function but requires that the computation of the forward model is fast. So we computed and saved the synthetic spectrum with all combinations of model parameters (two rupture durations and the ratio of rupture lengths, the amplitude of the rupture length is cancelled when performing the low-frequency normalization). Compared to computing the multitaper spectrum for each event, looking up the spectrum from the table is much faster. We found that the minimum misfit point of the Monte-Carlo sampling can often fit the data better than that of the downhill-simplex method, indicating that the uncertainties from the downhill-simplex method could be slightly overestimated and the Monte-Carlo sampling is a more reliable method for finding the approximate global minimum given the high-dimensional model space.

The mean deviations νf of the spectral ratios are assumed to be frequency dependent, and are estimated from the difference between the data and the synthetics produced by the minimum misfit model at each frequency, averaged over all stations for each pair. The mean deviations increase monotonically with frequency. Since the multitaper algorithm smooths the spectra in the spectral domain...
six stations in total are included in the inversion. We compared the
walk is located at
the high dimensional posterior distributions of model parame-

\[ \text{The denominator downweights pairs separated by large distance and}
\]

only functions for
The denominator downweights pairs separated by large distance and

\[ \text{and spectral coherency (coh) between the waveforms, frequency-}
\]

dependent signal-to-noise ratios (SNR) computed from the spectra
of noise before the \( P \) arrival, and the inter-event distances (\( d \)). The
weights \( w \) were defined as

\[ w(d, i, j) = \frac{\text{corr}(i) \cdot \text{coh}(i, j) \cdot \text{SNR}(i, j)}{0.005d + 0.5} \cdot w_{\text{last}} \cdot w_{\text{ind}}. \]  

(B2)

inversion results from the random walk in the parameter space, the
first 1960 \( N_m \) samples were treated as a ‘burn in’ phase and were
discarded, after which only one sample is recorded for every 35 \( N_m \)
samples to reduce the dependence between samples. As an example of
the inversion results, the 2-D marginal posterior distributions of
rupture length and rupture velocity of two events from a repeating
cluster are shown in Fig. B1. Figs B1(a) and (d) are the posterior
distributions of the two rupture lengths of both events, from which
the different signs of directivity can be observed. From the rupture
length rupture velocity plots, we can observe that some rupture
lengths are strongly correlated with their rupture propagation
velocities (c and e) while others not (b and f). The correlations occur
for the larger rupture lengths (\( L_{NW} \) in Fig. B1a and \( L_{SE} \) in B1d),
which suggests that these rupture length–rupture velocity couples
(c and e) are resolved while the couples in b and f are not. The
correlations between rupture length and rupture velocity mean that
the inversion is sensitive mostly to rupture duration rather than rupture
length and rupture speed independently.

The relocated catalogue has many tightly clustered earthquakes
which are separated by large distances from their closest neighbours,

\text{averaging over frequency only involves the independent spectral samples) }

\[ L \propto \exp \left( - \sum_{i,j} w(i,j) \cdot \frac{\text{Ratio}^{\text{obs}}(i,j) - \text{Ratio}^{\text{th}}(i,j)}{v_{i,j}} \right). \]  

(B3)

The a posteriori pdf is the product of the likelihood function and
the prior pdf \( P_{\text{prior}} \) (as shown in Fig. 2)

\[ P_{\text{post}} \propto P_{\text{prior}} \cdot L. \]  

(B4)

We used a Markov chain Monte Carlo technique to sample the
high dimensional posterior distributions of model parameters.
A Markov chain of samples was constructed based on the
Metropolis algorithm (Tarantola 2004). The algorithm starts from
an initial point in the model space and then performs a random walk
monitored by an acceptance rule. Suppose at one step the random
walk is located at \( m_{\text{current}} \), where \( m \) is a vector in the model space
containing an array of model parameters. In going to the next step,
a perturbation \( dm \) is drawn from a proposal distribution and a
candidate for the next step is \( m_{\text{proposed}} = m_{\text{current}} + dm \). The Metropolis
algorithm accepts the proposed point \( m_{\text{proposed}} \) with a possibility of

\[ \min \left( P_{\text{post}}(m_{\text{proposed}})/P_{\text{post}}(m_{\text{current}}), 1 \right). \]  

We started the random walk from the minimum misfit point of the downhill-simplex method and
used a Gaussian distribution with an empirical covariance matrix as
a proposal distribution to generate \( dm \). For faster converging speed
\text{towards the target distribution, the covariance matrix of the proposal}
distribution was iteratively updated as sampled points accumulate
(Pasarica & Gelman 2010).

For an \( N \)-event multiplet, there are \( N_e = 4N \) model parameters
if both rupture lengths and velocities are varied. In generating the

\text{inversion results from the random walk in the parameter space, the
first 1960} \( N_m \) \text{ samples were treated as a ‘burn in’ phase and were
discarded, after which only one sample is recorded for every 35} \( N_m \)
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\[ P_{\text{post}} \propto P_{\text{prior}} \cdot L. \]  

(B4)

We used a Markov chain Monte Carlo technique to sample the
high dimensional posterior distributions of model parameters.
A Markov chain of samples was constructed based on the
Metropolis algorithm (Tarantola 2004). The algorithm starts from
an initial point in the model space and then performs a random walk
monitored by an acceptance rule. Suppose at one step the random
walk is located at \( m_{\text{current}} \), where \( m \) is a vector in the model space
containing an array of model parameters. In going to the next step,
a perturbation \( dm \) is drawn from a proposal distribution and a
candidate for the next step is \( m_{\text{proposed}} = m_{\text{current}} + dm \). The Metropolis
algorithm accepts the proposed point \( m_{\text{proposed}} \) with a possibility of

\[ \min \left( P_{\text{post}}(m_{\text{proposed}})/P_{\text{post}}(m_{\text{current}}), 1 \right). \]  

We started the random walk from the minimum misfit point of the downhill-simplex method and
used a Gaussian distribution with an empirical covariance matrix as
a proposal distribution to generate \( dm \). For faster converging speed
\text{towards the target distribution, the covariance matrix of the proposal}
distribution was iteratively updated as sampled points accumulate
(Pasarica & Gelman 2010).

For an \( N \)-event multiplet, there are \( N_e = 4N \) model parameters
if both rupture lengths and velocities are varied. In generating the

\text{inversion results from the random walk in the parameter space, the
first 1960} \( N_m \) \text{ samples were treated as a ‘burn in’ phase and were
discarded, after which only one sample is recorded for every 35} \( N_m \)
\text{samples to reduce the dependence between samples. As an example of
the inversion results, the 2-D marginal posterior distributions of
rupture length and rupture velocity of two events from a repeating
cluster are shown in Fig. B1. Figs B1(a) and (d) are the posterior
distributions of the two rupture lengths of both events, from which
the different signs of directivity can be observed. From the rupture
length rupture velocity plots, we can observe that some rupture
lengths are strongly correlated with their rupture propagation
velocities (c and e) while others not (b and f). The correlations occur
for the larger rupture lengths (\( L_{NW} \) in Fig. B1a and \( L_{SE} \) in B1d),
which suggests that these rupture length–rupture velocity couples
(c and e) are resolved while the couples in b and f are not. The
correlations between rupture length and rupture velocity mean that
the inversion is sensitive mostly to rupture duration rather than rupture
length and rupture speed independently.

The relocated catalogue has many tightly clustered earthquakes
which are separated by large distances from their closest neighbours,

\text{averaging over frequency only involves the independent spectral samples) }

\[ L \propto \exp \left( - \sum_{i,j} w(i,j) \cdot \frac{\text{Ratio}^{\text{obs}}(i,j) - \text{Ratio}^{\text{th}}(i,j)}{v_{i,j}} \right). \]  

(B3)

The a posteriori pdf is the product of the likelihood function and
the prior pdf \( P_{\text{prior}} \) (as shown in Fig. 2)

\[ P_{\text{post}} \propto P_{\text{prior}} \cdot L. \]  

(B4)

We used a Markov chain Monte Carlo technique to sample the
high dimensional posterior distributions of model parameters.
A Markov chain of samples was constructed based on the
Metropolis algorithm (Tarantola 2004). The algorithm starts from
an initial point in the model space and then performs a random walk
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containing an array of model parameters. In going to the next step,
a perturbation \( dm \) is drawn from a proposal distribution and a
candidate for the next step is \( m_{\text{proposed}} = m_{\text{current}} + dm \). The Metropolis
algorithm accepts the proposed point \( m_{\text{proposed}} \) with a possibility of

\[ \min \left( P_{\text{post}}(m_{\text{proposed}})/P_{\text{post}}(m_{\text{current}}), 1 \right). \]  

We started the random walk from the minimum misfit point of the downhill-simplex method and
used a Gaussian distribution with an empirical covariance matrix as
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distribution was iteratively updated as sampled points accumulate
(Pasarica & Gelman 2010).

For an \( N \)-event multiplet, there are \( N_e = 4N \) model parameters
if both rupture lengths and velocities are varied. In generating the
Figure C1. (a) Low-frequency-normalized spectra of a seven-event repeating earthquake cluster at station BVL and spectral differences between events across the change of station delay on 1989 February 04. The coloured solid curves are spectra of all the events in the repeating cluster, which are labelled chronologically to the right of the curves. The blue dashed curves are the spectra of events after the change less the spectra before, with the mean plotted as a green solid curve. (b) Mean spectral differences across 1989 February 04 at 169 repeating clusters in the catalogue (the green curve in (a) is one of many). As the mean of the green curves, the thick black curve is taken as the spectral correction for this change.

APPENDIX C: CORRECTIONS FOR TIME DEPENDENT STATION CHANGES

The repeating earthquakes in our catalogue often span decades and the normalized spectra of events belonging to the same cluster are very similar, which makes it possible to compute empirical corrections for spectral ratios between events that span changes in station response. Fig. C1(a) plots the normalized spectra of a repeating cluster at station BVL. The spectra of the first three events have more high-frequency energy than later events, and the spectral contents within each time window separated by the change are quite similar. Because the spectra of these events recorded at other stations do not show a change at this time, it leads one to suspect that the change lies with the station and not the earthquakes. We also plotted the spectral differences between all event pairs across a possible change of station response on 1989 February 04 (blue dashed curves) and their mean (green solid curve). Fig. C1(b) shows the mean spectral differences across 1989 February 04 at station BVL for 169 repeating clusters in the relocated catalogue. The similarity between the spectral differences of most repeating clusters confirm that the differences are caused by a change of station response rather than different spectral contents between events across the specific time in all 169 clusters. After checking all the stations, we found that most of the obvious changes of spectral content were associated with the time-dependent changes in station delays identified by Rubin (2002b). We then computed the differences across all the times at all the stations identified by Rubin (2002b), just as was done to obtain the dashed curves in Fig. C1(a), and took the average of all the differences across a particular time and station to obtain an empirical spectral correction for events spanning that time at that station. The corrections have different amplitudes and different degrees of consistency over the repeating clusters, so the mean deviations of the mean spectral corrections over repeating clusters were used as the lower limit of error estimates in the likelihood function (Appendix B).

Fig. C2 shows the spectral ratios of two events in a repeating cluster before and after applying the empirical station corrections. The non-zero amplitudes of the spectral ratios at several stations in the top panel (without corrections) seem to suggest that the spectral contents of the two events are different at these stations, and the spectral differences are inconsistent at adjacent azimuths. However, the corrections close most ‘gaps’ at high frequencies in the top panel and the spectral ratios in the bottom panel are nearly flat. The forward model can well fit the corrected spectral ratios in the bottom panel, including the small directivity signal that the blue event has more high-frequency energy at the NW stations.

Figure C2. Spectral ratios of a pair of repeating events before (top) and after (bottom) applying the spectral corrections. The blue and red curves are the ratios of spectra of event 1 to 2 and 2 to 1, respectively. The stations are sorted by azimuth (shown above the spectral ratios, 0 for SE and 180 for NW). The corrections were applied at stations 1, 3, 4, 6, 7, 10, 11 and 12, and are based on data from all the repeating clusters. The grey scale curves are trying to fit the blue spectral ratios.