

## DISCUSSION

W. J. Derner<sup>2</sup>

This contribution by the author is commendable if only for the quantities of the bearings that have been tested under actual misalignment and must also be noted for comparing theory to actual test data.

This discussor would like the author to more specifically define "truncation" and "cross contact."

It would have been most instructive and rewarding if the author had developed, experimentally, the actual contacts following the method presented by Goodelle and others. This discussor has had extensive experience with misalignment conditions, both experimentally and analytically, and has found that calculated or presupposed misalignment due to machine reliefs, actual bearing and housing part tolerances, and the adjusting of bearing heights to achieve shaft slopes do not always achieve the misalignment anticipated. For this reason I feel that more stringent methods are required to determine the actual shaft to housing slope, and footprint techniques are a more factual demonstration of the misalignment that actually existed.

The quantitative load distribution within the whole bearing must be considered in the same light, since practical deviations from theoretical models can effect significant changes in bearing life. It is also considered essential to account for the individual roller size and geometry variations since differences in diameter, crown, radius and crown symmetry can induce greater truncation effects than otherwise anticipated.

Would the author please comment on these latter variations as they quite probably existed in the bearing tested. Are we to consider them as special experimental bearings atypical of Timken production parts or were they truly production bearings that were used in a laboratory experiment?

Lastly, it is my conclusion that the value of this paper would be significantly improved by defining the geometry of the bearing parts including roller crown.

N. M. Wickstrand<sup>3</sup>

This paper is a valuable addition to the literature of rolling element bearings. The derivation of the misalignment parameter is similar to what was found by other researchers as has been given in references [1-3] inclusive. As far as this discussor knows, these are the principal published derivations. However at least two other independent studies have produced similar results although neither of them have been published.

A careful examination of the misalignment parameter as given in equation (17) develops a very annoying question. This parameter contains both  $\theta$  and  $\delta$  both of which can be linear (or nearly linear) functions of the applied load. Hence for the condition where all of the "misalignment" is due to shaft deflection the parameter becomes independent of load. This condition is very common with needle bearings where the load is rather high compared to the shaft diameter and bearing spacing.

The analysis of test data for 532 individual bearings seems to indicate that the method of this paper is valid in spite of the aforementioned comments. This is particularly convincing since tests numbers 7, 13, and 17 all seem to fit the pattern of the other tests. These tests are mentioned since their only misalignment is due to shaft bending.

This paper gives deflection as a function of load to the  $1/e$  power. The foregoing discussion is on the premise that  $e = 1.00$ . The author uses  $e = 1.10$  which does make a slight difference.

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If one checks the origin of the power rule in reference [6] and in earlier sources one will find that this assumes a definite range of load in intensities and roller and race geometry. It might have been advisable if this were mentioned in the paper. In similar manner it might have been advisable if the author qualified equation (5) and Fig. 3. These are both true for line contact as between two cylinders. When the roller and race are cones, the contact is trapezoidal and hence the result is only approximate. For the usual proportions of tapered roller bearings the equation and diagram are approximately correct.

It would be interesting if the author would comment as to how the misalignment effect can be independent of load.

### Author's Closure

Truncation occurs whenever the major axis of the contact ellipse exceeds the effective length of contact, i.e.,  $2a/l > 1$ . Cross contact is that condition in which the misalignment in the bearing under the loading conditions is sufficient to allow all the rollers to come into contact ( $\epsilon \geq 1$ ) such that diametrically opposite rollers are contacting the race at opposite ends.

The author can appreciate Mr. Derner's enthusiasm concerning the footprint technique with which the discussor is familiar, but in the author's misalignment tests, the skewed load zones observed on the cup races were sufficient to verify that the anticipated misalignments had been obtained.

The author agrees with Mr. Derner that the load distribution within the bearing is important; for this reason the actual load zones were measured and the bearing life adjusted accordingly in the analysis presented in the paper.

The bearings tested were standard production bearings. In this type of test it is common practice to randomize a large group of bearings so that the effect of geometry variations in each subgroup that is selected is minimized when comparing one test group to another. In other words the effect of geometry variations is averaged out over each subgroup, thus eliminating that as a variable.

Based on the model presented in the paper, it is possible, as Mr. Wickstrand suggests, that the misalignment effect can be independent of load. However, this is true if, and only if,  $e = 1$  and  $\nu = 0$ . For line contact conditions  $e = 1.1$  and  $\nu = 0$ ; for point contact or truncated contact  $1 \leq e \leq 1.5$  and  $\nu > 0$ . Therefore, in the analysis given, the case of  $e = 1$  and  $\nu = 0$  is excluded. It should be pointed out that the 532 bearings tested were under truncated contact.

As for the power rule the value of  $e = 1.1$  was actually determined by the author considering a range of  $10 < \frac{DP_0 \sin \alpha}{l \sin \gamma} < 10^6$

where  $D$  is the mean roller diameter,  $\gamma$  is  $1/2$  included roller centerline angle, and  $P_0$ ,  $\alpha$ ,  $l$  are defined in the nomenclature. It is obvious that a very wide range of load intensities and bearing geometry is covered.

Mr. Wickstrand is correct in saying that equation (5) and Fig. 3 have to be qualified since a trapezoidal contact is actually formed. Equation (10) and Fig. 4 should also be qualified since the contact area formed under point contact is quasi-elliptic. The approximation was made for the obvious simplification and has been the common practice of other investigators. However, it might be possible to relate these types of contact with a skewed load distribution that is formed under some amount of misalignment, e.g., note that the contact widths for the conditions given in Table 2 form a truncated quasi-elliptic contact.