$$\mathrm{Nu}(\xi) = \frac{(1+\beta_5) + 8\sum_n \frac{A_n}{\lambda_n{}^2} \exp(-\lambda_n{}^2\xi)}{\left(\frac{16}{1+\beta_5}\right)\sum_n \frac{A_n}{\lambda_n{}^4}\,[1-\exp(-\lambda_n{}^2\xi)]} \qquad (25)$$

Hence the asymptotic Nusselt number for the case of linear wall temperature-variation is

$$\mathrm{Nu}_a = \frac{(1+\beta_5)^2}{16\sum_n \frac{A_n}{\lambda_n{}^4}} \qquad (26)$$

The solution to cases of given wall heat fluxes also can be obtained from the result for the case of uniform wall temperature. For the case of uniform wall heat flux, the Nusselt number by appropriate manipulation is given as

$$\mathrm{Nu}(\xi) = \frac{(1+\beta_5)}{\left(\frac{16}{1+\beta_5}\right)\sum_n \frac{A_n}{\lambda_n{}^4} + {}^1/_2 \sum_n \frac{\exp(-\gamma_m{}^2\xi)}{\gamma_m{}^4 H'(-\gamma_m{}^2)}} \qquad (27)$$

where

$$H(s) = 2\sum_n \frac{A_n}{s+\lambda_n{}^2}$$

and $\gamma_m{}^2$ are the values satisfying $H(-\gamma_m{}^2) = 0$.

The asymptotic Nusselt number for this case can be seen from (27) as $\xi \rightarrow \infty$ to be identical to (26). This states that far downstream from the thermal entrance region, the Nusselt numbers are the same for both cases of linear wall-temperature variation and constant wall heat flux.

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DISCUSSION

D. C. Schluderberg[2]

Dr. Tien's paper has made a noteworthy contribution to the art by theoretically confirming test results obtained by Farbar and Morley showing that suspended solids have negligible effect upon the heat-transfer coefficient below solids-to-air ratios of approximately 1.0.

The data of Farbar and Morley also show that at higher solids loadings there is a pronounced effect of added solids upon the heat-transfer coefficient. This has been confirmed by AEC-sponsored work at The Babcock & Wilcox Company (Atomic Energy Division, Lynchburg, Virginia). For a given gas velocity and density, the Nusselt Number has been increased by as much as eight times through the addition of graphite particles to a flowing gas.

Theoretical solutions describing the results in this range of solids loadings are now being studied and correlations are being developed.

Author's Closure

The author wishes to thank Mr. Schluderberg for his comments. It is quite definite from experimental observations that there exists a pronounced effect on the heat-transfer coefficient at high solids-to-gas ratios. An analysis to explain this pronounced effect will be highly helpful for the understanding of the momentum and heat transport mechanisms in fluid-solids mixture.

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