Imaging quasi-vertical geological faults with earthquake data

P. Zheglova,1 J. R. McLaughlin,2 S. W. Roecker,3 J. R. Yoon4 and D. Renzi5

1Department of Earth Sciences, Memorial University of Newfoundland, St. John’s, NL, Canada. E-mail: pzheglova@mun.ca
2Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY 12180, USA
3Department of Earth and Environmental Sciences, Rensselaer Polytechnic Institute, Troy, NY 12180, USA
4Department of Mathematical Sciences, Clemson University, Clemson, SC 29634, USA
5Weill Cornell Medical College in Qatar, Doha, Qatar

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SUMMARY
We present a method for imaging quasi-vertically dipping faults with surface records of reflected $P$ waves from small earthquakes. Faults are boundaries between geological structures, such as tectonic plates, and are located in earthquake active regions such as Parkfield, California. The high degree of seismic activity enables the use of multiple seismic recordings in our fault identification algorithm. Major challenges occur because of the quasi-vertical orientation of the fault and the fact that the wave reflected by the fault and recorded by the surface receivers is not well modelled by the direct arrival of the propagating wave generated by the earthquake source. Our method uses the 2-D acoustic wave equation as the model for $P$-wave propagation. We assume that an approximate wave speed map on the reflection side of the fault is available and the source locations are known, for example, from traveltime tomography. We also assume that the source time function is known. The new features of our method arise because earthquake sources are located very close to the fault. This has two implications: (1) the direct arrival and the reflected wave arrive almost simultaneously, so that it is impossible to separate them on a seismogram using standard techniques, and (2) most of the reflections occur above the critical angle which introduces a distortion in the reflected wave. To overcome these difficulties we use a modelled incident wave to (1) remove the direct arrival from the data, and (2) remove the post-critical distortion from the reflected wave. We justify the distortion removal using the leading-order term of an asymptotic expansion, and an optimization procedure. To complete our algorithm we utilize some features of reverse time migration: (1) the use of full acoustic wave equation for modelling and backpropagation, and (2) zero-lag correlation of the backpropagated time reversed reflected and incident fields. We present numerical examples of fault reconstructions with synthetic data.

Key words: Body waves; Wave scattering and diffraction; Wave propagation.

1 INTRODUCTION

1.1 Background: imaging steeply dipping reflectors
The problem of imaging steeply dipping reflectors is of considerable interest. The applications include imaging of salt flanks in hydrocarbon exploration and imaging of vertically dipping geological faults in the areas of high seismic activity for the purpose of earthquake prediction and response modelling.

Faults often are boundaries between geological structures. One characteristic feature of major crustal faults is that they often accumulate enough offset so that the bounding crustal structures have significantly different material properties. The other characteristic is that geological faults are often associated with a high degree of seismic activity due to the movement driving the separation. This provides the opportunity to obtain seismic recordings, due to earthquakes, that can be utilized in reflection seismology algorithms. At the same time the high degree of seismic activity also drives the necessity to accurately image the region so that crustal response to earthquakes can be characterized. Later we give a review of some of the results for recovering subsurface structure.

Standard surface seismic profiles are suitable for imaging subsurface structures with shallow dips, but are not well suited to imaging steeply dipping faults (Mooney & Brocher 1987; Eberhart-Phillips et al. 1995). Layered material is readily observed in such profiles however, so vertical or steeply dipping faults may be observed as offsets in the layers. An example is the study by Feng & McEvilly (1993). In this study the San Andreas Fault (SAF) zone is observed as a break between the series of horizontal reflectors on both sides of the fault zone.

Depth migration methods with turning waves have been used by the exploration industry to image steeply dipping or overturned
crustal reflectors in the subsurface, such as vertical or overturned flanks of salt domes. An example of such a method is post-stack Kirchhoff depth migration (Ratliff et al. 1992; Albertin et al. 2002), where a certain pre-processing needs to be applied to the data to retain the refracted reflections in the stacked data. Whitmore (1983) and Baysal et al. (1984) showed that it is possible to image overturned reflectors with turning waves using reverse time migration. A number of migration methods based on one-way wave equation, capable of processing turning waves have been proposed. Claerbout (1985, pp. 272–273) proposed a modification to the post-stack phase-shift migration algorithm; this modification enables imaging overturned reflectors with two passes of a one-way wave equation. This algorithm is applied by Hale et al. (1992) to create a 3-D image of a salt dome with overhanging flanks. True amplitude modification of this method is used by Zhang et al. (2006). Other one-way wave equation methods use a combination of extrapolation in the downward and horizontal direction. One of these methods is the model proposed by Zhang & McMechan (1997; Jia & Wu 2009), one-way wavefield extrapolation in tilted coordinate systems (e.g. Higginbotham et al. 1985; Shan & Biondi 2008), and one-way wavefield extrapolation in Riemannian coordinate systems (e.g. Sava & Fomel 2005; Shragge 2006). The common idea behind all these methods is that they use turning, that is, upwardly refracted waves in areas with depth-dependent wave speed gradient to image the steeply dipping or overturned reflectors. The differences are in the choice of migration method, and the method to extrapolate the incident and reflected wavefields if a one-way wave equation migration method is chosen.

Pre-stack Kirchhoff depth migration of turning waves has been applied to imaging of steeply dipping reflectors in active fault zones. Louie et al. (1988) applied pre-stack Kirchhoff migration with refracted reflections to the data collected in the Parkfield area of the SAF by the Consortium for Continental Reflection Profiling (CORP) survey in 1977. They located and interpreted several dipping reflectors on both sides of the SAF trace, however they did not succeed in locating the vertical SAF surface. In another study by Louie & Qin (1991) this method was used to image the east branch of the Garlock Fault in California, dipping at 45°.

Hole et al. (2001) used Kirchhoff pre-stack depth migration on the data acquired in a 2-D survey, dubbed PSINE (Parkfield Seismic Imaging Ninety-Eight), which consisted of a 5 km line across the SAF trace. They imaged reflectors in the Parkfield area of the SAF to within about 1 km depth. They located a reflector dipping southwest from the surface trace of SAF and a reflector extending down from approximately 500 m below sea level directly beneath the trace of the Gold Hill Fault. Bleibinhaus et al. (2001) used Kirchhoff pre-stack depth migration to image steeply dipping reflectors in the subsurface up to 6 km in depth, and horizontally to moderately dipping reflectors in the deep subsurface (≈8–25 km), using the data from a 46-km-long active survey line extending across the SAF trace.

Another study that used Kirchhoff migration with data from both passive and active sources has been accomplished by Chavarria et al. (2003) to image reflectors in the SAF area near Parkfield. These authors migrated the secondary signals that appeared below the SAF trace, as well as a dipping reflector that they interpreted to be the fault known to intersect the SAFOD pilot borehole, and two other reflectors on the southwest side of the fault.

Migration is typically performed with active source data recorded at the surface or in boreholes. Migration with turning waves often requires large offsets between the source–receiver pairs and the reflector, especially when the depth-dependent wave speed gradient is not very large. This is often the case with crystalline basement rocks such as the Salinian granite and Franciscan rocks on the two sides of the SAF (Stradtlander & Brown 1997). Therefore the depth to which the methods based on turning waves can penetrate is limited by the wave speed gradient and the offset.

For vertical faults, fault zone head waves can provide a simple high-resolution tool to image the wave speed contrast across the fault surface (Lewis et al. 2007; Zhao et al. 2010).

With this review as background we now describe the setting that drives the work reported in this paper. In the areas of major faulting, for example, in the Parkfield area of the SAF, records of local microearthquakes are abundant. The presence of a sharp material property contrast across most active faults makes imaging with reflections of body waves propagating from earthquake sources a natural choice. Development of traveltimes methods in recent years that can invert simultaneously for the wave speed model and the hypocentre locations (Thurber & Rabinovitz 2000), as well as methods to obtain the source time function from the data (e.g. Pratt 1999; Roecker et al. 2010), makes it possible to image the fault location using seismic recordings from earthquake, that is, passive source data.

1.2 Introduction to our method for imaging with earthquake sources

The main new feature of the method proposed in this paper is that the earthquake sources are mostly located very close to the fault surface. The implications are the following: (1) the direct and reflected waves arrive almost simultaneously at the receivers, and they are not well separated, and (2) most of the reflected waves are reflected above the critical angle. Post-critically reflected waves undergo a frequency-dependent phase shift, which demonstrates itself in the time-dependent signal as a distortion in the waveform. This phenomenon is well studied for the cases of plane and spherical waves (e.g. Brekhovskikh 1960; Pujol 2003; Aki & Richards 2009). The distorted reflected signal does not correlate well with the incident signal. Our numerical experiments with synthetic data show that when the reflected data are not corrected to remove the post-critical phase shift in our method, the reconstructed fault appears shifted into the low wave speed region with respect to the true fault location. This discrepancy increases with the increase in the incidence angle.

The problem of post-critical incidence arises in at least two other seismic imaging applications, where wide angle reflection data are used: (1) shallow overburden-bedrock interface imaging, where wide angle surveys are often done to reduce the noise in the data, and (2) cross-well exploration. Pullan & Hunter (1985) investigated the effect of the post-critical incidence on the amplitude and phase of the reflected signal in the context of the ‘optimum window reflection’ technique (Hunter et al. 1984) for shallow overburden–bedrock interface imaging. They used synthetic seismograms computed in various two-layer media, and field seismograms collected in Winkler, Manitoba, to show that the waveform of the reflected wave can undergo a significant distortion when reflected at angles above the critical. Byun & Rector (1999) investigated the effects of post-critical reflection in cross-well imaging. They found that uncorrected post-critical reflections led to
cancellations of reflected signal during stacking and degraded the quality of stacked data. To the best of our knowledge no systematic theory-based methods have been proposed to correct for the waveform distortion in the post-critically reflected data.

In the remaining sections of this paper we address the separation of the direct and reflected arrivals, and the post-critical phase-shift removal, in more detail. We propose a method that incorporates the use of modelled incident (direct) wave both to remove the direct arrival from the data and correct for the phase shift in reflections above the critical angle. Our investigation is in the 2-D acoustic case. That is we assume that wave propagation is governed by the acoustic wave equation:

\[
\nabla \cdot \left( \frac{1}{\rho} \nabla u(x, z, t) \right) = \frac{1}{c^2} \frac{\partial^2 u(x, z, t)}{\partial t^2} + \delta(x - x_0)\delta(z - z_0)f(t),
\]

with the initial conditions

\[
u(x, z, 0) = 0, \quad \left. \frac{\partial u(x, z, t)}{\partial t} \right|_{t=0} = 0, \tag{2}
\]

where the horizontal coordinate \(x\) is directed transversely to the fault, depth \(z\) is downwards, \(u(x, z, t)\) is the total field, \(\rho\) is density, \(c\) is wave speed, \((x_0, z_0)\) are the source coordinates and \(f(t)\) is the source wavelet. We ignore the fact that the \(P-P\) reflection coefficient in the acoustic and elastic media is not the same. We also ignore the anisotropic radiation pattern of the earthquake source, and assume that the source is spatially isotropic. In the numerical examples we also assume that density is constant throughout the medium. Adaptation of our method to more realistic media governed by the linear equations of elasticity remains a future goal.

Our post-critical phase-shift removal algorithm is based on high-frequency asymptotic forms of the incident and reflected acoustic waves from a point source in two dimensions. To derive such approximations, the problem of reflection of circular waves from a planar interface in a piecewise homogeneous medium is considered theoretically. For the incident wave in such a medium the asymptotic form is well known. For the reflected wave the asymptotic form is derived from its integral representation in terms of plane waves. The accuracy of the phase-shift correction computed by our algorithm depends on the accuracy of the approximations of the incident and reflected waves by their asymptotic forms. To assess the latter accuracy, quantitative bounds on the remaining terms in the asymptotic expansions have been established.

Our method uses a few features of reverse time migration. Namely, we backpropagate the time-reversed records of the reflected wave recorded at the receivers on the surface of the earth, into a background medium, using the full acoustic wave equation. While here we demonstrate our method with synthetic data, the long-range goal is for the background medium to be constructed from the available wave speed models of the region of interest. The second similarity is the use of the zero-lag correlation with the incident field. We compute the incident field also in the background medium and backpropagate it from virtual receivers on the opposite side of the fault. The third common feature is the stacking of the zero-lag correlations from several sources.

The remainder of the paper is organized as follows. In Section 2 we describe our imaging method: removal of the direct arrival (Section 2.1); theoretical results necessary for the post-critical phase-shift removal (Section 2.2); our algorithm to remove the phase shift from the reflected data (Section 2.3); and the details of our imaging method (Section 2.4). Section 3 contains the details of our forward modelling numerical examples. Section 4 contains numerical examples.

2 Outline of the fault imaging method when reflections can occur above the critical angle

The set up for the fault reconstruction problem is shown in Fig. 1. \(c_1, \rho_1, c_2\) and \(\rho_2\) are slowly varying wave speed and density profiles on the two sides of the fault. It is assumed that earthquakes primarily occur on the side of the fault where the wave speed is lower, which we denote \(c_1\), that is in general \(c_1 < c_2\). We also assume that density is correlated with the wave speed, that is, \(\rho_1 \leq \rho_2\). The recorded data are acoustic pressure at the surface.

We assume that the following data are available for our problem: (1) an approximate wave speed model on the slower side of the fault, obtained, for example, from traveltime tomography, (2) approximate hypocentre locations, (3) an approximate source wavelet for each hypocentre and (4) the fault trace on the surface. From the surface trace of the fault we identify the receivers on the reflection side of the fault.

2.1 Removal of the direct arrival on the reflection side of the fault

We illustrate removal of the direct arrival from the total recorded wave on the medium shown in Fig. 1. The recorded data \(u(x, z, t)\) contain both the direct arrival and the reflected wave on the reflection side of the fault.

The direct arrival \(u^{inc}(x, z, t)\) is computed numerically in the background medium, which is constructed for the whole imaging region by extending the wave speed \(c_1\) and density \(\rho_1\) from the slower side throughout the model. The resulting medium contains no fault and approximates well the wave speed and density on the reflection side of the fault. The incident wave is propagated numerically from the known source and saved at the receivers on the reflection side of the fault. We subtract the calculated direct arrival from the data, so that only the reflected wave, \(u^{ref}(x, z, t)\), is retained:

\[
u(x, z, t) - u^{inc}(x, z, t) = u^{ref}(x, z, t).
\]

To illustrate the need for the subtraction of the incident wave, Figs 2(a)-(c) show examples of the modelled recorded signal on the reflection side of the fault, the corresponding direct arrival and the reflected wave obtained by subtraction of the direct arrival from the recorded signal, for the true medium shown in Fig. 1. In this calculation \(c_1 = 4\) km s\(^{-1}\), \(c_2 = 6\) km s\(^{-1}\) and \(\rho = \) constant. The source is located in the low speed medium 0.11 km from the fault, at the depth 4.1 km below the surface; the receiver is located at the surface, 0.6 km from the fault. The source wavelet is the Ricker

![Figure 1. Piecewise homogeneous true medium with straight vertical fault; receivers located on the reflection side of the fault are marked by Δ. \(R_0\) and \(R_0\) are the distances travelled by the incident and reflected waves, and \(\theta_0\) is the angle of specular reflection.](https://doi.org/10.1093/gji/ggq036)
wavelet with peak frequency 9 Hz. The critical angle of reflection in this calculation is 0.72973 ± 0.23π. The specular angle of reflection is 0.44π, thus the reflection is post-critical. The reflected wave undergoes a phase shift of ≈0.8π, so that its shape is distorted.

Note that the low amplitude break with reversed polarity, which appears before time ≈1 s on the seismograms in Figs 2(a) and (c), is the head wave [or lateral wave in the terminology of Brekhovskikh (1960)]. It is shown in Zheglova (2010) that in the high-frequency asymptotics the amplitude of the head wave has the same order of magnitude as the error in the asymptotic representation of the reflected wave, therefore in the estimates presented in the next section the head wave is included in the error term.

2.2 Estimates needed for post-critical phase-shift removal

In this section we give the leading-order terms and the error estimates for the asymptotic expansions of the incident and reflected waves from a point source in two dimensions. In this paper we only present the main results; for the proofs and precise error bounds the reader is referred to Zheglova (2010). The theoretical results given later were obtained by considering wave propagation in an infinite medium consisting of two homogeneous half-planes separated by the straight vertical fault (Fig. 1). The analysis is performed in the frequency domain. The resulting estimates are valid for high frequencies, that is when the wavelength λ on the reflection side of the fault is much smaller than the distance between the source and receiver, R0, leading to

\[ k_1 R_0 \gg 1, \]

where \( k_1 \) is the wavenumber of the slow medium. Additionally we use the following assumptions:

(i) \( c = \begin{cases} c_1, & x > 0 \\ c_2, & x < 0 \end{cases}, \quad \rho = \begin{cases} \rho_1, & x > 0 \\ \rho_2, & x < 0 \end{cases} \) such that \( c_1 < c_2, \rho_1 \leq \rho_2; \)

(ii) the index of refraction, \( n = c_1/c_2 \) is bounded away from 0 and 1: \( 0 < n_s \leq n \leq n^* < 1 \) for some \( n_s, n^* \);

(iii) \( 0 \leq \theta_0 \leq \pi/2 \), i.e. all incidence angles including normal and grazing are allowed;

(iv) the source position is \((x_0, z_0)\), where \( x_0 > 0, z_0 > 0 \), that is, the source is located in the slow medium;

(v) the receiver position is \((x, z)\) where \( x > 0, z < z_0 \), that is, the receiver is located in the slow medium above the source.

**Theorem 1.** Under assumptions (i)–(v) for \( \theta_0 < \theta_c \), the reflected wave in the frequency domain is given by

\[
\hat{u}^\text{ref}(x, z, \omega) = V(\theta_0) \sqrt{\frac{2\pi}{k_1 R_1}} e^{ik_1 R_1} \hat{f}(\omega) + \frac{E_1}{k_1 R_1},
\]

where

\[
\theta_0 \text{ is the angle of specular reflection}; \\
\theta_c = \arcsin n \text{ is the critical angle}; \\
R_1 = \sqrt{(x + x_0)^2 + (z - z_0)^2} \text{ is the distance travelled by the reflected wave}; \\
V(\theta_0) \text{ is the reflection coefficient given by}
\]

\[
V(\theta_0) = \frac{\rho_2/\rho_1 \cos \theta_0 - \sqrt{n^2 - \sin^2 \theta_0}}{\rho_2/\rho_1 \cos \theta_0 + \sqrt{n^2 - \sin^2 \theta_0}}
\]

where \( 0 < V(\theta_0) < 1 \);

\( E_1 \) is an order 1 error term.

**Theorem 2.** Under assumptions (i)–(v) for \( \theta_0 > \theta_c \), the reflected wave in the frequency domain is given by

\[
\hat{u}^\text{ref}(x, z, \omega) = e^{-\text{sgn}\omega} \sqrt{\frac{2\pi}{k_1 R_1}} e^{i(k_1 R_1 + z)} \hat{f}(\omega) + \frac{E_1}{k_1 R_1},
\]

where

\[
e^{-\text{sgn}\omega} \text{ is the reflection coefficient computed by formula (4) in which } \phi \text{ is given by}
\]

\[
\phi = 2 \arctan \left( \frac{\sqrt{n^2 - \sin^2 \theta_0}}{\rho_2/\rho_1 \cos \theta_0} \right),
\]

and \( \text{sgn}\omega \) is the sign function defined by

\[
\text{sgn}\omega = \begin{cases} 1, & \omega > 0 \\ -1, & \omega < 0 \end{cases};
\]

\( E_2 \) is an order 1 error term.

**Theorem 3.** Under assumptions (i)–(v) for \( \theta_0 = \theta_c \), the reflected wave in the frequency domain is given by

\[
\hat{u}^\text{ref}(x, z, \omega) = \sqrt{\frac{2\pi}{k_1 R_1}} e^{i(k_1 R_1 + z)} \hat{f}(\omega) + \frac{E_3}{(k_1 R_1)^{3/2}},
\]

where \( E_3 \) is an order 1 error term.

Note that for \( \theta_0 = \theta_c \), the reflection coefficient is 1.

The incident wave in the frequency domain in a homogeneous 2-D medium is given by

\[
\hat{u}^\text{inc} = \hat{f}(\omega) \pi H_0^{(1)}(k_1 R_0),
\]

and the leading-order term and the error order estimate for the incident wave is

\[
\hat{u}^\text{inc} = \hat{f}(\omega) \sqrt{\frac{2\pi}{k_1 R_0}} e^{ik_1 R_0} e^{i\pi/2} + \frac{E_4}{(k_1 R_0)^{3/2}},
\]

where \( E_4 \) is an order 1 error term. This result immediately follows from a well known asymptotic approximation of the Hankel function \( H_0^{(1)} \) for large arguments. We give it here for completeness.

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The results in Theorems 1–3 were obtained following the general approach employed by Aki & Richards (2009) and Brekhovskikh (1960) where similar results for 3-D spherical waves are derived.

The key features in the results above are as follows:

(i) The leading terms in the above-mentioned representations of the incident and reflected waves are $O \left( \frac{1}{\epsilon_i^N} \right)$.

(ii) The error term of the incident wave has the order $O \left( \frac{1}{\epsilon_i^N} \right)$.

(iii) The error term of the reflected wave has the order $O \left( \frac{1}{\epsilon_i^N} \right)$ when $\theta_0 \neq \theta_c$, and order $O \left( \frac{1}{\epsilon_i^N \epsilon_c^N} \right)$ when $\theta_0 = \theta_c$.

(iv) For $\theta_0 \neq \theta_c$ the constants $E_1$ and $E_2$ in the error terms contain factors of the form $\sin^2 \theta_0 - \mu^2 / \epsilon_c^2$ and $(\sin (\theta_0 - \theta_c))^2$ that approach $\infty$ as $\theta_0 \rightarrow \theta_c$. We conjecture without proof that these factors give rise to the lower order error estimate when $\theta_0 = \theta_c$.

In other words, as $\theta_0 \rightarrow \theta_c$, the order of the error approaches $O \left( \frac{1}{\epsilon_i^N \epsilon_c^N} \right)$.

2.3 Removal of the distortion due to reflection at angles above the critical angle

We use representations (3), (5), (7) and (8) to derive the phase-shift correction algorithm. The corresponding representations in the time domain of the reflected wave take the following form:

$$u^{\text{ref}}(x, z, t) = \begin{cases} \sqrt{\frac{2}{k_1}} \mathcal{H}(\theta_0) \sin \left( x, z, t - \frac{R_i - R_0}{c_1} \right) + \text{error}, & \theta_0 \leq \theta_c \\ \sqrt{\frac{2}{k_1}} \cos \phi \rho \sin^2 \left( x, z, t - \frac{R_i - R_0}{c_1} \right) + \sin \phi \mathcal{H} \left( x, z, t - \frac{R_i - R_0}{c_1} \right) + \text{error}, & \theta_0 > \theta_c, \end{cases}$$

where $\mathcal{H}$ is the Hilbert transform defined as

$$\mathcal{H} \rho(t) = P. V. \int_{-\infty}^{\infty} \frac{\rho(s)}{s-t} \, ds.$$ 

In this integral $P. V.$ denotes Cauchy principal value.

From the frequency domain representation (5) it is clear that for $\theta_0 > \theta_c$, the leading-order term for $u^{\text{ref}}$ has a frequency-dependent phase shift $-\phi \text{ sgn} \omega$, which translates into a distortion of the reflected waveform in the time domain as shown in (9). Our method is to make a change in $u^{\text{ref}}$ in formula (5) at each frequency to obtain a new function $\tilde{u}^{\text{ref}}(x, z, \omega)$:

$$\tilde{u}^{\text{ref}}(x, z, \omega) = e^{i \phi \text{ sgn} \omega} u^{\text{ref}}(x, z, \omega),$$

(10)

followed by taking the inverse Fourier transform of the new functions $\tilde{u}^{\text{ref}}(x, z, \omega)$ to obtain the undistorted time trace $\tilde{u}^{\text{ref}}(x, z, t)$:

$$\tilde{u}^{\text{ref}}(x, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{u}^{\text{ref}}(x, z, \omega) e^{-i \omega t} \, d\omega.$$ 

This is done at each receiver location on the reflection side of the fault. From (5) and (10) in the frequency domain above the critical angle the phase-corrected reflected wave becomes:

$$\tilde{u}^{\text{ref}}(x, z, \omega) = \frac{2\pi}{k_1 R_0} u^{\text{inc}}(x, z, \frac{1}{2} t) \tilde{f}(\omega) + \text{error},$$

so that in the time domain the recovered undistorted reflected wave takes the form of a delayed scaled incident wave:

$$\tilde{u}^{\text{ref}}(x, z, t) = \frac{R_0}{R_i} u^{\text{inc}} \left( x, z, t - \frac{R_i - R_0}{c_1} \right) + \text{error}.$$

It is this wave that we utilize to backpropagate from all the receivers on the reflection side of the fault. In the case of plane waves, or as $\omega \rightarrow \infty$, if the recorded wave is from subcritical reflection our algorithm produces $\phi = 0$ and no phase correction is required.

To find the phase correction $\phi$ we again utilize the leading-order terms of representations (3), (5), (7) and (8). We take the quotient of the leading-order terms of $u^{\text{ref}}$ and $u^{\text{inc}}$ to obtain

$$\frac{\tilde{u}^{\text{ref}}(x, z, \omega)}{u^{\text{inc}}(x, z, \omega)} = \begin{cases} V(\theta_0) \sqrt{\frac{x}{x_i}} e^{i \phi} e^{\frac{1}{2} \pi}, & \theta_0 \leq \theta_c \\ e^{-i \phi \text{ sgn} \omega} \sqrt{\frac{x}{x_i}} e^{i \phi} e^{\frac{1}{2} \pi}, & \theta_0 > \theta_c. \end{cases}$$

Now we take $\text{Im}(\ln(\tilde{u}^{\text{ref}} / u^{\text{inc}}))$ to obtain

$$D(\omega) = \text{Im} \left[ \frac{\tilde{u}^{\text{ref}}(x, z, \omega)}{u^{\text{inc}}(x, z, \omega)} \right] = \begin{cases} k_1 (R_1 - R_0), & \theta_0 \leq \theta_c \\ k_1 (R_1 - R_0) - \phi \text{ sgn} \omega, & \theta_0 > \theta_c, \end{cases}$$

where the phase of the linear logarithm is unwrapped to obtain a continuous function $D(\omega)$. As the dependence of $D$ on $\phi$ is linear, the above expression defines a system of linear equations for $\phi$ in which each equation corresponds to a certain frequency. This very simple formula is marred slightly by the fact that $R_1$ is unknown. Since however $\omega R_i / c_1$ appears in each equation, one can select $\phi$ that minimizes the following functional which is independent of $R_i / c_1$:

$$\sum_{j=2}^{N} \left( D(\omega_j) + \phi \right) \omega_j - (D(\omega_j) + \phi)^2, \quad (11)$$

where $\omega_j, j = 1..N$ are positive, and represent frequencies for which the corresponding wavenumbers $k_j$ are sufficiently small; $k_j \ll k_0$.

We use the leading-order terms of $u^{\text{ref}}(x, z, \omega)$ and $\tilde{u}^{\text{ref}}(x, z, \omega)$ in the derivation of formula (11) for finding $\phi$, however in calculations with synthetic or measured data the full Fourier transform of the reflected wave $u^{\text{ref}}$, and the computed incident wave $u^{\text{inc}}$ is utilized.

2.4 The imaging method

As described earlier, our fault identification algorithm involves removal of the direct arrival from the recorded wave on the reflection side of the fault and phase correction of the reflected wave. Additionally, we note that when we compute the direct arrival we save it at locations on the transmission side of the fault. In this study we choose these virtual receiver locations to be symmetric to the receivers on the reflection side of the fault with respect to the surface fault trace. The imaging method consists of the following steps:

(i) Backpropagate the time-reversed phase-corrected reflected wave $\tilde{u}^{\text{ref}}$ from the receivers on the reflection side of the fault in the background medium.

(ii) Backpropagate the time-reversed direct arrival from the virtual receiver locations on the transmission side of the fault in the background medium.

(iii) Compute the zero-lag correlation in time of the backpropagated fields determined in steps (i) and (ii) at each point $(x_i, z_i)$ of the computational domain according to the formula:

$$C(x_i, z_i) = \sum_{n=1}^{N_{\text{max}}} \tilde{u}^{\text{ref}}_{\text{bp}}(x_i, z_i, t_n) u^{\text{inc}}_{\text{bp}}(x_i, z_i, t_n). \quad (12)$$
(iv) For each depth $z_j$, find the range position $x_j^*$ where the zero-lag correlation from step (iii) is maximized:

$$C(x_j^*, z_j) = \max_i C(x_i, z_j).$$

The set $\{(x_j^*, z_j), j = 1...j_{\text{max}}\}$ is the recovered fault location.

(v) Zero-lag correlations from several sources are summed to increase the accuracy of the reconstruction:

$$C_{\text{sum}}(x_i, z_j) = \sum_{s=1}^{j_{\text{max}}} C_s(x_i, z_j),$$

where $s$ is the source number. Then step (iv) is applied to $C_{\text{sum}}(x_i, z_j)$, that is, the set $\{(x_j^*, z_j), j = 1...j_{\text{max}}\}$ is the recovered fault location, where

$$C_{\text{sum}}(x_j^*, z_j) = \max_i C_{\text{sum}}(x_i, z_j).$$

### 3 Forward Modelling Method

Calculation of the incident wave and steps (i) and (ii) of our fault imaging method require solution of the forward acoustic wave equation in the background medium; thus it needs to be solved three times for each source. In addition, to generate synthetic data we solve the forward wave propagation problem (1)–(2) in the true medium.

This section describes the computational domain, the numerical scheme, our implementation of the outgoing boundary conditions, mesh parameters and interpolation procedures that we use to solve these four forward problems. We also explain our method for choosing frequencies that we use in the calculation of the phase correction $\phi$.

To solve the above wave propagation problems, we set in all examples the density $\rho$ to be constant throughout the computational domain. The source is modeled as the Ricker wavelet point source with the peak frequency of 9 Hz. In the case of the backpropagation problem the maximum absolute frequency content of the computed incident wave: $\omega > \pi f_{\text{peak}}$, and

$$|\hat{u}^{\text{inc}}(x, z, \omega)| \geq 0.1 \max_{\omega < \omega_{\text{peak}}} |\hat{u}^{\text{inc}}(x, z, \omega)|.$$

The above conditions provide $\sim 40$ frequencies for the chosen record length of 3 s with time sampling interval of $\Delta t = 0.001$ s. The above specified range of frequencies has been determined by visually examining the graph of $D(\omega)$ and making sure that $D$ depends approximately linearly on $\omega$. While we did not do it in this study, the choice of suitable frequencies can be automated with an optimization procedure.

### 4 Numerical Results

In this section we present numerical examples. First we present examples of fault reconstructions without noise. Then we present examples of fault reconstructions without noise.
4.1 Numerical results without noise

Here we present three numerical examples. In Example 1 we perform a reconstruction of a straight vertical fault separating regions of constant wave speed, as shown in Fig. 4(a). The wave speed in the low- and high-speed region is \( c_1 = 4 \text{ km s}^{-1} \) and \( c_2 = 6 \text{ km s}^{-1} \), respectively. The reconstruction is done with a single source located at \((-1.09, 3.5)\).

In Examples 2 and 3 we reconstruct the fault that shifts towards the low-speed region as it dips (Figs 4b and c), using data from 6 sources. The true medium in Example 2 is piecewise constant with the wave speed \( c_1 = 4 \text{ km s}^{-1} \) and \( c_2 = 6 \text{ km s}^{-1} \) in the low- and high-speed region, respectively. In Example 3 the high- and low-wave speed profiles are depth dependent and are shown in Fig. 4(d). The shape of the fault is described by the following smooth piecewise polynomial function:

\[
x(z) = \begin{cases} 
-1.2, & -1.6 \leq z \leq 2.5 \\
33.35 - 36.28z + 12.44z^2 - 1.38z^3, & 2.5 \leq z \leq 3.5 \\
-0.51, & 3.5 \leq z \leq 7.2. 
\end{cases}
\]

The source locations for Examples 2 and 3 are listed in Table 1.

Fig. 5 shows the image of the zero-lag correlation and the recovered fault location for Example 1. We infer from Fig. 5 that our method can produce a very good recovery in this benchmark example with data from a single source. Since the shape of the fault is simple, it is possible to calculate theoretical value of \( \phi \) predicted by geometric optics, formula (6), and compare it with the value of \( \phi \) obtained in our phase-shift removal algorithm, that is, by minimizing functional (11). Table 2 shows both values of \( \phi \) for each receiver. Reflections arriving at receivers 1–9 are post-critical. At receiver 10 the value \( \phi = 0 \), predicted by geometric optics, corresponds to subcritical reflection. The values of \( \phi \) computed by our algorithm are consistent with the values of \( \phi \) predicted by geometric optics at receivers 1–5, and get progressively less accurate at receivers 6–10, where the reflection angle gets within 14° of the critical angle. This is consistent with the results of Section 2.2, where we showed that the approximation of the circular reflected wave by its leading-order term is the least accurate near the critical angle.

Fig. 6 shows the image of the stacked zero-lag correlations from all sources and fault reconstruction for Example 2. For comparison, Figs 7 and 8 show reconstructions with the data from single sources 1 and 5. Source 1 is shallow, therefore the upper segment of the fault is recovered very well. With the deeper source 5 the lower segment of the fault above source is recovered best. We note that a significantly better image of the whole fault is obtained from the zero-lag correlation stack (Fig. 6). We also note that we obtained a good recovery with a small number of sources.

Fig. 9 shows the image of the zero-lag correlation stack and fault reconstruction for Example 3. We do not include any reconstructions of the fault with single sources for this example, but they can be found in Zheglova (2010).

<table>
<thead>
<tr>
<th>Source</th>
<th>Location ((x_0, z_0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((-0.4, 2.6))</td>
</tr>
<tr>
<td>2</td>
<td>((-0.4, 3.5))</td>
</tr>
<tr>
<td>3</td>
<td>((-0.4, 4))</td>
</tr>
<tr>
<td>4</td>
<td>((-0.4, 4.5))</td>
</tr>
<tr>
<td>5</td>
<td>((-0.4, 5))</td>
</tr>
<tr>
<td>6</td>
<td>((-0.4, 6.5))</td>
</tr>
</tbody>
</table>

Figure 4. True media: (a) Example 1, (b) Example 2 and (c) Example 3. (d) Wave speed profiles in Example 3.
Table 2. Values of $\phi$ calculated by our distortion removal algorithm and predicted by the geometric optics (formula 6).

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Coordinates</th>
<th>$\theta_r$</th>
<th>Calculated $\phi$</th>
<th>Predicted $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-1.1, -0.6)</td>
<td>1.5194</td>
<td>2.8788</td>
<td>3.0036</td>
</tr>
<tr>
<td>2</td>
<td>(-0.6, -0.6)</td>
<td>1.3991</td>
<td>2.5206</td>
<td>2.6790</td>
</tr>
<tr>
<td>3</td>
<td>(-0.1, -0.6)</td>
<td>1.2836</td>
<td>2.3006</td>
<td>2.3620</td>
</tr>
<tr>
<td>4</td>
<td>(0.4, -0.6)</td>
<td>1.1755</td>
<td>1.9894</td>
<td>2.0556</td>
</tr>
<tr>
<td>5</td>
<td>(0.9, -0.6)</td>
<td>1.0762</td>
<td>1.5415</td>
<td>1.7609</td>
</tr>
<tr>
<td>6</td>
<td>(1.4, -0.6)</td>
<td>0.98659</td>
<td>0.82141</td>
<td>1.4755</td>
</tr>
<tr>
<td>7</td>
<td>(1.9, -0.6)</td>
<td>0.90641</td>
<td>0.50629</td>
<td>1.1933</td>
</tr>
<tr>
<td>8</td>
<td>(2.4, -0.6)</td>
<td>0.83517</td>
<td>0.37712</td>
<td>0.90062</td>
</tr>
<tr>
<td>9</td>
<td>(2.9, -0.6)</td>
<td>0.77205</td>
<td>0.33110</td>
<td>0.55852</td>
</tr>
<tr>
<td>10</td>
<td>(3.4, -0.6)</td>
<td>0.71617</td>
<td>0.36885</td>
<td>0</td>
</tr>
</tbody>
</table>

$\theta_r$, the angle of reflection; calculated $\phi$, calculated by our algorithm using functional (11); predicted $\phi$, predicted by geometric optics, obtained from formula (6).

The straight parts of the fault in Figs 6 and 9 are imaged better than the shifting parts. In Fig. 6 the location of the shift is shown correctly and the reconstruction gives a clear indication of its shape, but it does not render correctly the incline of the shift. In Fig. 9 the shape of the shift is rendered correctly, but the steepness of the shift and its location are not reconstructed quite correctly. One reason for this is that our imaging method is naturally better suited to image straight vertical reflectors, as it takes advantage of the symmetry of the incidence and reflection angles that straight faults provide.

The other reason is that reflections from non-straight interfaces can include multiply scattered waves, multiple reflected arrivals from several parts of the fault arriving at the same receiver and refractions. Our algorithm is designed to make good use of singly reflected waves but it is not well suited to take into account multiples and waves refracted across the fault. Also, our algorithm does not distinguish between several reflected arrivals and treats them all as a single reflected wave. Typically the waves that are singly reflected dominate in amplitude, so that they have dominant weight in the calculation of $\phi$, and produce the largest values of the zero-lag correlation. Thus the straight parts of the fault from which waves are singly reflected are imaged better.

4.2 Sensitivity of the method to uncertainty in source location

In practice exact knowledge of the earthquake source locations is not achievable. To determine sensitivity of the proposed method to errors in the source location we conducted a series of experiments where modelled incident waves from randomly perturbed sources were used to recover the fault with our method. In these tests we used the fault configuration of Fig. 4(a). We assumed 20 sources located along two straight lines at $x = -1.09$ km and $x = -0.8$ km. The depths of the sources ranged from 2 to 6.5 km with a spacing of 500 m. We assumed a normal distribution of the absolute source
Figure 7. Example 2, source 1: (a) image of the zero-lag correlation computed according to formula (12), step (iii) of our method; (b) reconstruction of the fault location with the imaging condition (13), step (iv). The source location is (−0.4, 2.6).

Figure 8. Example 2, source 5: (a) image of the zero-lag correlation computed according to formula (12), step (iii) of our method; (b) reconstruction of the fault location with the imaging condition (13), step (iv). The source location is (−0.4, 5).

Figure 9. Example 3: (a) image of the sum of the zero-lag correlations from sources 1–6 computed according to formula (14), step (v) of our imaging method; (b) reconstruction of the fault location with imaging condition (15), step (v). The source locations are given in Table 1.

location perturbation with an SD ranging from 100 to 500 m, and a uniform distribution of the direction of perturbation. The results for SDs of 100, 300 and 500 m are shown in Figs 10–12 (the rms deviations from the true sources are, respectively, 107.3, 246.2 and 500.3 m). We found that with the current range of frequencies in the source the method produces good recoveries with SD of up to 150 m, which is about 2 per cent of the depth of the deepest source. Above this margin the method is somewhat sensitive to how many sources are actually well located. For example we obtained a good fault recovery with SD of absolute perturbation of 300 m, and a rather unacceptable recovery with SD of 500 m. We find that our method is more sensitive to perturbation when we cancel the direct arrival and we compute the phase correction $\phi$, and less sensitive in the backpropagation step. We obtained improvements in recovery with 500 m SD by using the incident wave from exact sources to cancel direct arrivals and to compute $\phi$ (Fig. 13). This suggests that robustness of our method can be improved by increasing the number of sources, and by elimination of the modelled incident wave from the data processing steps of our method. We are currently exploring the possibility of achieving separation of the direct and the reflected arrivals and computation of $\phi$ without the use of a modelled incident wave. We will report on this in a future paper.

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4.3 Sensitivity to uncertainty in model parameters

In general the wave speed model recovered by tomography methods is not precise. The minimum structure regularization that is typically used in traveltime tomography algorithms produces smooth wave speed models and the resolution depends on ray coverage. To study the sensitivity of our method to uncertainty in the background model we added random variations to the models from Figs 4(a) and (c), and reconstructed the fault using the homogeneous and depth-dependent background media of Examples 1 and 3, respectively. The variations in the true wave speed had zero mean and a von Karman covariance function of order 0.3. Characteristic lengths in the range and depth directions were chosen to be 0.4 and 0.2 km, which is 90 per cent and 45 per cent of the peak wavelength. The rms wave speed variations of 3, 5, 7 and 10 per cent of the low wave speed for the model in Fig. 4(a) were chosen, and 1 and 3 per cent of the wave speed for the model in Fig. 4(b). In all tests we obtained good fault recoveries. The results for the two media with rms variations of 10 and 3 per cent, respectively, are shown in Figs 14 and 15.

4.4 The effect of the fault zone

In many cases faults represent a narrow zone of fractured or partially consolidated rocks at least at shallow depths. At deeper depths the
assumption of a sharp interface is well supported by observations (Ben-Zion & Sammis 2003). The presence of a fault zone may have significant effect on our method, due to the following factors:

(i) if the wave speed in the fault zone differs significantly from the surrounding rock it needs to be accounted for in the background model to ensure the correct speed of the backpropagated waves;
(ii) in the wide angle regime most of the energy is trapped inside the fault zone, and the waves propagating outside the fault zone are evanescent and
(iii) the walls of the fault zone produce multiple reflections.

We conducted a series of tests where a low-velocity fault zone was introduced into the true models of Figs 4(a) and (b). We used the wave speed of 3.2 km s$^{-1}$ in the fault zone and varied its width and shape. We found that our method works well with very narrow fault zones ($\sim 100$ m), and with relatively wide fault zones (over 1 km wide). In the latter case the fault zone wave speed is used as the background wave speed and multiple reflections from the fault zone boundaries are muted. We note that muting is possible in this case, since multiples are well separated from the incident wave and the primary reflections. Muting the multiples is especially effective when imaging deeper parts of the fault zone. One possible reason

Figure 13. Straight fault recovery with perturbed sources, when exact incident wave is used for direct arrival removal and computation of $\phi$: (a) zero-lag correlation stack; (b) reconstruction of the fault location. SD of the absolute source location perturbation is 500 m.

Figure 14. Straight fault recovery in the variable medium with rms wave speed variation of 10 per cent: (a) zero-lag correlation stack; (b) reconstruction of the fault location.

Figure 15. Shifting fault recovery in the variable depth-dependent medium with rms wave speed variation of 3 per cent: (a) zero-lag correlation stack; (b) reconstruction of the fault location.
currently our method is implemented in two dimensions and the wave propagation is assumed to be governed by the acoustic wave equation.

we tested our method on synthetic examples with noise-free data in piecewise homogeneous and depth-dependent media with two types of fault configurations. we also assessed numerically the sensitivity of our method to various kinds of noise in the data, and investigated to some degree performance of our method in the presence of a fault zone.

the questions that this study does not address and that we consider our future work directions include the following:

(1) elimination of the computed incident wave from the data processing steps of our method to improve robustness to uncertainty in the source location. this can be achieved by formulating the arrival separation and phase-shift correction steps as an optimization problem, where the incident and the reflected wave traveltime and the phase correction \( \phi \) are fitted to the recorded waveforms at each receiver, using our asymptotic formulae. it is a fully non-linear optimization problem with a highly oscillatory objective function, however the forward computation is fast, so that it can be handled efficiently by bayesian inversion combined with a global optimization engine. preliminary tests of this approach with synthetic data suggest that the problem is more sensitive to the incident arrival time than to the reflected arrival time and \( \phi \).

(2) development of a procedure to construct the background medium that accurately represents the true wave speed in the low-speed region from a reconstruction of the true wave speed that is not exact or too smoothed out in the vicinity of the fault, such as occurs in a traveltine tomography reconstruction.

(3) further investigation of our method in the presence of a fault zone.

(4) generalization of our method to 3-D elastic media and incorporation of the anisotropic source radiation pattern into the modeling process.

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