Nordic Hydrology 6, 1975, 189–206
Published by Munksgaard, Copenhagen, Denmark

ESTIMATION OF THE STORAGE COEFFICIENT OF THE CAMBRIAN SANDSTONE IN THE BILLINGEN – FALBYGDEN AREA, VÄSTERGÖTLAND, SWEDEN

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The storage coefficient for a confined aquifer in lower Cambrian sandstone has been calculated, partly from pumping tests and partly from correlations of water-level fluctuations with changes in atmospheric pressure. The different methods of calculation were in close agreement, although the values differed slightly depending on drawdown or recovery and on the horizontal distance to the border of the confining bed. In the pumping tests, the aquifer was undergoing conversion from confined to unconfined condition, which made it possible to calculate the specific yield of the sandstone too.

In order to evaluate the occurrence and availability of ground water in the Billingen – Falbygden area in the county of Västergötland, Sweden, (Fig. 1), the Geological Survey of Sweden (SGU) has performed an extensive hydrogeological investigation there. The investigation was commissioned by AB Atomenergi (the Atomic Energy Company of Sweden) and was directed by Åke Hörnsten, chief hydro-geologist at the Geological Survey of Sweden. An essential part of this investigation consisted of efforts to study the geohydrological parameters of the occurring aquifers.

The purpose of the present paper is to compare calculations of the storage

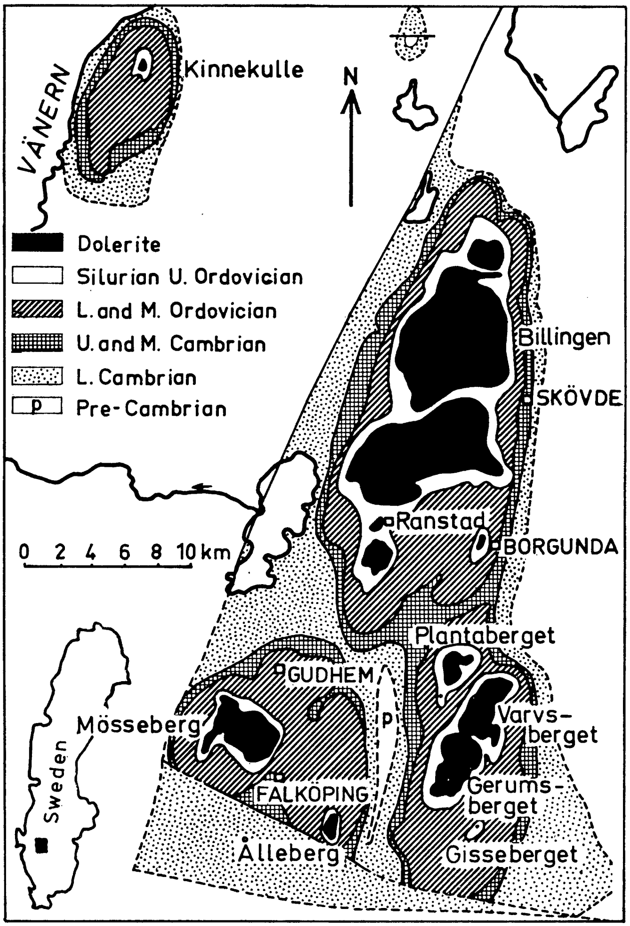

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Fig. 1.
Geographical map of the Billingen – Falbygden area (after Munthe 1905).

coefficient from pumping tests with calculations from barometric efficiency. The calculations refer to the confined aquifer in the Cambrian sandstone within the investigated area, and derive from data obtained from the two sites, Ranstad and Gudhem (see Fig. 1).

Table 1.
Geologic and hydrogeologic division of the sedimentary sequence of Billingen.

Geological Division	Thickness m	Physical Character	Hydrogeological Division	Thickness m
Permian-Carboniferous dolerite	45	Fine to coarse grained, columnar jointed	Hydrogeological properties uncertain	45
Silurian shale	11.2	Shales and mudstones with intercalations of shales	Aquifer	46.0
Upper Ordovician shale	33.6	Shales with scattered beds of limestones	Aquifer	46.0
Middle Ordovician limestone	29.3	Mudstones and marly limestones, layers of bentonite. Bedded limestones	Aquiclude	1.5
Lower Ordovician limestone	19.5	Bedded limestones, in the basal parts in some places clayey shales	Aquifer	46.1
Upper Cambrian alum-shale Middle Cambrian alum-shale	9.0 14.2	Shales developed as alum-shales with content of bitumenous limestone (stinkstone)	Aquiclude	23.2
Lower Cambrian sandstone	31.2	In the upper parts thick-banked and built up of almost pure quartz-sand. In the lower parts thin-banked and interbedded with clayey material	Aquifer	31.2
Pre-Cambrian gneiss		Medium to coarse grained gneiss	Aquiclude	

GEOLOGICAL AND HYDROGEOLOGICAL CONDITIONS

Geology

The investigated area is built up of sedimentary rocks of Cambro-Silurian age, which are partly covered by Post-Silurian dolerites (see Fig. 1). The Cambro-Silurian sequence of the Billingen-Falbygden area was first described by Linnarsson in 1869. Later Munthe (1905) and Westergårdh (1928) published regional descriptions of the stratigraphy and the areal extensions of the different layers and the tectonics. In 1960, Thorslund & Jaanusson gave an account of the geology and the stratigraphy of the sedimentary sequence.

The extension of the sedimentary rocks is shown in Fig. 1, and the stratigraphy is summarized in Table 1, left.

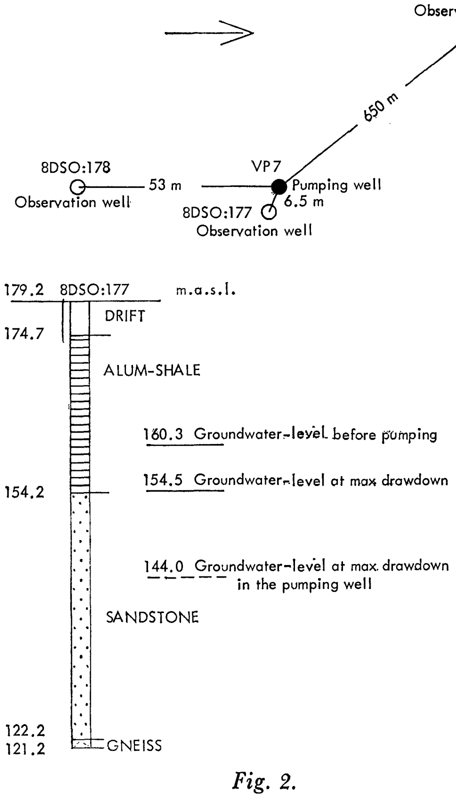

Fig. 2.

Situation map and geological profile at the pumping site at Ranstad.

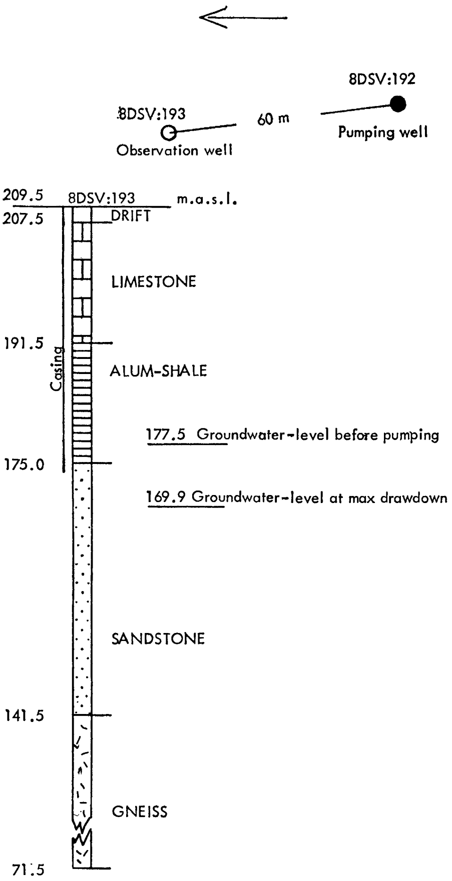

Fig. 3.
Situation map and geological profile of the pumping site at Gudhem.

Hydrogeology

From a hydrogeological point of view, another division of the sedimentary sequence may be distinguished. This division aims at a differentiation into more or less water-bearing units, aquifers vs. aquicludes (see Table 1). The water-bearing formation discussed in this paper comprises sandstone of Lower Cambrian (Table 1) which is fairly fine-grained and has an average porosity of about 10 %, determined in the laboratory on homogeneous specimens.

At the investigated sites the sandstone aquifer is confined between the underlying Pre-Cambrian gneisses and the overlying Cambrian alum-shale, both rocks being regarded as having low permeability compared with the sandstone.

The hydrogeological conditions at the investigated sites, Ranstad and Gudhem, are presented in Figs. 2 and 3.

PUMPING TESTS

General conditions

At Ranstad and Gudhem (see Fig. 1) pumping tests were made in order to estimate the geohydrological properties of the sandstone aquifer. The pumping tests were carried out with a constant pumping rate for two months and the recoveries were measured for the same length of time. Pumping rate, distances to observation wells and other relevant data for the tests are given in Table 2.

Analysis and results

The ground water in the sandstone is confined. A very short time after pumping started, the water level at the pumping wells had declined below the confining bed and the aquifer underwent conversion from confined to unconfined conditions. The dewatering after conversion presumably does not signi-

Table 2.

Pumping rate, distances from pumping well and max. drawdown for the pumping tests at Ranstad and Gudhem.

Site	Bore hole	Distance from pumping well m	Maximum drawdown m	Pumping rate m^3/sec.	Remarks
Ranstad	VP7	-	16.30	$7.5 \cdot 10^{-3}$	Pumping well
	8DSO:177	6.5	5.75		Observation well (weeping well)
	8DSO:178	53	4.70		Observation well
	VP5	650	0.80		Observation well
Gudhem	8DSV:192	–	not known	$8 \cdot 10^{-4}$	Pumping well
	8DSV:193	60	7.40		Observation well

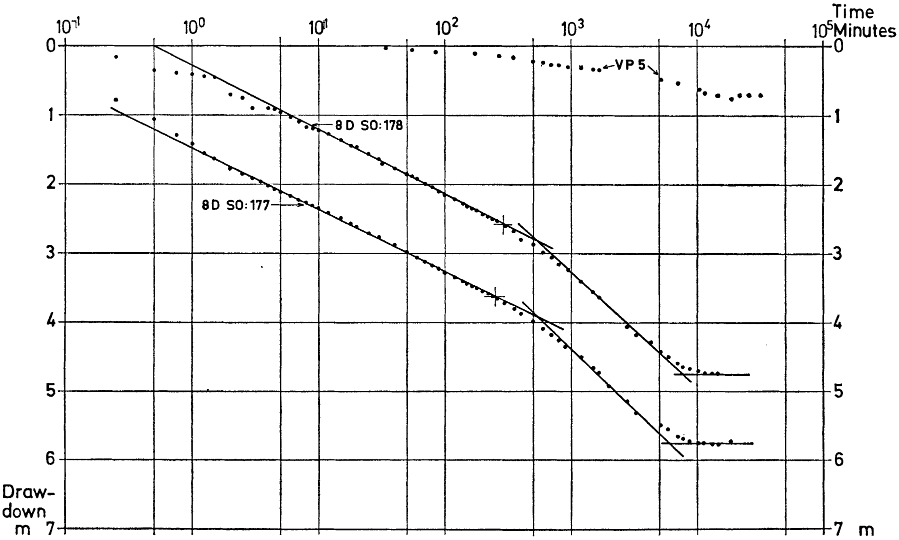

Fig. 4.

The drawdown vs. time in a semilogarithmical plot of the observation wells 8D SO:177, 8D SO:178 and VP 5 at Ranstad.

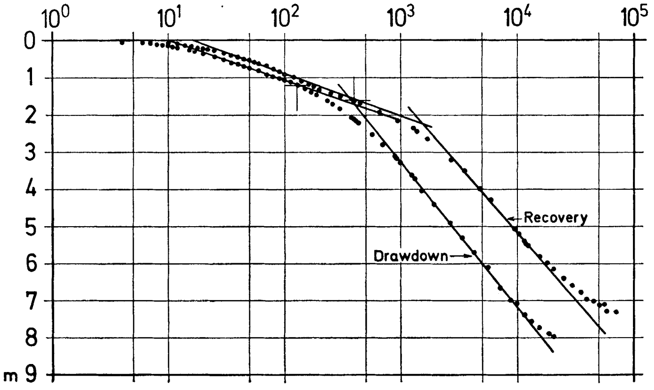

Fig. 5.

The drawdown and recovery vs. time in a semilogarithmical plot of the observation well 8D SV:193 at Gudhem.

ficantly reduce the transmissivity of the aquifer. The drawdowns obtained, corrected for the barometric efficiency in the observation wells, are shown in semilog plots in Fig. 4 for Ranstad and in Fig. 5 for Gudhem. These figures display marked changes in slopes, indicating the conversion from confined to unconfined conditions.

The drawdown in an aquifer undergoing conversion from confined to unconfined conditions is given by the following equations (Moench & Prickett 1972):

$$s_1 \equiv \frac{Q}{4\pi T} \exp\left[v\left(\frac{S_1}{S_2} - 1\right)\right] \cdot W(u_1) \qquad r > R \tag{1}$$

$$s_2 \equiv \frac{Q}{4\pi T} W(u_2, v) \qquad r < R \tag{2}$$

$$u_1 = \frac{r^2 \cdot S_1}{4Tt} \tag{3}$$

$$u_2 = \frac{r^2 \cdot S_2}{4Tt} \tag{4}$$

$$v \equiv \frac{R^2 \cdot S_2}{4Tt} \tag{5}$$

where

$s_1 \equiv$ drawdown below the initial water level during the confined conditions

$s_2 \equiv$ drawdown below the top of the aquifer during the unconfined conditions

$r =$ radial distance measured from the pumped well

$R \equiv$ radial distance to the point of conversion measured from the pumped well

$t \equiv$ time since pumping started

$Q \equiv$ pumpage from the well

$T =$ aquifer transmissivity

$S_1 \equiv$ storage coefficient under confined conditions when $r > R$

$S_2 =$ storage coefficient under confined conditions when $r < R$

$W(u_1) =$ well-function according to Theis (1935)

$W(u_2, v) \equiv$ well-function according to Moench & Prickett (1972)

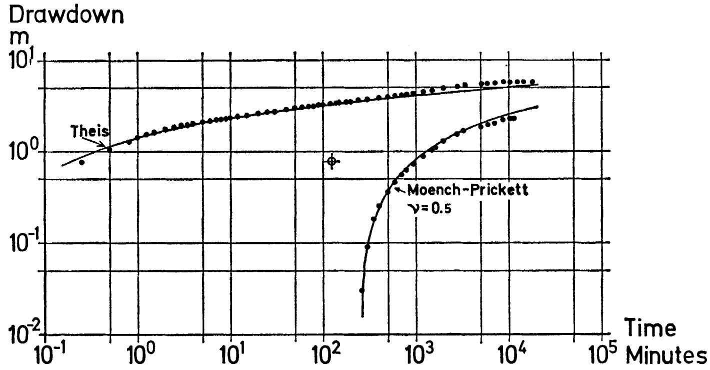

Fig. 6.

The drawdown vs. time in observation well 8D SO:177 at Ranstad compared with Theis type curve (confined conditions) and Moench & Prickett type curve (unconfined conditions).

The analysing method is also described by Moench & Prickett (1972). This method uses log-log plots of the drawdown vs. time and matching against type curves for the unconfined condition. The semilog plots are used to determine the time for the conversion from confined to unconfined condition. The drawdown vs. time in observation wells in log-log plots at Ranstad and Gudhem is shown in Figs. 6, 7, and 8. In these figures, the drawdown vs. time is also shown only for the unconfined condition with matching type-curve according to Moench & Prickett. Calculated values of transmissivity T and storage coefficients S_1 and S_2 are presented in Table 3.

Attempts have also been made to evaluate T, S_1, and S_2 from semilog plots, as in the method described by Jacob (1940). In estimating the transmissivity, T, the slope of the line in the semilog plot for the confined condition gives a value which differs from the real one by factor of exp $\left[v\,(\frac{S_1}{S_2}-1)\right]$. Since the ratio $\frac{S_1}{S_2}$ is smaller than 1, the factor can be set equal to e^{-v}. As v is constant for the given test condition, the transmissivity T can be calculated from the slope of the line after conversion to unconfined condition in accordance with Jacob (1940) by the following equation:

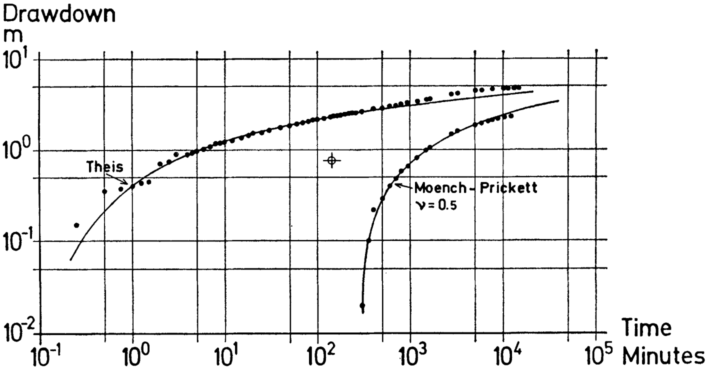

Fig. 7.
The drawdown vs. time in observation well 8D SO:178 at Ranstad compared with Theis type curve (confined conditions) and Moench & Prickett type curve (unconfined conditions).

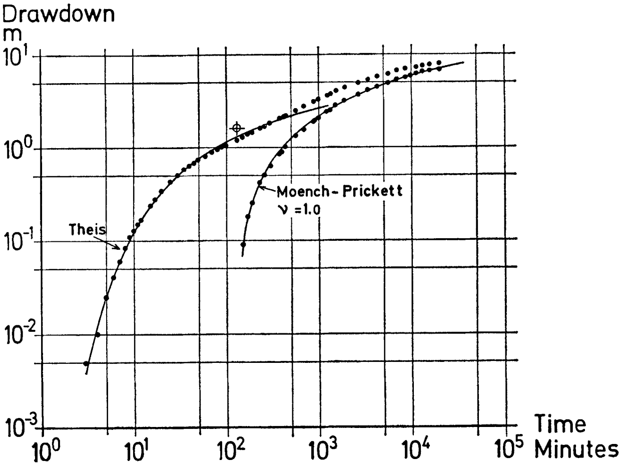

Fig. 8.
The drawdown vs. time in observation well 8D SV:193 at Gudhem compared with Theis type curve (confined conditions) and Moench & Prickett type curve (unconfined conditions).

Table 3.

Calculated values of transmissivity T, storage coefficient under confined conditions (S_1) and under unconfined conditions (S_2) from pumping tests at Ranstad and at Gudhem.

Locality	Well	Transmissivity m^2/sec		Storage Coefficient under confined conditions		Storage Coefficient under unconfined conditions		Remarks
		semilog plot	loglog plot	semilog plot	loglog plot	semilog plot	loglog plot	
Ranstad	VP 7	$7.9\ 10^{-4}$						Drawdown
	8DSO 177	$7.8\ 10^{-4}$	$7.8\ 10^{-4}$	$5.5\ 10^{-5}$	$6.6\ 10^{-5}$			Drawdown
		$6.8\ 10^{-4}$	$7.5\ 10^{-4}$	$42.\ 10^{-5}$	$51.\ 10^{-5}$			Recovery
	8DSO 178	$8.3\ 10^{-4}$	$7.9\ 10^{-4}$	$2.0\ 10^{-5}$	$1.7\ 10^{-5}$	$12.\ 10^{-3}$	$9.5\ 10^{-3}$	Drawdown
		$6.8\ 10^{-4}$	$7.5\ 10^{-4}$	$2.3\ 10^{-5}$	$3.2\ 10^{-5}$	$6.5\ 10^{-3}$	$6.4\ 10^{-3}$	Recovery
Gudhem	8DSV 193	$3.7\ 10^{-5}$	$4.0\ 10^{-5}$	$5.5\ 10^{-5}$	$2.9\ 10^{-5}$	$0.4\ 10^{-3}$	$0.4\ 10^{-3}$	Drawdown
		$3.5\ 10^{-5}$	$3.5\ 10^{-5}$	$2.1\ 10^{-5}$	$4.0\ 10^{-5}$	$1.1\ 10^{-3}$	$1.5\ 10^{-3}$	Recovery

$$T = 0.183 \frac{Q}{\Delta s_2} \qquad (6)$$

where Δs_2 = the drawdown difference per log cycle of time during the unconfined conditions

In estimating the storage coefficient S_1 under confined conditions, Jacob's formula (1940) is applicable, which gives the following expression:

$$S_1 = \frac{135 \cdot T \cdot t_0}{r^2} \qquad (7)$$

where t_0 = the time intercept on the zero drawdown axis in min.

By estimating the slope of the line for the confined conditions and the time t_c for conversion ($r = R$), the storage coefficient S_2 for unconfined conditions can, according to equation (4), be calculated as follows:

$$S_2 \doteq \frac{240 \cdot T \cdot t_c}{r^2} \cdot \log \frac{\Delta s_2}{\Delta s_1} \qquad (8)$$

where t_c = time in min for conversion from confined to unconfined conditions

Δs_1 = the drawdown difference per log cycle of time during the unconfined conditions.

Calculated values of the transmissivity T and the storage coefficients S_1 and S_2 from semilog plots are presented in Table 3. Comparisons between the values calculated by log-log plots and semilog plots show close agreement, even for the recovery data.

In the later parts of the pumping tests, a stationary stage was developed at Ranstad, and a deviation from the line in the semilog plot was observed at Gudhem. This behaviour was assumed to depend on vertical leakage through the overlying alum-shale and on the natural gradient of the piezometric level.

The low T value at Gudhem indicates a smaller number of fractures than at Ranstad.

BAROMETRIC EFFICIENCY

General relations

Water level fluctuations caused by changes in atmospheric pressure have been described by different authors, for example Leggette & Taylor (1937), Thomas & Taylor (1946), Taylor & Leggette (1949), Tuizaad (1954), Maxwell & Devaul (1962), Andersen (1965), Andersen & Haman (1970) and Gustavsson (1972).

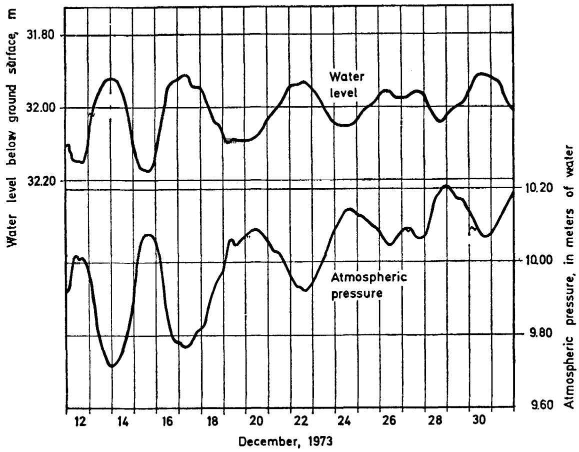

Fig. 9.
The water level response to atmospheric pressure changes in the well 8D SV:193, Gudhem.

An expression relating the barometric efficiency to aquifer and water properties was presented by Jacob (1940). The barometric efficiency, B, expresses the ratio between the change in water level, Δ h, to the change in atmospheric pressure, Δ p.

$$\text{Thus, } B = \frac{\Delta h}{\Delta p} \cdot \gamma$$

where γ is the density of water.

The barometric efficiency is related to the storage coefficient, S, by the following expression given by Jacob (1940):

$$S = \frac{\alpha \cdot \gamma \cdot b}{E_w \cdot B} \tag{9}$$

where

α = porosity of the aquifer
b = aquifer thickness
E_w = bulk modulus of elasticity of water ($2.07 \cdot 10^9$ N/m^2)

Observations and calculations

Groundwater level fluctuations of the sandstone aquifer have been registered by automatic water level recorders in the wells at Ranstad (8D SO:178) and at Gudhem (8D SV:193) (see Fig. 1) during the winter 1973–1974. The atmospheric pressure was registered at Borgunda (see Fig. 1), and official meteorological station.

An example of the response of water level fluctuations to the changes in atmospheric pressure is given in Fig. 9. As seen from this figure, an increase in atmospheric pressure causes the water level to decline, while a decrease in atmospheric pressure makes it rise.

Correlations of the barometric pressure changes to water level changes of the two bore holes are shown in Figs. 10 and 11. The positive or negative atmospheric pressure changes are plotted as abscissa against the corresponding

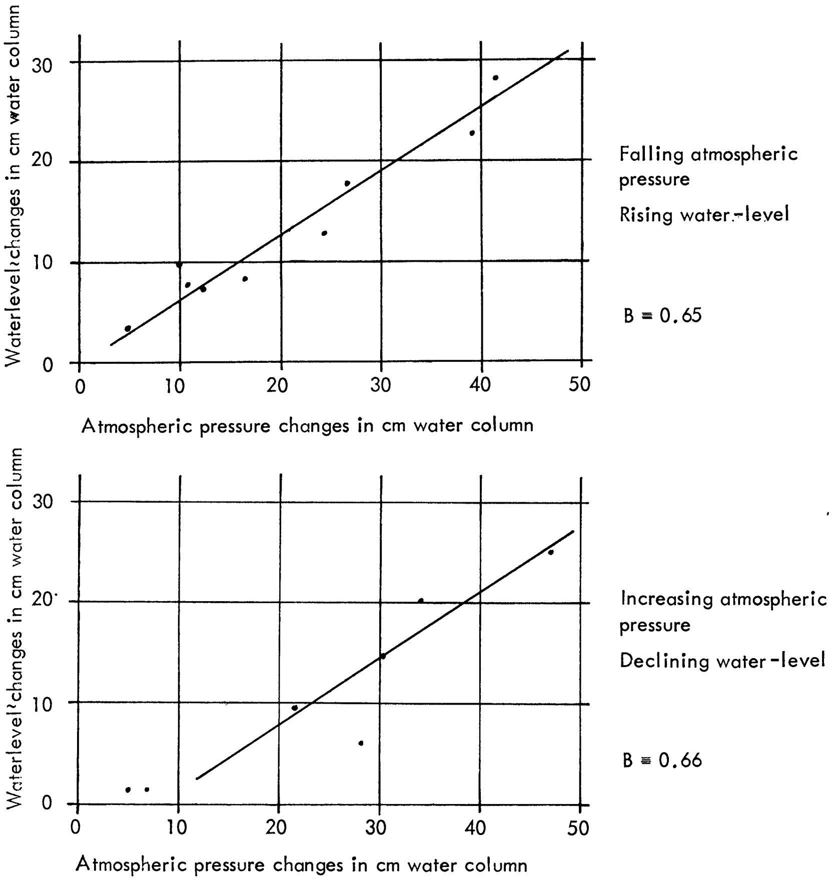

Fig. 10.

Correlation of atmospheric pressure changes at Borgunda with water level changes at Ranstad.

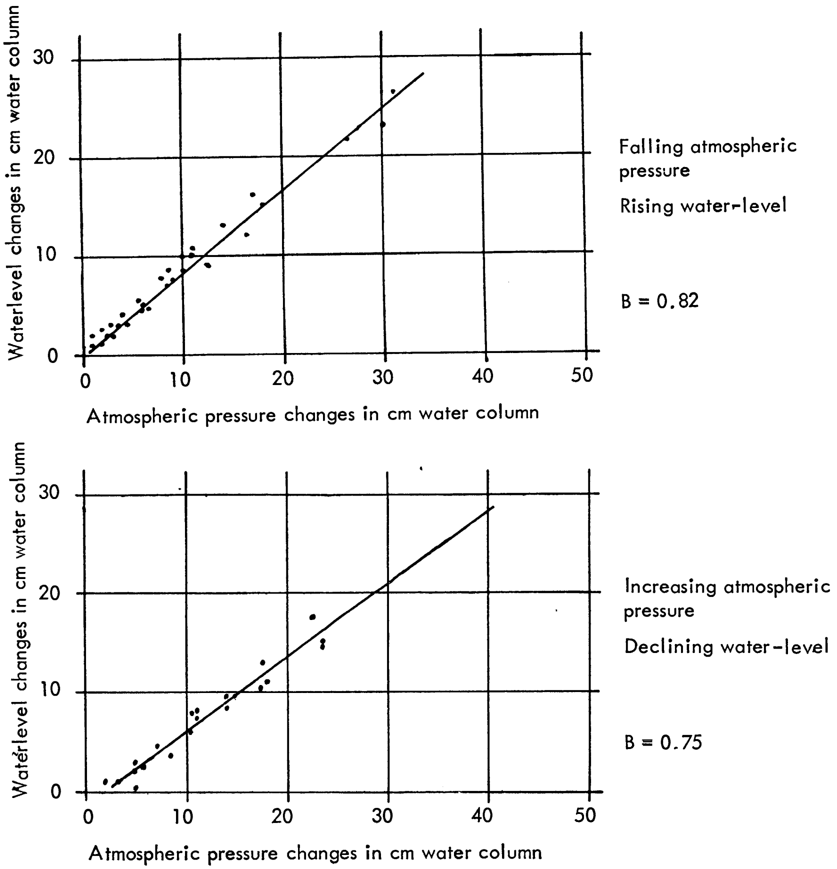

Fig. 11.

Correlation of atmospheric pressure changes at Borgunda with water level changes at Gudhem.

water level changes. Thus, the changes in pressure have been calculated as the difference between the maximum and minimum points for each rising and falling period respectively. The barometric efficiency, which is obtained from the slope of the correlation line, shows different values for rising and falling atmospheric pressure, especially from Gudhem. It should be pointed out that the values from Ranstad were obtained during the later part of the drawdown-period of the pumping test. The values of the barometric efficiency and the calculated value of the storage coefficient according to eq. (9) are presented in Table 4.

Table 4.
Calculated values of barometric efficiency and storage coefficient.

Site	Barometric efficiency	Storage coefficient	Remarks
Ranstad	0.65	$2.5 \cdot 10^{-5}$	Falling pressure-rising water level
Ranstad	0.66	$2.4 \cdot 10^{-5}$	Increasing pressure-declining water level
Gudhem	0.83	$1.9 \cdot 10^{-5}$	Falling pressure-rising water level
Gudhem	0.73	$2.1 \cdot 10^{-5}$	Increasing pressure-declining water level

Remarks on the results

While the results from the Gudhem bore show a quite distinct difference in barometric efficiency at falling and rising atmospheric pressure respectively, the results obtained from the Ranstad bore are not so easily interpreted. The results can possibly be explained by the smaller number of observations, and the fact that the ground water was influenced by the pumping test during the observation period.

Andersen (1965) describes the influence on barometric efficiency in a well affected by extraction of ground water from the well. The barometric efficiency calculated from rising atmospheric pressure and falling water level was thereby greater than it would be if calculations from falling atmospheric pressure and rising water level were made. A difference of 10–15 % was noticed.

Brown et al. (1972) have pointed out that the transmission of atmospheric pressure changes into a confined aquifer occurs only through the well, and that the rate of the redistribution of pressure is dependent both on the permeability and on the elastic properties of the aquifer materials. Therefore it is not possible to draw a curve showing changes in atmospheric pressure as a simple and strictly mathematical dependence.

However, the results obtained in the present investigation confirm that the barometric efficiency under natural (non-influenced) conditions (Gudhem bore) is greater when calculated from falling atmospheric pressure and rising water level than if calculated from rising atmospheric pressure and falling water level. Thus, the equalization of the barometric efficiency at Ranstad could possibly be caused by the withdrawal of ground water.

CONCLUSIONS

The storage coefficient calculated from the pumping test for the confined condition varies, depending on drawdown or recovery. As can be seen from Table 3, the coefficient is smaller at the drawdown than at the recovery. This has also been observed at a pumping test on the Kristianstad plain in southern Sweden (VIAK AB 1973). Calculation of the storage coefficient from the barometric efficiency gives different values depending on rising or falling atmospheric pressure, but here the variations are smaller than at the drawdown/recovery. A small difference in barometric efficiency is noticed between the two sites Ranstad and Gudhem. According to Ferris et al. (1962), this may depend on different horizontal distances to the border of the confining bed or on discontinuities in the confining layer. Comparisons between the values obtained from the pumping tests and the barometric efficiency show good agreement.

Thus, in areas of confined ground water conditions and availibility of suitable wells, it is possible to estimate the storage coefficient by synchronous observations of the piezometric level changes and the atmospheric pressure changes. Together with ordinary step drawdown tests of the wells to estimate the transmissivity and the well loss, the method constitutes a useful and simple way to estimate the hydrogeological parameters S and T. The authors' experience is that it is usually possible to get the use of suitable wells for the required measurements by keeping in contact with the local well drillers.

ACKNOWLEDGEMENT

The authors wish to thank AB Atomenergi for making available all the data used in this work.

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Abbreviation: SGU = Sveriges Geologiska Undersökning (The Geological Survey of Sweden).

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Received March 1975.

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