# Automated fuzzy decision and control system for reservoir management

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## ABSTRACT

Traditional approaches to the management of an artificial reservoir involve the use of linear, dynamic, nonlinear or stochastic programming. Hence a purely model-based approach would be extremely difficult and thus, recently, a large number of papers devoted to the solution of reservoir management problems based on fuzzy logic approaches have appeared. In this work, two management problems of a reservoir are addressed with fuzzy logic: the definition of the water flow to supply to the user, a typical decision problem; and the regulation of the dam gate, which is a typical control problem. Both problems have been integrated in an Automated Fuzzy Decision and Control System (AFDCS) that is able to identify the ordinary and drought operating conditions. Fuzzy rules are developed from a database derived from the traditional experience of operators and they have been optimized with a genetic algorithm. Two cost functionals are used, able to weight user's desiderata (water demand) with water waste (water spills and evaporation). Three strategies are developed for a case study and are validated in different scenarios, using Monte Carlo simulations and worst case situations. Results show a good performance of AFDCS to alleviate the consequences of drought and to control of the dam gate.

**Key words** | decision support system, drought, fuzzy logic, optimization, reservoir operation

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## INTRODUCTION

The management of an artificial reservoir involves different aspects – technical, economic, social, political, etc. – and conflicts can arise from the allocation of water resources because stakeholders have different goals to achieve. Indeed the artificial reservoir can have, generally, multiple purposes: civil, agricultural, industrial and energetic that are often conflicting and, recently, are growing with different scales and needs (Gleick 1993). Another aspect that increases the complexity of water resource management of an artificial reservoir is that, in the vast majority of cases, they were built between the second half of the 19th century and the first half of the 20th century, following old design approaches, often without any optimization techniques. Therefore, in the last 50 years, many researchers and operators studied and, in too few cases, applied (Simonovic 1992) optimization techniques to existing structures facing different main issues as demand changes and conflicting users (Miller 2009).

The optimization analysis involved in artificial reservoirs is based on very different considerations, rather hard to express in mathematical terms both for complexity (multi-objective problems) and for not-deterministic variables (uncertainties due to natural processes as rainfall, flow, temperature, etc.).

As well described by Chen & Chang (2010), studies on water resources distributions explored two types of uncertainties: (1) uncertainties within the models themselves based on their underlying data and hypotheses; and (2) uncertainties that arise during decision-making such as influential stakeholders with different interests. The first ones are essentially addressed with a probabilistic approach facing the randomness both of rainfall and hypothesis (Yeh 1985); the second ones use decision-making with different techniques (Koutsoyiannis *et al.* 2002; Sechi *et al.* 2005). The latter are more democratic and are generally dealt with

by a 'bottom-up' approach (Miller 2009) that is strongly preferable in the environmental politics process (Chen & Chang 2010). In this context, a purely model-based approach is difficult to use for the modelling of the decision-making because it does not allow different aspiration levels to co-exist in the decision-making or take into account suggestions by non-experts. The research in the management of artificial reservoirs developed different optimization techniques that can be implemented in the decision-making process.

Classical approaches to optimization problems in water resources management involve the use of linear, dynamic, nonlinear or stochastic programming (Jain *et al.* 1988; Jacovkis *et al.* 1989; Loucks 1992; Labadie 2004).

More recently, neuro-dynamic programming (Bing-Yuan 2003; Farias *et al.* 2006) has been proposed where evolutionary concepts have been used to accelerate the training phase of a neural network. Different optimization techniques based on genetic algorithms (Reddy & Kumar 2006) also coupled with synthetic inflow generation (Kim *et al.* 2008) have been proposed.

Moreover, in the last two decades, a large number of papers devoted to the solution of reservoir management problems based on fuzzy logic approaches have appeared, both with reference to reservoir operating rules (Savic & Simonovic 1991; Russel & Campbell 1996; Shrestha *et al.* 1996; Mousavi *et al.* 2007) and management (Panigrahi & Mujumdar 2000; Cancelliere *et al.* 2002; Cavallo *et al.* 2003, 2004, 2007; Taslima & Simonovic 2004; Karmakar & Mujumdar 2006 Chen & Chang 2010). The fuzzy approach has proved to be very effective, both for its 'native' capability to deal with nonlinear models and for the possibility of taking into account heuristic and political rules. The advantage of the fuzzy implementation is the possibility of defining a linguistic meaning for the rules resulting from the mathematical optimization and also to add empirical rules, thus combining heuristic and rigorous mathematical treatment.

The presence of uncertainties in a fuzzy approach in water resources applications was faced in different ways, i.e. a *compromise* programming technique based on a fuzzy approach to withstand the problem of resource planning for long-range water management (Bender & Simonovic 2000), a *fuzzy-stochastic* linear programming

proposed to model the case of uncertain evaporation losses (Nazemi *et al.* 2002) or dynamic programming (DP), integrated with multicriteria decision-making, applied to derive operating rules for a single multi-objective reservoir operation problem under imprecise objectives (Akbari *et al.* 2010) and a *fuzzy-boundary interval-stochastic* programming that can deal with uncertainties expressed as probability distributions and fuzzy-boundary intervals (i.e. the lower and upper bounds of some intervals may rarely be acquired as determinist values and so they are expressed in fuzzy sets) (Li *et al.* 2010). In Panigrahi & Mujumdar (2000), an original steady state policy derived from a stochastic dynamic programming (SDP) model is proposed as an alternative to the expert knowledge that is generally available with experienced reservoir managers. In Cancelliere *et al.* (2002), a neural network approach coupled to a dynamic programming for deriving irrigation reservoir operating rules was proposed using a penalty term enforcing optimal releases series to follow the real system managing criteria, explicitly taking into account the social and management constraints to release water demand when reservoir storage volume exceeds a fixed threshold.

In Celeste & Billib (2009), the performance of different stochastic models used to define optimal reservoir operating policies were investigated, based, essentially, on Implicit (ISO) and Explicit Stochastic Optimization (ESO) as well as on the Parameterization–Simulation–Optimization (PSO) approach. The ISO models include multiple regression, two-dimensional surface modelling and a neuro-fuzzy strategy, the ESO models include the well-known and widely used SDP techniques, an approach that incorporates probabilistic inflow methods directly into the optimization problem, and PSO models are a variant of the Standard Operating Policy (SOP). The study shows that all ISO and PSO models performed better than SDP and the SOP and also provided release rules similar to the ones found by perfect forecast optimization. Recently, Ganji & Pouyan (2011) have proposed a modification of a simple model of DP based on a novel intelligent state dropping (ISD) mechanism, in which the ISD mechanism is designed based on fuzzy logic theory that allows the shortfall in supplying demand to be reduced and improves the reservoir operation performance indices, i.e. the reservoir

reliability indices, as compared with the results by the DP and SDP models.

In this paper, an original PSO approach, specifically a fully Automated Fuzzy Decision and Control System (AFDCS), based on two fuzzy devices, is developed for a multipurpose reservoir. The AFDCS automatically distinguishes the *ordinary conditions* from the *emergency conditions* as defined in Cavallo *et al.* (2004). The approach adopted is similar to Panigrahi & Mujumdar (2000) and Cancelliere *et al.* (2002), but with different modelling and operational aspects. Indeed the AFDCS is based on a full description of the reservoir model, based on a hybrid model (Gollu & Varaiya 1989) and non-linear Proportional-Integral (PI) strategies (Ho *et al.* 1998; Tan *et al.* 2001) for reservoir gate regulation, which first allows the decision strategy to be defined and then to control water releases. Further novel aspects with respect to the above works are the use of different fuzzy input variables and a Monte Carlo technique to tune the parameters of fuzzy rules systems.

The paper presents a wide extension of the results obtained in Cavallo *et al.* (2012), since a new objective function was added in order to also penalize evaporation losses, so that the trade-off between users' demand and losses (both due to spills and to evaporation) can be addressed directly, and new scenarios were analysed in order to test the effectiveness of operation strategies in different operation conditions.

Specifically, uncertainties are addressed using different scenarios by using Monte Carlo simulations in order to assess reliability and effectiveness of decision strategies (Willis *et al.* 1984; Basson & van Rooyen 2001). In particular, a periodic, Auto Regressive (AR)-lognormal and a more complex periodic Auto Regressive Moving Average with eXogenous input (ARMAX)-lognormal model of the inflow are identified and used to assess the performances of strategies selected by using genetic algorithms on a fuzzy decision system (FDS).

Complementary to the Monte Carlo approach, a worst case analysis has been carried out by artificially increasing the water demand by 10% and reducing the water inflow by 10%.

All operation policies have been tested on a case study using a MATLAB/SIMULINK integrated environment.

## MATERIALS AND METHODS

In this section, different building blocks of the AFDCS phases are presented: the reservoir modelling, the automatic smart decision and control system design, the water operation strategies, the identification of the water inflow and Monte Carlo technique and the performance indices used to compare water release policies.

### Reservoir modelling

The mathematical model of the dynamics of the reservoir is described by the differential equation:

$$\dot{V} = q_{\mathrm{in}}(t) - q_{\mathrm{ev}}(t) - q_{\mathrm{out}}(t) \tag{1}$$

where $V(t)$ is the reservoir volume at the generic time instant $t$, expressed in months, that depends on the geometry of the reservoir, $q_{\mathrm{in}}(t)$ is the water inflow in the reservoir, $q_{\mathrm{ev}}(t)$ is the evaporation and $q_{\mathrm{out}}(t)$ is the water outflow from the reservoir, resulting from the users' demand. In particular $V = \int_0^h A(\sigma)\,\mathrm{d}\sigma$, where $A(h)$ is the area of the water surface and $h$ is the water height in the reservoir. The evaporation $q_{\mathrm{ev}}(t)$ is usually modelled via an evaporation coefficient $k_{\mathrm{ev}}(t)$ deduced from reservoir's losses at time instant $t$: $q_{\mathrm{ev}}(t) = k_{\mathrm{ev}}(t)A(h(t))$. The volume is physically lower bounded by the so-called 'dead volume', hence $A(h) \neq 0$ and $h$ cannot go below a minimum level $h_{\min}$. Obviously, $h$ is also upper bounded by volume of the basin with $h_{\min} < h < h_{\max}$. Thus, the model of the reservoir can be written:

$$\dot{h} = -k_{\mathrm{ev}}(t) + \frac{1}{A(h)}\left(q_{\mathrm{in}}(t) - q_{\mathrm{out}}(t)\right) \tag{2}$$

Then, hybrid modelling concepts (Gollu & Varaiya 1989) must be used to describe with a single, parametrized model, the reservoir, both in normal operation and in the presence of water spills. Specifically, three states are considered for this model:

1. The standard condition, with water level between $h_{\min}$ and the maximum level. For this state, Equation (2) holds.

2. The condition of 'reservoir full', where the water level is fixed at its maximum value and the difference between water inflow and flow released to users (plus evaporation) is lost as 'water spills'.

3. The condition of 'reservoir empty', where the water level is fixed at $h_{min}$ and no water is supplied to the users (in this case, due to evaporation, the actual water level may go below $h_{min}$, but this occurrence is infrequent and of minor consequence, hence it is not directly addressed in the paper).

## Water operation strategies

The life cycle of the reservoir can be divided into: (1) ordinary management condition; and (2) emergency management condition (Cavallo *et al*. 2004).

The first condition refers to the case where, in a given time interval, the total available water volume is not less than that required. In this case, there is enough water to satisfy the user's demand, and the decision strategy must select whether to supply all the water the users ask for or to save some water for possible future needs. Note that, due to evaporation losses, too conservative strategies would result in water waste without fulfilling future users' demand. The second management condition takes place in the drought period when the system enters an 'emergency operation condition'. In this case, reduced water flows are supplied while still trying to reduce discomfort to the users, resulting from unsatisfied water demand. According to the model derived in the previous section, ordinary management occurs when the reservoir is in states 1 or 2, while state 3 is associated to 'emergency'.

The overall dam management is usually based on the SOP, which simply tries to satisfy all water demand, if there is enough available water stored. This policy, although often used by reservoir managers, may cause periods of severe shortage during droughts. The SOP strategy is the simplest reservoir operating rule that prioritizes immediate water release up to the target demand. In such cases, different operation policies, the so-called hedging strategies, that allows the saving of water in order to mitigate potential future shortages, can be proposed.

Basically, hedging aims at modulating the water demand, starting from an *ideal* water demand $q_{out}^{id}(t)$, which is the true demand from the users, and defining a new *feasible* water demand $q_{out}^{feas}(t)$ so that requested water is usually released when available but, if a drought is expected, lower water reference levels $q_{out}^{feas}(t)$ are imposed. Different techniques can be used to produce a modified, feasible, water demand, e.g. by using analytical techniques (Draper & Lund 2004), probabilistic approaches (You & Cai 2008) or inflow forecasting (Wang & Liu 2013). In this paper a Fuzzy Operation Policy (FOP) has been developed, with the objective of addressing both the issues of simplicity and flexibility of the heuristic approach and the issue of availability of synthetic parameters that can be further optimized.

The FOP decides automatically how much water the operator must release while an automated fuzzy control system regulates the dam gate in order to release the *actual* water demand $q_{out}^{act}(t)$ just defined. The two fuzzy decision and control systems are described in the following sections.

## Automated fuzzy decision and control system

The main parts of the AFDCS are illustrated in Figure 1 where a 'FDS' and a 'Fuzzy Controller System' (FCS) use two feedback loops and the reservoir and actuator systems (dam gates) are indicated with R.

Although in principle the water released depends on the gate operation, the key idea is to separate the decision maker and the controller into two different designs. The FDS ignores the operations on the dam gate and decides the flow to release, assuming that the gate will be operated in an ideal way so as to result in water release to users perfectly equal to the one computed by the automatic decision system. Next, the controller, R, will operate the gate in order to guarantee that the released water is as close as possible to the one computed by the FDS.
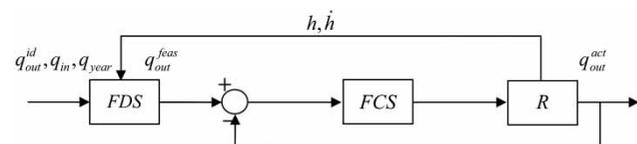


**Figure 1** | Automated fuzzy decision and control system.

The first FDS decides how much water the operator must release, compatible with the current and possibly future conditions. Moreover, the outer loop employs a FDS to define, in real-time, the 'reduced flow reference' in the case of 'emergency management conditions'.

The FCS controls the operation of the dam gate, as accurately as possible, with the 'reference' signal generated by the FDS. The FCS is in charge of assuring the accurate tracking of the FDS commands, whatever the water release policy. Thus, the first step is to design the FDS as if it were able to command an actual water release. Next, the FCS will be responsible for operating the dam gate so as to guarantee that the released water is exactly (within a prescribed accuracy) that imposed by the FDS. The fuzzy modelling of two devices has been carried out with MATLAB/FUZZY LOGIC Toolbox (The MathWorks Inc. 2004a).

## Fuzzy decision system design

As is known in the literature (Zadeh 1965; Dubois & Prade 1980), fuzzy systems gained popularity due to their ability to turn numeric input through linguistic knowledge into numeric output. Thus, in spite of recent advances (first of all, automated membership functions definition), the core of fuzzy logic theory is still linguistic rules set.

In a pioneering paper, the definition of a fuzzy inference system (FIS) for the water reservoir management, Panigrahi & Mujumdar (2000) highlighted the role of knowledge engineer in expressing the knowledge on some linguistic form required by fuzzy logic, i.e. translating expert knowledge in the form of 'if-then' rules. However, nowadays, an approach based only on expert knowledge is considered outdated, due to its exceedingly subjective character.

In this study, trying to take into account knowledge reservoir management operator, the following Sugeno-type rule system (Sugeno 1985; Zimmermann 2001) is obtained with $l$-th rule with $l = 1, ..., r = 9$ rules (then the rules will also be optimized in a second step):

$$R^{(l)}: \text{IF } x_1 \text{ is } P_1^{(l)} \text{ AND } x_2 \text{ is } P_2^{(l)} \text{ AND } x_3 \text{ is } P_3^{(l)}$$
$$\text{AND } x_4 \text{ is } P_4^{(l)} \text{ THEN } y = \rho^{(l)} y^{\text{id}} \quad (3)$$

where

- $x_i \in U_i \subset R$  $x_1 = q_{\text{out}}^{\text{id}}$, $x_2 = h$, $x_3 = \dot{h}$, and $x_4 = q_{\text{year}} = \int_0^{1 \text{ year}} q_{\text{in}}(\tau) \, d\tau$;
- $y \in S \subset R$ is the output linguistic variable in the universe of discourse, expressed as product of a coefficient $\rho^{(l)} \in [0, 1]$ by an ideal output $y^{\text{id}} \in S$;
- $P_i^{(l)}$ is the fuzzy set referred to the $i$-th input linguistic variable with $i = 1,...,4$. Specifically, $P_1 = \{\text{Moderate, Intense}\}$, $P_2 = \{\text{Low, High}\}$, $P_3 = \{\text{Negative, Zero, Positive}\}$, $P_4 = \{\text{Drought}\}$;
- $\rho^{(l)} \in C_1 \subset [0, 1]$ is a crisp multiplier for the $l$-th rule, $l = 1, ..., 9$, assuming values in the set $C_1 = \{\text{Nothing, Very-Little, Little, Much, VeryMuch, Emergency}\}$. This is a 'reduction factor' of the output with respect to an 'ideal' output and it is $\rho^{(l)}$ chosen so as to reduce the ideal user's water demand $q_{\text{out}}^{\text{id}}$.

In particular, an example of a decision rules system is

$$R^{(l)}: \text{IF } q_{\text{out}}^{\text{id}} \text{ is Intense AND } h \text{ is Low AND } \dot{h} \text{ is Zero}$$
$$\text{THEN } q_{\text{out}} = \text{VeryMuch } (q_{\text{out}}^{\text{id}}) \quad (4)$$

The number of Membership Functions (MFs) and rules must be kept reasonably small, since although by increasing the number of classes a greater accuracy may be achieved, the dimensionality of the optimization problems increases hugely (Panigrahi & Mujumdar 2000). These actions are essential to develop the smart core of fuzzy system for water reservoir management and can be achieved in different ways starting from statistical data with a stochastic approach (Mousavi *et al.* 2004; Celeste & Billib 2009), neural clustering (Cancelliere *et al.* 2002; Mehta & Jain 2009), or heuristically from expert knowledge (Russel & Campbell 1996).

In this study, similarly to the last method, starting from expert knowledge, the shape and the rules are chosen and reported in Table 1.

The design of the FOP has been carried out with few 'trial and error' repeated simulations. Although 21 parameters have to be suitably selected, the simplicity of the fuzzy approach and the possibility to exploit the physical meaning of all the parameters, a 'reasonable' choice has easily been obtained. Specifically, referring to the input and output variables:

- the required current (ideal) water outflow has been taken into account in the input fuzzy set $P_1$, considering only

**Table 1** │ Fuzzy inference system rules

| Input | | | | Output |
|---|---|---|---|---|
| $P_1$ ($q_{out}^{id}$) | $P_2$ ($h$) | $P_3$ ($dh/dt$) | $P_4$ ($q_{year}$) | $C_1$ |
| Moderate | Low | – | – | Little |
| Moderate | High | Negative | – | VeryLittle |
| Moderate | High | Zero | – | Nothing |
| Moderate | High | Positive | – | Nothing |
| Intense | Low | Negative | – | VeryMuch |
| Intense | Low | Zero | – | VeryMuch |
| Intense | Low | Positive | – | Much |
| Intense | High | – | – | Nothing |
| – | Low | – | Drought | Emergency |

two possible requests (Moderate and Intense) (two parameters for each MF);

- Low and High MF for the input fuzzy set $P_2$ depend on two parameters each, as well as Negative and Positive for the fuzzy set $P_3$, and Drought for $P_4$;
- Zero MF for the fuzzy set $P_3$ depends on a single parameter (its centre is fixed to the value 0, only the variance is considered as a parameter);
- all the MFs of the output fuzzy set $C_1$ depend on a single parameter for the specific Sugeno structure of the fuzzy member function; so the output parameters are six.

The final selection of the MFs for the FOP in the case studied in this paper is shown in Figure 2, first column.

In order to improve the heuristic FOP strategy, two objective functions have been defined, and two optimization problems have been solved. The optimization of the 21 parameters was carried out with a genetic algorithm (GA) (Goldberg 1989). The problem is a nonlinear and constrained optimization problem, since, in order to preserve linguistic meaning of fuzzy rules presented in Table 1, i.e. for example, to avoid that a membership function labelled 'High' comes 'before' (i.e. it is associated to smaller numerical values) the membership function 'Low', it is necessary to constrain all the variables. Thus, the following upper and lower bounds (based on the values resulting from the FOP) have been imposed on the six output variables $C_1$: $0.95 \le c_1 \le 1.00$, $0.80 \le c_2 \le 1.00$, $0.60 \le c_3 \le 0.80$, $0.40 \le c_4 \le 0.60$, $0.05 \le c_5 \le 0.20$,

$0.20 \le c_6 \le 0.40$. Obviously, the choice of the bounds is itself a problem that can be handled, e.g. by a trial and error approach.

The first Objective Function (OF1) defined in optimization procedure is:

$$y = \int \varpi(q_{sp}) \left( q_{out}^{id}(t) - q_{out}^{feas}(t) \right)^2 dt \qquad (5)$$

where $\varpi(q_{sp})$ is a fuzzy weighting function which penalizes situations with high spills $q_{sp}$. This is done to consider the case that saving more water can alleviate droughts but increases water waste due to spills.

The GA solution to the problem discussed in the next section, obtained with a population size of 40 individuals, defines a new decision strategy, named OFOP1 (Optimized Fuzzy Operation Strategy). OFOP1 starts the GA optimization using the FOP solution as a starting guess, because in this way, the optimization solver is allowed to start from a 'good' starting guess, and trivial local minima are *a priori* avoided. The MFs after optimization are shown in Figure 2, second column.

Objective Function (5) only partially penalizes water losses, since it ignores evaporation. Moreover, the penalty on water spills is multiplicative. Thus, a new objective function (OF2) minimizing both water deficit and all water waste, additively, is considered. It takes into account the time integral of the conflicting objectives: the squared deficit and losses due to evaporation and spills,

$$y = \int \left[ \left( q_{out}^{id}(t) - q_{out}^{feas}(t) \right)^2 + q_{evap}^2(t) + q_{spills}^2(t) \right] dt \qquad (6)$$

Index (6) generates the new operation policy defined OFOP2. The MFs after optimization are shown in Figure 2, third column.

The optimization phase has been carried out with MATLAB/OPTIMIZATION Toolbox (The MathWorks Inc. 2004c).

## Fuzzy control system design

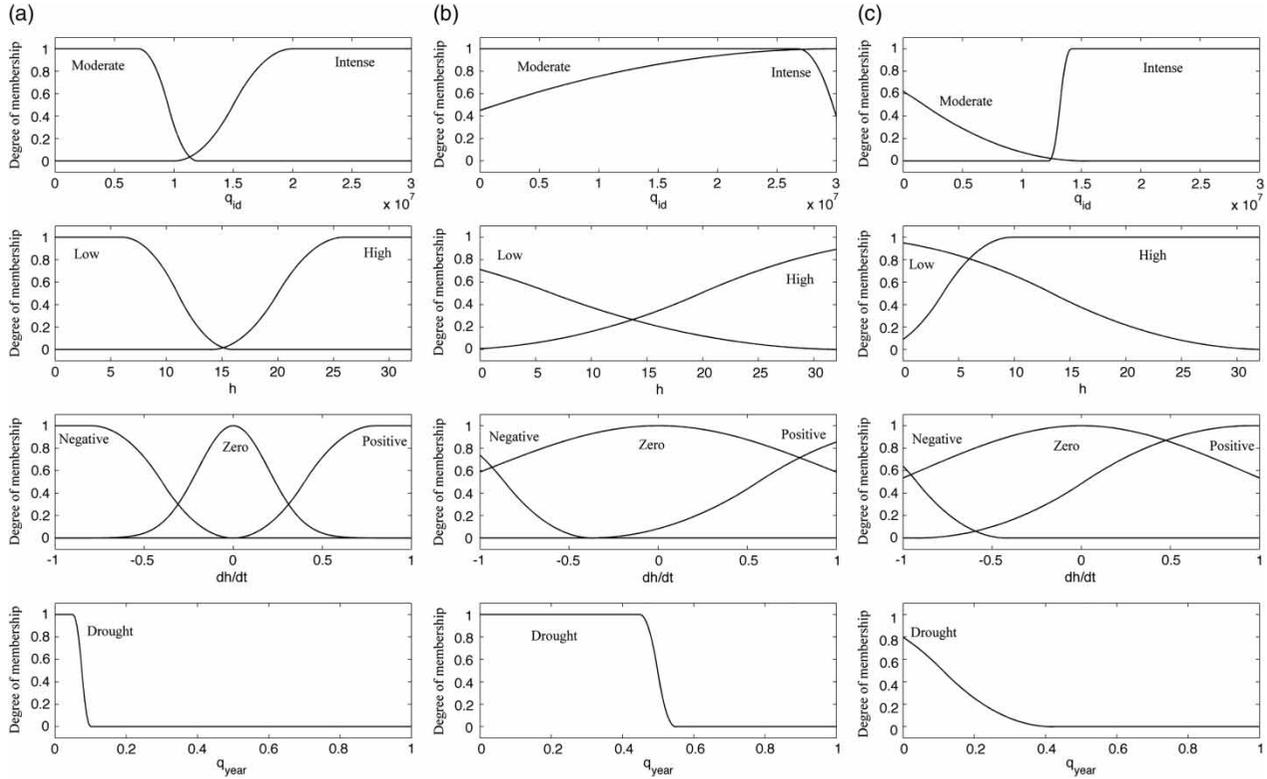The definition of a control strategy for the dam gate can be formulated as a tracking problem, i.e. a reference

**Figure 2** | Membership functions of fuzzy inference system: column (a) FOP, column (b) OFOP1 and column (c) OFOP2.

outflow $q_{\text{out}}^{\text{feas}}(t)$ is defined, and a control action is sought such that the actual water outflow $q_{\text{out}}^{\text{act}}(t)$ after a transient follows the reference profile with the smallest possible error:

$$e(t) = q_{\text{out}}^{\text{feas}}(t) - \tilde{q}_{\text{out}}^{\text{act}}(t) \tag{7}$$

where $\tilde{q}_{\text{out}}^{\text{act}}(t)$ is a filtered version of the water outflow, in order to filter its variability. A simple first-order linear filter has been selected, with time constant of 1 h. A fuzzy PI strategy (Lee & Chae 1993; Cavallo 2005) has been considered for solving the control problem:

$$u(t) = K_{\text{P}}(e)e(t) + K_{\text{I}}(e) \int_0^t e(t)\,\mathrm{d}\tau \tag{8}$$

where $K_{\text{P}}(e)$, $K_{\text{I}}(e)$ are Proportional and Integral nonlinear control gain. The gains are chosen as $K_{\text{P}}(e) = K_1 K_{\text{F}}(e)$, $K_{\text{I}}(e) = K_2 K_{\text{F}}(e)$, where $K_1$ and $K_2$ are constant

gains resulting from optimization and $K_{\text{F}}(e)$ is a fuzzy function resulting from a Sugeno FIS with the following simple rules:

- if $e$ is 'positive' then $y = K_1^1 e$
- if $e$ is 'zero' then $y = K_1^0 e$
- if $e$ is 'negative' then $y = K_1^2 e$

where $K_1^1 = 1$, $K_1^0 = 0$, $K_1^2 = 10$ and the input MFs are shown in Figure 3.

The motivation for choosing different gains for positive and negative errors is because negative errors means that the reservoir is actually releasing more water than required, thus increasing water waste. In this case, control authority is increased in order to counteract this problem. The shape of the overall nonlinear control gain of the fuzzy function $K_{\text{F}}(e)$ is shown in Figure 4. The effect of the FCS will be discussed in the case study section.
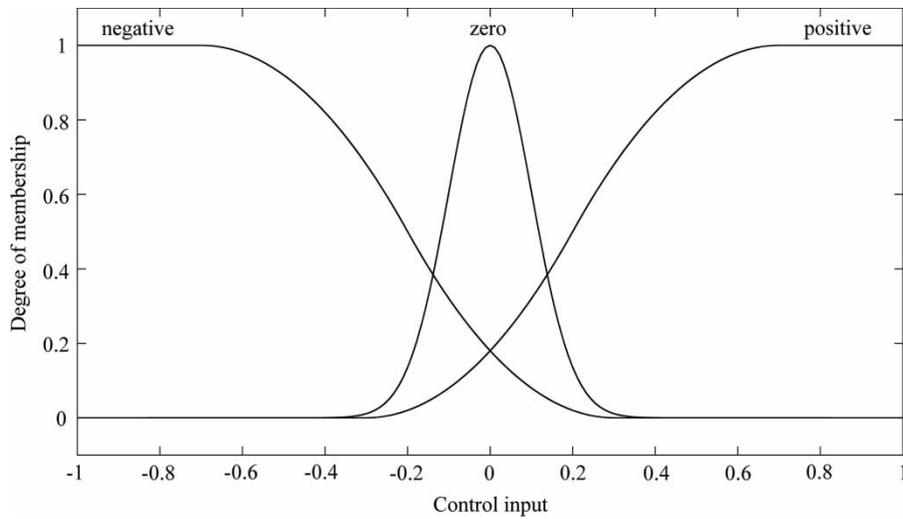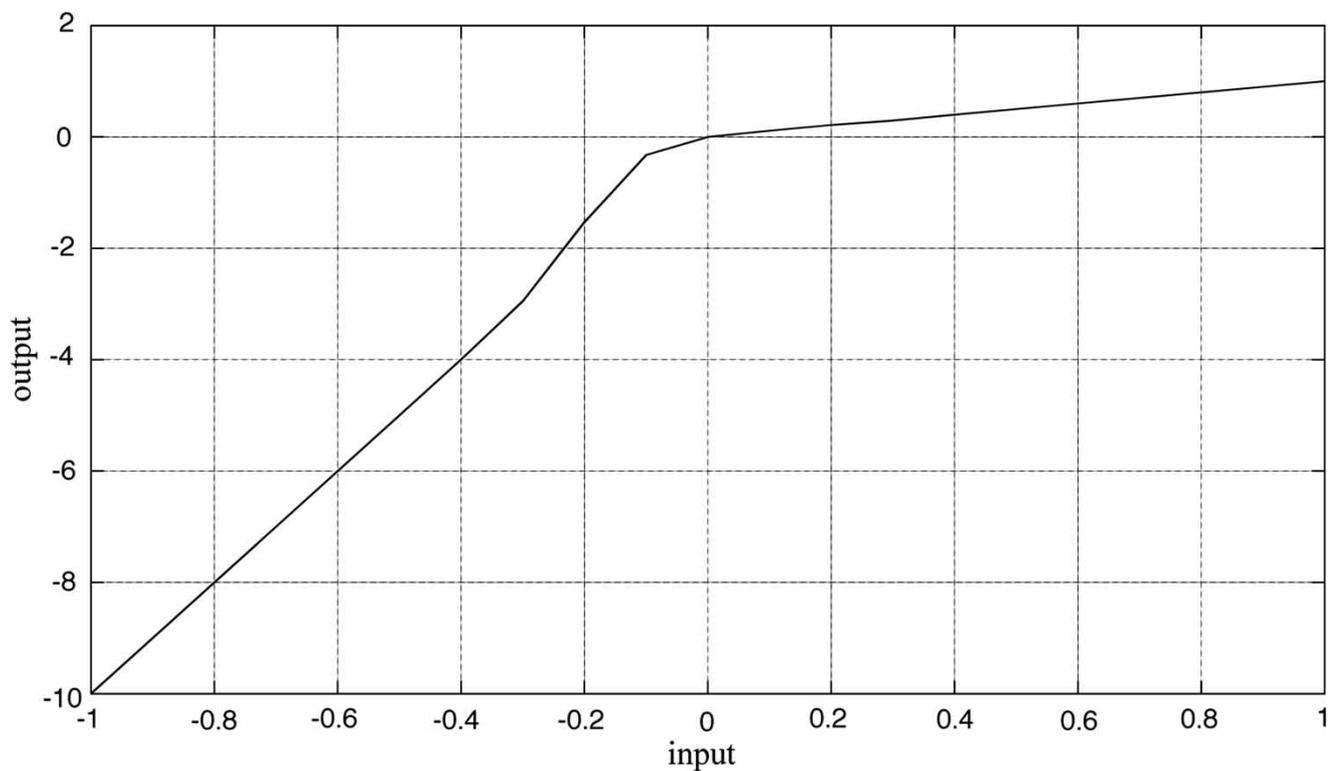
**Figure 3** | FCS input MFs.



**Figure 4** | Fuzzy gain input–output map for FCS.

### Identification of the inflow

Obviously, the results depend not only on the ability of the genetic algorithm to seek for 'good' sub-optimum, but also on the inflow historical data entering the system. Since naturally only one (although, rather long) $q_{in}^{hist}(t)$ time series exists, in this study, in order to check the effectiveness of the proposed strategy, new input

data $q_{\text{in}}^{\text{sim}}(t)$ are 'generated' by a Monte Carlo technique starting from statistics of the inflow process. This can be accomplished by identifying a dynamical model of the inflow time history (Box & Jenkins 1970), and using a random generator to produce simulated inflow processes, i.e. vector of random numbers preserving the statistics of the original process (Ljung 1987). Thus the decision strategy is defined on the whole original set of data, and its performances are assessed by checking its behaviour when inputs generated by the identified model are used as new inputs.

The first step in identifying a dynamic system or a time history is to prefilter the data. Generally, all that can be easily extracted from the data, as mean and trends, is removed and the time series $\hat{Q}(t)$ to be identified is next normalized by removing the seasonal mean and variance (Box & Jenkins 1970). However, it is reasonable to guess the only removing mean and variance still leaves a more complex component of seasonality in the data, e.g. periodic components with a period equal to one year (yearly repetitions) or to half-year (dry/wet season period), or also multi-year periods, to be detected. In order to test this hypothesis and simultaneously removing the above seasonalities, the following approach has been followed. A black-box input–output model is fitted to the data, assuming as input a fictitious periodic signal, $u(t)$, with a period of one year. The operation is performed by using the System Identification Toolbox of Matlab (The Math-Works Inc. 2004b), which implements a large set of techniques based on classical concepts (Ljung 1987). Using the classical prediction error as optimality criterion, the ARMAX (3,1,2), with delay 3 in the input, the following is obtained:

$$q(t) = 0.86(\pm 0.066)q(t-1) + 1.15(\pm 0.044)q(t-2)$$
$$- 0.35(\pm 0.066)q(t-3) + 0.0064(\pm 0.0038)u(t-3)$$
$$+ \xi(t) - 0.49(\pm 0.01)\xi(t-1) + \xi(t-2) \qquad (9)$$

where $q(t)$ denotes the time history to identify, $u(t)$ is the fictitious input defined above and $\xi(t)$ is a white Gaussian noise. Fitness of the model (9) to available historical data has been assessed by using suitable whitening tests (e.g. Anderson's test) and normality tests (e.g. Kolmogorov–Smirnov test for normality).

The ARMAX model thus deduced is used for simulation, by feeding the identified system with a Gaussian pseudo-white noise with variance computed from the model error variance. A plot of a realization of the simulated inflow vs. the true data is shown in Figure 4.

## Case study

The methodology developed in this paper has been applied to the case of the management of Pozzillo reservoir, on the Salso River in Sicily (Italy). Pozzillo reservoir is a multipurpose system (hydroelectric, irrigation and municipal), the basin area is about 577 km$^2$ and net storage is $123 \times 10^6$ m$^3$.

Annual streamflow series, shown in Figure 5, presents a high variability between the maximum value equal to $139.5 \times 10^6$ m$^3$, which occurred on 1972, and the minimum value zero.

Analysing the stochastic distribution of inflow data, it is possible to recognize some recent drought events from 1970 to 1971; from 1973 to 1976 and from 1994 to 1995, but the most severe drought period was experienced during the years 1988–1990. All these drought events highlighted the need to re-design operating rules in order to mitigate the worst irrigation deficits (Cancelliere *et al.* 2002). In Table 2 the irrigation and hydro potable water demand was reported, with a minimum value of mean monthly demand equal to $1.0 \times 10^6$ m$^3$ and maximum value equal to $25.6 \times 10^6$ m$^3$.

Then, as described in the previous section, the SOP was modified to include hedging in the FOP strategy to distinguish between ordinary and emergency management conditions whilst trying to reduce negative consequences for users in drought situations. It is designed with an heuristic estimation of all parameters according to the rules described in the above section, based only on expert knowledge. The input MFs of inference fuzzy system are shown in the first column of Figure 2, while the values of the output are $C_{1\_\text{FOP}} = \{1.00, 0.95, 0.80, 0.40, 0.30, 0.20\}$.

The OFOP strategies also consider water waste (spills and evaporations). The result of optimization, obtained with OF1 and OF2, are reported in the input MFs, showed in the second and last column of Figure 2, in which it is possible to observe that the shape of each MF is changed
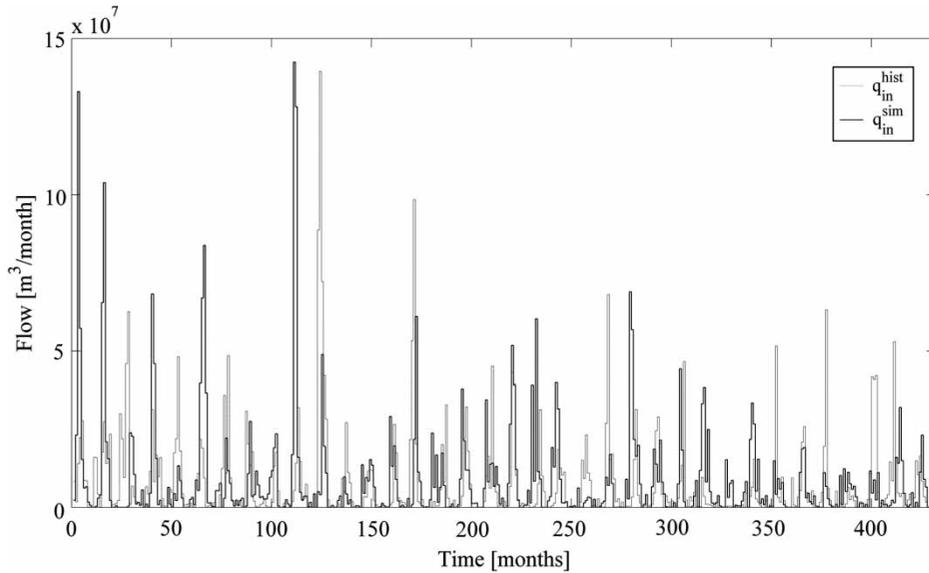
**Figure 5** | Measured and a simulated inflow.

**Table 2** | Annual water demand for irrigation and hydro potable purposes $q_{out}^{id}(t)$

|  | Jan | Feb | Mar | Apr | May | June | July | Aug | Sept | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Demand ($10^6$ m$^3$) | 1.0 | 1.0 | 1.0 | 1.0 | 3.5 | 11.6 | 24.6 | 25.5 | 17.3 | 5.5 | 1.0 |

while also preserving the meaning of each attribute. The values of the output, obtained with the constrained GA, are $C_{1\_OFOP1} = \{0.98, 0.97, 0.64, 0.58, 0.11, 0.36\}$ and $C_{1\_OFOP2} = \{0.99, 0.87, 0.61, 0.58, 0.2, 0.37\}$.

Each policy has its advantages and drawbacks. In order to evaluate the effectiveness of the proposed strategies, the following performance indices are considered. In particular, the following performances indices are defined:

- **VR**, Volumetric Reliability: $\left( \sum_{t=1}^{n} q_{out}^{feas}(t) / \sum_{t=1}^{n} q_{out}^{id}(t) \right) \times 100$

- **SSD**, square root of the Sum of Squared Deficits: $\sqrt{\sum_{t=1}^{n} \left( q_{out}^{feas}(t) - q_{out}^{id}(t) \right)^2}$

- **DF**, Deficit Frequency: $\frac{1}{n} \left( \sum_{t=1}^{n} d(t) \right) \times 100$

- **TS**, Total Spills $\sum_{t=1}^{n} Sp(t)$

- **TE**, Total Evaporation $\sum_{t=1}^{n} q_{ev}(t)$

where

$n = $ number of months

$Sp(t) = q_{out}(t) - q_{out}^{id}(t)$ when $q_{out}(t) > q_{out}^{id}(t)$

$$d(t) = \begin{cases} 0 & \text{when} & q_{out}^{feas}(t) = q_{out}^{id}(t) \\ 1 & \text{when} & q_{out}^{feas}(t) < q_{out}^{id}(t) \end{cases}$$

The above indices are only indirectly related to the objective functions OF1 and OF2, hence the only way to assess the effectiveness of the proposed approach is via simulations.

A complete historical 432-months water inflow $q_{in}^{hist}(t)$ and monthly evaporation rates $k_{ev}$ data set, for the years 1962–1998, has been used. All strategies have been designed on the complete set, without using historical data for validation. This choice is motivated by the data behaviour discussed above, that makes the classic 'half data for training and half for validation' approach not feasible. Instead, the effectiveness of the strategies has been assessed by using a set of 20,000 Monte Carlo runs $q_{in}^{sim}(t)$, as explained in the previous section.

Moreover, using drought data for the design may make the automatic decision system robust against a water crisis. In order to test this idea, simulations have also been run in pessimistic cases of water inflow reduction and/or water demand increase, although only one of the worst cases (both 10% inflow reduction and 10% demand increase) is discussed in this paper.

## RESULTS

First, consider the case of SOP: in Figure 6, is it possible to observe several months in which the reservoir does not succeed in fulfilling the water demand $q_{out}^{id}(t)$. Specifically, during the drought in months around 110, 150, 250, and especially in months 320–340, where the severest water
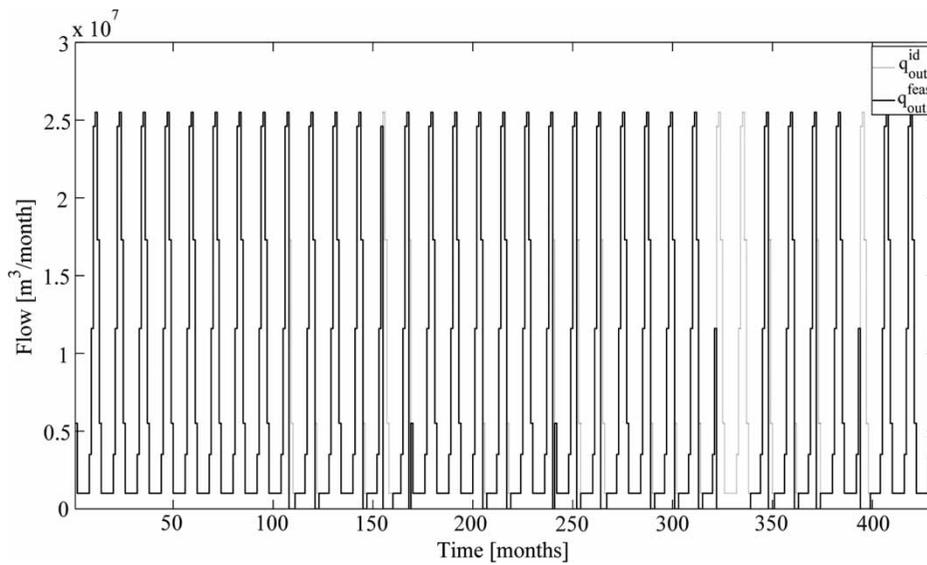


**Figure 6** │ Simulation results in the period 1962–1998 (SOP: Standard Operation Policy).
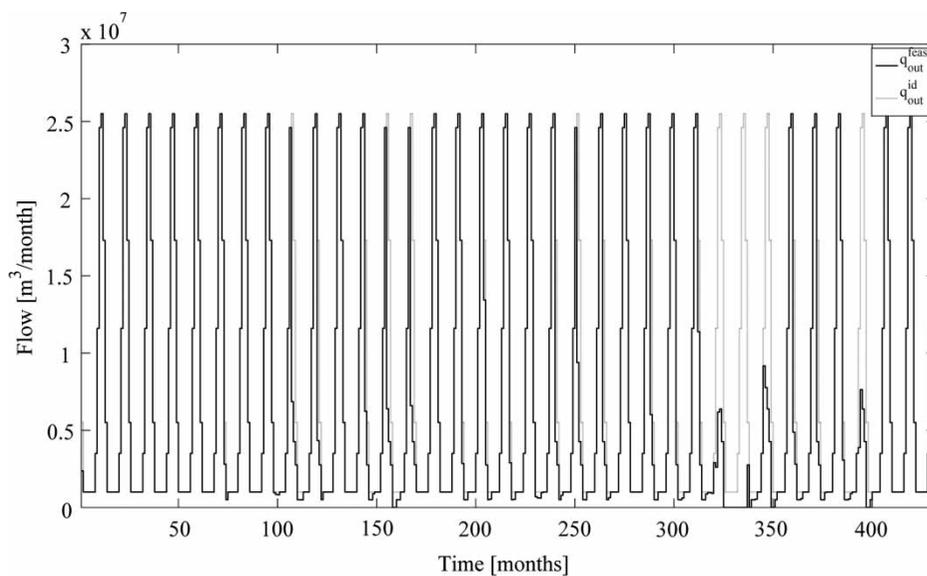


**Figure 7** │ Simulation results in the period 1962–1998 (FOP: Optimized Fuzzy Operation Policy).

crisis occurred. This simulation highlights that the SOP strategy may result in long periods of complete absence of water releases.

In Figure 7, it is shown that a significant improvement is made possible by the use of the FOP strategy. By preserving water in the wet months and releasing it in the drought months, the cases of complete drought have been almost alleviated with a significant reduction of crisis periods, but

a closer insight shows the appearance of a new phenomenon: in the middle of the dry season (summer) water release is suddenly reduced, in order to save water for the future. The FOP strategy moreover behaves poorly in the months 320–340, where the most severe drought occurred.

A possible alternative may be to save more water in wet months, when available, in order to avoid abrupt reduction of release during summer. This is basically done by the two
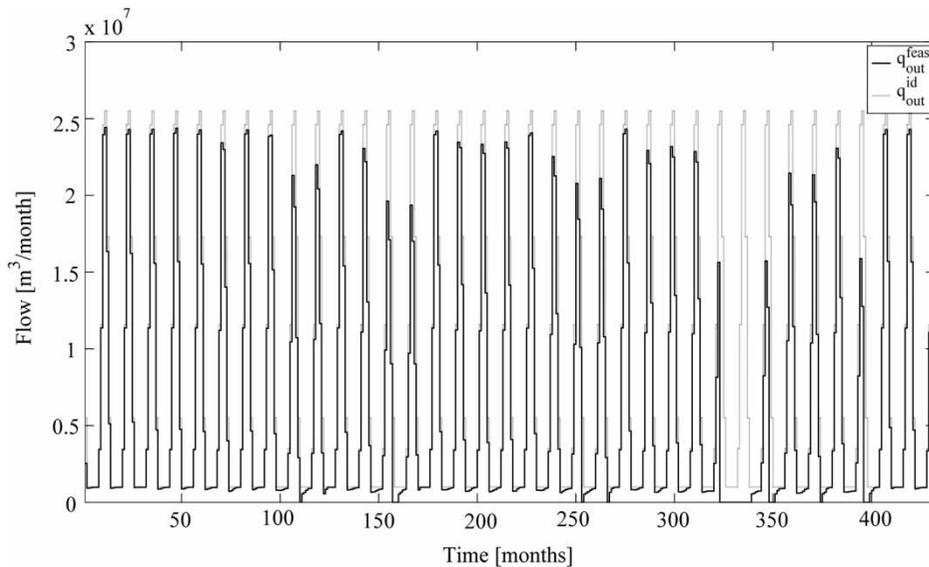


**Figure 8** │ Simulation results in the period 1962–1998 (OFOP1: Optimized Fuzzy Operation Policy with OF1).
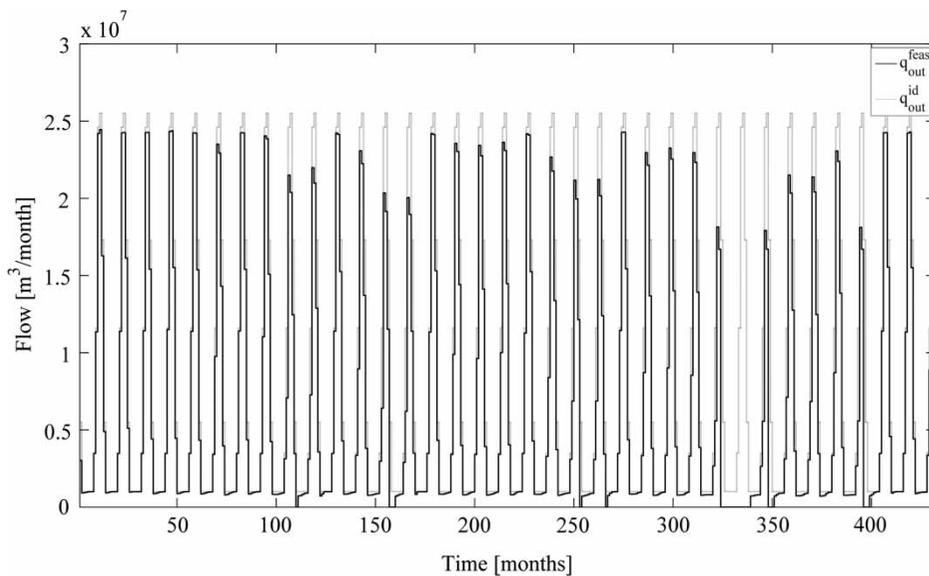


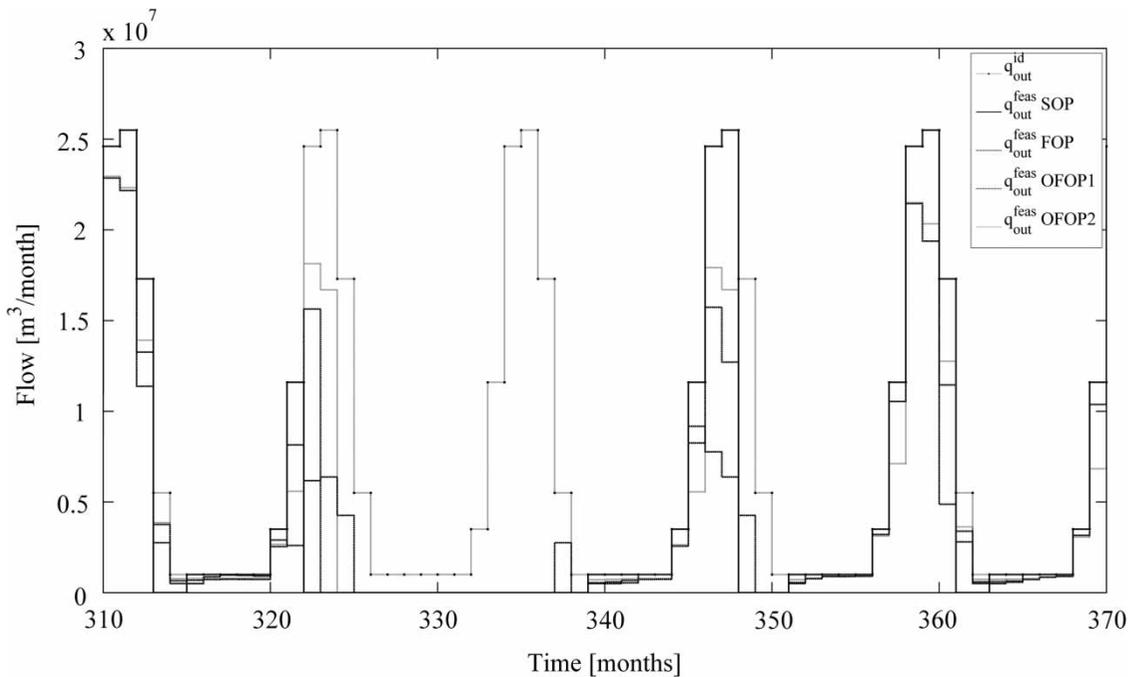**Figure 9** │ Simulation results in the period 1962–1998 (OFOP2: Optimized Fuzzy Operation Policy with OF2).

**Figure 10** │ Comparison of simulation results in the period from month 310 to month 370.

optimized strategies, shown in Figures 8 and 9. Basically, in OFOP, water crises are prevented by preserving water even during winter and releasing it in the drought months. In fact, it is possible to observe that during wet months, when the water demand is smaller, the $q_{out}^{id}(t)$ is *almost* completely satisfied (but *never* completely satisfied), and in summer drought months, the water released to the user does not change abruptly.

Finally, in Figure 10, a comparison between the four strategies (SOP, FOP, OFOP1 and OFOP2) is illustrated, and it is possible to note the superiority of the optimized strategies reducing discomforts to the users (i.e. unsatisfied water demand), compared to the SOP. Consider, for instance, the months 320–330, it is evident that the SOP is unable to release water after month 322, FOP is able to yield one more month of water (but well below the user demand), while OFOP2 is able to give water for one more month and with higher outflow, while OFOP1 can reach even the next month 325, although with reduced water release. Naturally, the improved result obtained with OFOP strategy has the drawback that the user is generally given less water than required because the fuzzy strategies save some resources for possible future shortages.

However, a comparison between OFOP1 and OFOP2 is not very clear from the analysis of the figures only, thus the indices presented in the previous section have been computed and are presented in Table 3. It is clear that, although water losses increase with the proposed strategies, with an increasing of TS and TE indices, the most important parameter for the user, the sum of squared deficit SSD, gives the best score to the proposed approaches. It may seem strange that OFOPs, that explicitly penalizes water losses, has more losses than SOP and FOP. The point is that OFOPs penalize losses *versus* deficit, while SOP and FOP have low losses as they have less stored water. Moreover, the value of 100% of DF index for the OFOPs is simply the result of the optimized strategies,

**Table 3** │ Performances indices computed with historical data during 1962–1998

| Operation policy | VR (%) | SSD ($10^5$ m$^3$) | DF (% months) | TS ($10^7$ m$^3$) | TE ($10^7$ m$^3$) |
|---|---|---|---|---|---|
| SOP | 85.4 | 916 | 13.4 | 49.9 | 23.2 |
| FOP | 82.9 | 882 | 37.3 | 51.7 | 27.2 |
| OFOP1 | 80.8 | 734 | 100 | 52.5 | 27.3 |
| OFOP2 | 81.3 | 668 | 100 | 52.7 | 27.6 |

**Table 4** | Performances indices computed with 20,000 Monte Carlo simulations

| Operation policy | VR (%) m | VR s | SSD (10⁵ m³) m | SSD s | DF (% months) m | DF s | TS (10⁷ m³) m | TS s | TE (10⁷ m³) m | TE s |
|---|---|---|---|---|---|---|---|---|---|---|
| SOP | 92.3 | 2.8 | 644 | 142 | 7.0 | 2.2 | 57.5 | 21.2 | 26.1 | 1.6 |
| FOP | 90.1 | 3.0 | 631 | 115 | 26.0 | 3.9 | 60.2 | 21.5 | 29.0 | 1.0 |
| OFOP1 | 86.5 | 2.73 | 475 | 109 | 100 | 0 | 67.6 | 22.7 | 29.5 | 1.3 |
| OFOP2 | 86.2 | 2.72 | 461 | 109 | 100 | 0 | 69.0 | 22.6 | 29.7 | 1.3 |

**Table 5** | Performances indices computed in worst case simulations

| Operation policy | VR (%) | SSD (10⁵ m³) | DF (% months) | TS (10⁷ m³) | TE (10⁷ m³) |
|---|---|---|---|---|---|
| SOP | 75.9 | 1,324 | 19.7 | 31.8 | 20.3 |
| FOP | 74.4 | 1,197 | 40.3 | 31.8 | 24.3 |
| OFOP1 | 72.5 | 1,080 | 100 | 32.6 | 24.3 |
| OFOP2 | 72.8 | 1,062 | 100 | 32.6 | 24.2 |

never giving the user the required water, since, as discussed above, less water is always released. For the same reasons, the VR index is also naturally higher for the SOP and FOP strategy with respect to the other optimized operation policies because in these cases the water not released in order to save resources for possible drought periods can evaporate and spill and, consequently, the VR is lower.

In order to perform a more objective test, as mentioned above, a campaign of 20,000 Monte Carlo runs has been performed on the three proposed strategies. The results obtained from historical data input are confirmed by the Monte Carlo approach that presents better values for all performance indices because historical data input are strongly affected by heavy drought periods. The results are shown in Table 4, in terms of means *m* and standard deviation *sd* estimates.
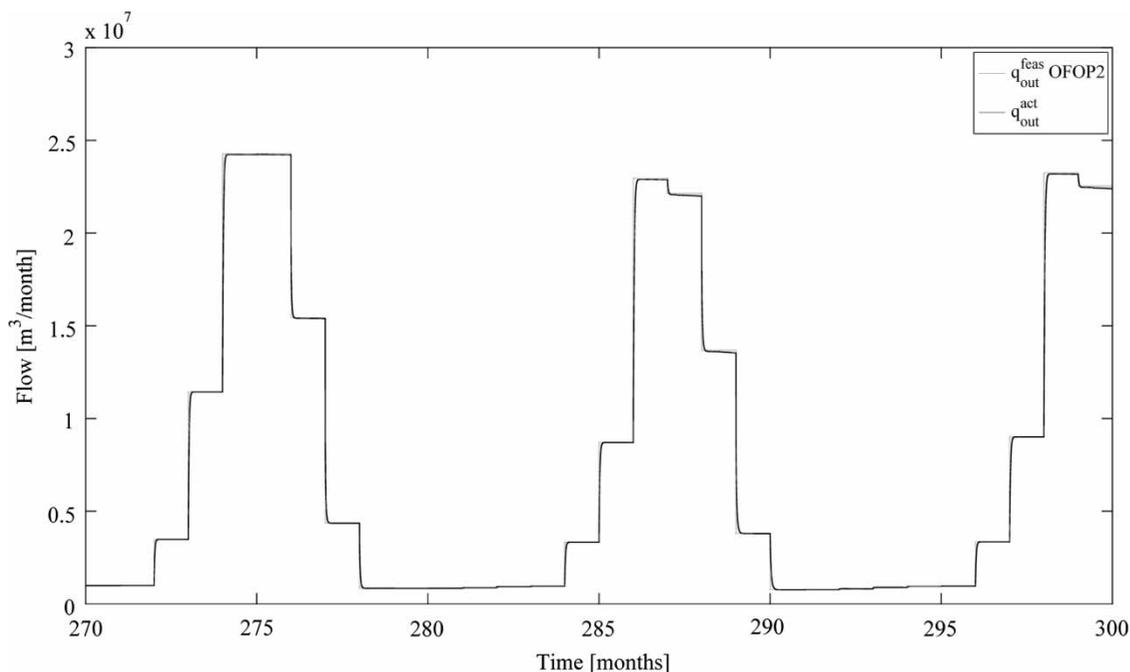


**Figure 11** | Simulation with FCS.

The superiority of the OFOP approach from the point of view of the minimization of the SSD index, both in terms of mean and variance, is apparent w.r.t. the SOP.

Finally, worst case results (reported in Table 5) still confirm the superiority of the proposed strategies in terms of reduction of cumulated water deficit with an acceptable increasing of TS and TE indices.

For the design of the FCS, since the structure of the fuzzy gain has already been discussed above, only the constant gains $K_1$ and $K_2$ are to be chosen. Using a semi-automatic approach (a rough optimization of the parameters via optimization and a fine-tuning via trial and error methods) the values $K_1 = 0.03$, $K_2 = 400$ has been chosen.

The results of the simulation in the months 270–320 of the strategy imposed by the OFOP2 are shown in Figure 11 and compared with the actual released water outflow $q_{out}^{act}(t)$ by operating the dam gate with the fuzzy PI controller. It is apparent that the actually released water $q_{out}^{act}(t)$ is very close to the prescribed water reference $q_{out}^{feas}(t)$, thus confirming the validity of the proposed approach.

## CONCLUSIONS

Different decision strategies, a SOP, a FOP and two different Optimized Fuzzy Operation Policies (OFOPs), have been analysed and compared for the problem of handling the water management of an artificial reservoir in a fully automatic way. The SOP releases water whenever possible, regardless of future water demand. The fuzzy strategies consider the possibility of saving water for future shortages. The OFOP supplies water based on reservoir dynamics (water height and its trend) and external factors (water inflow), thus foreseeing future possible shortages and reducing the water release, even if there is currently some available water, if it seems that saving water can alleviate foreseen future droughts. Moreover, a PI fuzzy controller operates the reservoir gate. The system is integrated in a unique mathematical and software environment for dealing with decision and control problem for automatic reservoir management.

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