this inlet configuration could have given better performance if the blade design had been conducted along the shroud streamline, a better prediction of τ could have been made by that approach. Furthermore, we find from equation (17) that at the shroud and at the test condition of $\phi = 0.091$, the leading edge static pressure coefficient above vapor pressure is $k = 0.02$, rather than the value of 0.045 that is obtained on the 50 percent streamline as given in Fig. 13. Point 3 is virtually unavoidable with a sharp blade and can be understood in the light of the cavity phenomena discussed already. Even without the cavity formation, the strong, localized, three-dimensional flow effects at a blade leading edge are not well described by the approximate blade-to-blade analysis that was used. This undoubtedly caused the incipient cavitation at high τ observed in Fig. 12. Point 4, the tip-vortex phenomenon, has been found to be one of the prime factors affecting cavitating performance of this type of impeller, as is well described in the observations of reference [10].

Further observation illustrates clearly how the two-phase phenomenon discussed earlier assists in the maintenance of high head ratio at low flows.

Fig. 14 shows the suction specific speeds that resulted from these tests. At the point discussed above; viz., $\psi/\psi_{NC} = 0.7$ and $\phi = 0.091$, $S = 25{,}000$. Of particular note here is the fact that the operation was satisfactorily steady for all flows at this value of head ratio. This would seem to bear out the long, narrow-passage theory discussed earlier, especially with regard to stability.

The fact that cavitating performance was indeed possible has been discussed with regard to the geometrical capabilities of the pump. However, thermodynamic property variations are known to give rise to different results in different fluids, once bubble formation is admitted [11]. No comparative water tests were performed. Additional work in this area would be desirable, as well as shroud static pressure measurements to verify the theory presented herein.

Conclusions

The attention of the designer has been called to the fact that pump flow passages can be designed to meet pressure criteria which are directly connected with the overall suction parameters. In particular, test results have demonstrated that by these methods low-specific-speed pumps can be designed for considerably higher suction specific-speed limits than are generally used.

The low blade angles that arise from a low NPSH design with light initial loadings in single-phase flow give long, narrow passages, which produce stable cavitating operation over a wide range of flow rates.

Finally, criteria for the velocity distributions that reduce the possibility of flow separation can be used to determine the acceptable inlet geometry of a pump as well as the shapes of the impeller passages in the higher pressure regions.

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DISCUSSION

Glenn M. Wood[2]

Mr. Cooper is to be complimented for his presentation of a fundamental approach to the design of centrifugal pumps. This approach parallels the design methods we are using at Pratt & Whitney Aircraft, although the detailed procedure is quite different.

One of the principal drawbacks of this paper is the lack of any detailed velocity and pressure measurements in the tests of the pump and inlet design described to permit a direct correlation of theory and experiment. Mr. Cooper has also selected a most difficult meridional flow passage to analyze, as shown in Fig. 1 of the paper. With regard to the inlet flow passage, what entrance flow conditions were assumed at the labeled H to I boundary? It would appear that such entrance flow conditions would be extremely important. Also, would the author please elaborate on the criteria he used in estimating the separated flow region in the inlet along the H boundary.

The author also did not clearly indicate the methods used to obtain the velocity and pressure distributions presented in the paper. Mr. Cooper made the general statement that an electrical analog was used, but gave no hint as to how this was accomplished. I believe that such details regarding the solution would be of great interest.

I also have had great difficulty in interpreting what the author is presenting in Figs. 4, 6, and 7 as contrasted to the results shown in Figs. 5, 8, and 9. First, with regard to Figs. 4, 6, and 7, there is no indication of change in the curves of the plotted variables when the solidity is changed from 3 blades to 6 blades. On the other hand, the results plotted in Figs. 8 and 9 depict the change of the blade solidity. I am also puzzled by the sharp break in the blade angle versus radius ratio shown in Fig. 5. Why was such a selection made and did it not complicate the blade fabrication procedure?

With regard to the cavitation results shown in Figs. 12, 13, and 14, I have difficulty making any correlation between these data and the analytical predictions. Certainly for all values of flow coefficient there exists a considerable amount of cavitation in the inlet of the pump before any performance drop off with reduced NPSH occurs. Therefore, I can not see any relation between the prediction of the onset of cavitation and these test results. How do the thermodynamic properties of the trimethylpentane compare with those for water?

In conclusion, it would be appreciated if the author would elaborate upon the analytical procedures followed in obtaining the results presented to permit a more ready evaluation of his extensive work.

Author's Closure

Mr. Wood's questions are important to the understanding of the theory used in this paper, and the following explanations should help to clarify these points.

The two-dimensional electric analogue solution of the inlet

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(meridional) velocity field in Fig. 1 was obtained by imposing constant values of potential along the S and H boundaries of the conducting medium (teledeltos paper). Lines of constant potential between these boundaries are then analogous to streamlines. At locations I and II the boundary condition of uniform, parallel, radial flow was imposed by cutting the paper normal to the direction of the lines of constant potential. The boundary of the separated region b was simply estimated as its location has negligible effect on the flow pattern at the impeller. Because the analog applies only to inviscid flow, a constant total pressure P_0 is assumed throughout the inlet flow field.

At the point of blade number change within the impeller, the static pressures and relative velocities do undergo sudden changes. These are not visible in Figs. 6 and 7 because the blade-to-blade differences in p and W are small in the region of concern, and the input head P_i is increasing (see equations (31) and (33)). Since P_i is prescribed in Fig. 4, the blades had to be set at a higher angle, Fig. 5 to prevent the reduction of P_i that would have occurred from blockage of the splitter blades. This change in angle was incorporated into the impeller over a finite, small distance and is not noticeable to the naked eye.

The occurrence of pump pressure-rise deterioration due to cavitation could have been better predicted if the blade-to-blade impeller analysis performed along the 50 percent streamline also had been applied to the lower pressure regions that existed along the shroud streamline. Additional two and three-dimensional blade-leading-edge and tip-vortex effects could be responsible for most of the bubble activity at higher NPSH.