S to P Conversion as an Aid to Crustal Studies

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Summary

Studies of S to P conversion at the base of the crust of waves from a number of local deep earthquakes have indicated a crustal thickness of 24 km ± 3 km and 31 km ± 4 km beneath the New Zealand Seismograph Stations of Wellington (WEL) and Gisborne (GNZ) respectively. Considerable spread in the data indicates that results obtained from isolated earthquakes can be inconclusive, and that the concept of a Mohorovičić Discontinuity without small scale irregularities may well be inadequate.

1. Introduction

The occurrence of mode conversion at the Mohorovičić Discontinuity has been firmly established. Cook, Algermissen & Costain (1962) have reviewed studies of P to S converted waves, in which the thicknesses of various layers in the crust have been determined. Báth & Stefánsson (1966), recording a distant earthquake on a long-period instrument at Umeå, identified a phase which preceded the S with a lead time that was consistent with its having undergone S to P conversion at the base of a crustal slab 40 km thick beneath the station. Pachadzhanova (1962) used a similar technique to find the thickness of the sedimentary layer in Tadzhikistan.

This paper combines results from many earthquakes recorded at a single station to give an estimate of the crustal thickness, and shows that with enough high quality readings it may be possible to extend the method to find crustal velocities and local variations in the depth and attitude of the Mohorovičić Discontinuity. The notation of Báth and Stefánsson of SP for the S to P converted wave will be retained.

This is a preliminary paper outlining the method involved; SP phases have been identified at other stations, and more detailed studies of crustal thickness beneath New Zealand are under way.

2. Occurrence of S to P conversion

The short-period records at New Zealand stations sometimes exhibit a precursor to the main S arrival, both for earthquakes in the Fiji region, at distances of 20° to 25°, and for local deep shocks. This precursor appears as a regular train of waves, with period similar to that of the S phase, rather longer than that of the P coda. Its time of arrival cannot be explained by SP or any other known phase. Fig. 1 shows the short-period vertical record at Wellington, for an earthquake on 1964 September 4, at a depth of 204 km and a distance of 1°56 degrees. The lead time is 3·8 s.

The P velocity in the lower crust and the S velocity in the upper mantle define a critical incidence angle, beyond which there is no transmitted P wave. If S waves are to arrive steeply enough at the base of the crust, they must come either from distances greater than about 20°, or from earthquakes of great enough depth. This study will confine itself to the latter, for reasons given in the following section.
FIG. 1. Short-period vertical record at WEL of an earthquake on 1964 September 4 at a distance of 1.56° and a focal depth of 204 km. The Sp phase is visible 3.8 s ahead of S.

Báth & Stefánsson (1966) assumed a homogeneous crust which in four of their five models was taken as a parallel-sided slab. Their final model incorporated a dip in the plane discontinuity, and this possibility will be examined here. Fig. 2 shows the model in the general case.

Incidence angles at the base of the crust were tabulated as a function of focal depth and distance using S velocity models for the upper mantle given by Anderson (1965), Toksöz, Chinnery & Anderson (1967), and Ibrahim & Nuttli (1967). The proximity of the focus for local deep earthquakes implies a divergence of up to 10° for the two incident rays in Fig. 2, while for distant shocks a plane wavefront can be assumed. This distinction calls for a more careful treatment of the local earthquakes.

FIG. 2. Geometry of S to P conversion. The angle θ varies with azimuth.
The lead time is given approximately by

\[ \Delta T = H \cos \theta \left( \frac{1}{v_s \cos i_s} - \frac{1}{v_p \cos i_p} + \frac{\sin[(I_s + I_p)/2 + \theta] \times [\tan i_p - \tan i_s]}{V \cos[(I_s - I_p)/2]} \right) \]

where \( v_s \) and \( v_p \) are the crustal velocities, \( V \) is the mantle \( S \) velocity, and \( I_p, I_s, i_p, i_s, H \) and \( \theta \) are as shown in Fig. 2. This formula is an attempt to allow for the curved incident wavefront; the error in lead time is considerably less than 0.1s for rays not diverging by more than about 10°.

3. Application of method to two New Zealand stations

\( Sp \) phases from both local and distant earthquakes observed at Wellington (WEL) and Gisborne (GNZ) have been investigated in some detail. At Wellington the three-component WWSSN seismograph enabled the longitudinal polarization of the \( Sp \) phase and the subsequent transverse motion of the true \( S \) to be established. This procedure could not be carried out at Gisborne, where a three-component recorder has only recently been installed, but the phase readings of \( Sp \) that were used were all noted as unidentified phases in the routine reading for the New Zealand Seismological Report. These seismograms were re-examined for this study and the lead times accurately determined. Although particle motion could not be established at Gisborne, it would appear that the phase \( Sp \) is a satisfactory explanation of the unidentified arrivals.

While the distance from the station to the point of \( S \) to \( P \) conversion is generally small (50 km or less) in the local case because the wavefront is curved, for the more distant earthquakes it can be up to 150 km. The crustal model included a plane Mohorovičić Discontinuity, and this assumption would hardly be realistic over such large distances. For this reason, and also because the distant case may be further complicated by branching in the travel-time curve, this preliminary analysis was confined to local shocks.

The map of the North Island of New Zealand in Fig. 3 shows the positions of the earthquakes studied at WEL (southern group) and GNZ (northern group), and the extent of the crustal area examined near each station. Details of the earthquakes studied at WEL are presented in Table 1, which gives the location relative to WEL, with the observed lead times, and those calculated for a parallel-sided slab 24 km thick, using locally determined velocities.

### Table 1

**Earthquakes studied at Wellington. Calculated lead times are for a crust 24 km thick with velocities of 6.6 and 3.7 km s\(^{-1}\). Standard deviation of residuals = 0.8s.**

<table>
<thead>
<tr>
<th>Date</th>
<th>Depth km</th>
<th>Distance degrees</th>
<th>Azimuth N° E</th>
<th>Lead Time (seconds)</th>
<th>residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962 Jun 15</td>
<td>190</td>
<td>1.57</td>
<td>276</td>
<td>4.9</td>
<td>0.9</td>
</tr>
<tr>
<td>1962 Oct 20</td>
<td>220</td>
<td>1.42</td>
<td>303</td>
<td>5.0</td>
<td>3.6</td>
</tr>
<tr>
<td>1964 Feb 12</td>
<td>183</td>
<td>1.41</td>
<td>314</td>
<td>3.5</td>
<td>-0.3</td>
</tr>
<tr>
<td>1964 May 20</td>
<td>102</td>
<td>1.42</td>
<td>260</td>
<td>5.6</td>
<td>-0.9</td>
</tr>
<tr>
<td>1964 Sep 4</td>
<td>204</td>
<td>1.56</td>
<td>311</td>
<td>3.8</td>
<td>-0.0</td>
</tr>
<tr>
<td>1964 Oct 18</td>
<td>151</td>
<td>1.08</td>
<td>298</td>
<td>3.7</td>
<td>1.2</td>
</tr>
<tr>
<td>1964 Oct 26</td>
<td>182</td>
<td>1.34</td>
<td>315</td>
<td>3.8</td>
<td>0.0</td>
</tr>
<tr>
<td>1964 Nov 2</td>
<td>100</td>
<td>0.84</td>
<td>337</td>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>1965 Apr 5</td>
<td>226</td>
<td>1.66</td>
<td>346</td>
<td>4.9</td>
<td>3.7</td>
</tr>
<tr>
<td>1965 Jun 25</td>
<td>153</td>
<td>1.17</td>
<td>302</td>
<td>3.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>1965 Jul 25</td>
<td>118</td>
<td>1.31</td>
<td>265</td>
<td>4.4</td>
<td>4.9</td>
</tr>
<tr>
<td>1965 Dec 11</td>
<td>177</td>
<td>1.56</td>
<td>351</td>
<td>3.9</td>
<td>4.2</td>
</tr>
<tr>
<td>1966 Feb 28</td>
<td>121</td>
<td>1.20</td>
<td>339</td>
<td>3.0</td>
<td>4.5</td>
</tr>
<tr>
<td>1966 Apr 2</td>
<td>188</td>
<td>1.27</td>
<td>316</td>
<td>3.6</td>
<td>-1.5</td>
</tr>
</tbody>
</table>
3.1 Details of calculation

The angles of incidence at the base of the crust depend on the crustal model assumed, because the wavefront is curved and the distance travelled in the crust must be taken into account. $S$ and $Sp$ rays were traced through the crust independently, in order to determine the incidence angles $I_p$ and $I_s$ (Fig. 2).
An attempt was made to find values for five crustal parameters by the method of least squares: \( P \) and \( S \) velocities, thickness below the station, and two angles specifying the slope of the Mohorovičić Discontinuity. The computer program was so written that any number of these could be restrained at specified values if desired. The \( S \) velocity in the upper mantle was held at 4.7 km \( s^{-1} \), the locally determined value (Hamilton 1966). Only those earthquakes whose \( Sp \) wave was generated less than 75 km from the station were used in the analysis, because of the earlier assumption that the discontinuity was plane.

### 3.2 Results from lead-time calculations

The theoretical potential of the method in solving for five parameters could not be realized, because of the large spread in the lead-time residuals as shown in Table 1. Attempts to find a dipping plane discontinuity to satisfy the observed lead times more closely were inconclusive, possibly because of the limited range of azimuths involved. Neither the use of different \( S \) velocity models for the upper mantle nor the inclusion of a granitic layer in the crustal model had any significant effect in reducing the residuals. The local \( P^* \), \( S^* \) velocities of 6.6 km \( s^{-1} \) and 3.7 km \( s^{-1} \) (Hamilton 1966) were therefore used, and the thickness of the parallel-sided crustal slab was found to be 24 km ± 1 km at Wellington, from 14 observations, and 31 km ± 2 km at Gisborne, from 19 observations.

The standard deviation of the residuals in Table 1 is 0.8s. This rather large value would seem to indicate that the concept of a plane Mohorovičić Discontinuity may be inadequate.

### 3.3 Errors

The most significant error arises from uncertainty in the adopted hypocentres. For local earthquakes the incidence angles are critically dependent on the distance and depth to the focus. The error was estimated by a worst case analysis, whereby each focus was adjusted in latitude, longitude and depth by the appropriate standard errors in location. Although these changes were small (e.g., 0.06° in latitude) they resulted in a change of up to 10 per cent in the calculated lead times.

Accurate location of hypocentres is thus essential for studies of this type. Note also that the standard error measures only the spread of the data, and not its accuracy, so the true error may be larger. The error in calculated lead time due to mislocation could be as great as 0.3s, but could not explain fully the scatter in observed values.

Uncertainties of 0.1 km \( s^{-1} \) in the velocities increase the errors in crustal thickness to ±3 km at WEL and ±4 km at GNZ.

### 4. Comparison with earlier results

The determination of crustal thickness at GNZ is consistent with that of Thomson & Evison (1962), who found a thickness of 30–40 km over most of New Zealand, using Rayleigh wave dispersion, and that of Reilly (1962), who obtained an average thickness of 32–39 km and a normal thickness at sea level of 29–34 km, by using formulae relating to Bouguer gravity anomalies. These thicknesses are rather larger than the proposed value for Wellington, as is Garrick’s (1968) value of 36 km, obtained from a refraction profile. It should be noted that surface wave methods yield a mean thickness over the propagation path, not for a small area. Gravity anomaly methods show up variation in thickness, but lack an accurate absolute calibration and depend on the assumption that Bouguer anomalies are due entirely to changes in crustal thickness. In the region examined near Wellington the Bouguer anomaly varies from −25 to −50 mgal, and the isostatic anomaly from −50 to
Near Gisborne the variations are both -50 to -75 mgal. The discrepancy between the present value at Wellington and that of Garrick may be due to the geographic locations; the Sp results refer to a region 30-50 km out to sea, in the north-west quadrant from Wellington, while the refraction data refer to the mountainous Wellington province to the north-east of the station.

5. Conclusions

$S$ to $P$ conversion has been identified on short-period records of local deep earthquakes and events at a distance of 20–25 degrees. While calculations based on one earthquake may be inconclusive, use of a number of events can produce more reliable values of crustal parameters. Using the local shocks, mean values have been obtained for the thickness of the crust near Wellington (24 km ± 3 km) and Gisborne (31 km ± 4 km). Excessive spread in the data prevented the determination of velocities or the detection of a dip in the Mohorovičić Discontinuity. When velocities can be obtained in this way, they apply to the lower portion of the crust, as the technique is relatively insensitive to variations nearer the surface.

The method outlined above is applicable where stations are situated near well-located deep earthquakes, especially if these occur over a wide range of azimuths. The existence of a close network of stations in the North Island of New Zealand makes it a particularly appropriate region for these studies. It may also be possible to extend the method to determine small-scale variations in crustal thickness, using the variation in lead-time residual with the position of the conversion point.

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References

