The Possible Existence of a Neutral Meson with Isotopic Spin 0

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The possible existence of a neutral meson with isotopic spin 0 is suggested. Its effects on pion physics and some ways to test its existence are discussed.

§ 1. Introduction

We now have rather good evidence for the charge independence of all strong interactions. In particular, the low energy nuclear physics has supplied us with the strongest evidence in favour of this symmetry property. Although we have many arguments in favour of accepting the symmetrical theory of pions, which of course is charge independent, direct tests of this symmetrical theory have not been sufficiently accurate so exclude all other charge independent theories of undiscovered mesons. In particular, experimental evidence does not exclude the existence of a neutral meson with isotopic spin 0 in addition to the usual charge-triplet pion (and K-mesons). Considering the existence of both charge triplet and singlet hyperons (Σ and Λ), it is quite attractive to postulate the existence of a charge singlet meson. Assuming such a meson to exist, we denote it by \( \pi' \), which must not be confused with the neutral member \( \pi^0 \) of charge-triplet pion. It is the purpose of this paper to discuss possible effects of this \( \pi' \) in pion physics, as well as its possible detection.

§ 2. Properties of \( \pi' \)

We are interested mainly in such a \( \pi' \) as can affect the low energy meson physics. So we consider only two types of mesons, the usual pion which obeys the symmetrical theory and the new meson \( \pi' \) which obeys the neutral theory. To avoid immediate contradiction with our experience, we would like to attribute the following properties to \( \pi' \):

i) The mass \( \mu' \) of \( \pi' \) is nearly equal to the mass \( \mu^0 \) of \( \pi^0 \), e.g., \( \mu^0 \lesssim \mu' \lesssim \mu^0 + 30 \text{ Mev.} \)

ii) The spin, the isotopic spin and the strangeness are all zero.

iii) It interacts with baryons as strongly as the pion does.

iv) It decays into two \( \gamma \)-rays with the lifetime \( \lesssim 10^{-14} \text{ sec.} \)

Unless we use the Panofsky effect (which we shall discuss later), we have so far

* We do not discuss here another case, \( \mu' \geq 2 \mu^0 \), which has been considered by Nambu, ref. 10.)
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had no accurate method to measure the mass of neutral mesons. Therefore we may conclude that the above conditions imposed on $\pi'$ are sufficient to make $\pi'$ and $\pi^0$ indistinguishable. As for the Panofsky effect, one can find a special model which avoids the simultaneous emission of $\pi^0$ and $\pi'$ after the $\pi^-$ capture. One of such models is given by a special assumption

$$\mu' \gtrsim \mu,$$

where $\mu$ is the charged pion mass. Other possibilities will be discussed later.

An interesting feature of $\pi'$ is worth mentioning. Since the isotopic spin of $\pi'$ is zero, the pion-nucleon (or briefly $\pi-N$) scattering with total isotopic spin $I=3/2$ has nothing to do with this $\pi'$. In fact, there is beautiful agreement\(^1\) between the pion theory and the experiments for the $\pi-N$ scattering with $I=3/2$. This is why we have assumed $\pi'$ to have the isotopic spin 0. On the other hand, the $\pi-N$ collision in the $I=1/2$ state can lead to a final state in which the nucleon is accompanied by a $\pi'$. The symmetrical theory of pions alone seems to be in disagreement\(^1\) with experiment in this case. The introduction of $\pi'$ may serve to remove this discrepancy (also see ref. 5)).

Let us discuss the $\pi-N$ collision with $I=1/2$ in a little more detail. We have here two eigen states for each value of the total charge\(^0\). For example, the charge wave functions for $I=1/2$ and $I_3=-1/2$ may be written in obvious notation

$$\begin{align*}
(\cos \xi) \sqrt{2} \left( \frac{2}{3} \langle \pi^- p \rangle - \langle \pi^0 n \rangle \right) + (\sin \xi) \langle \pi' n \rangle, \\
(\cos \xi) \langle \pi' n \rangle - (\sin \xi) \sqrt{2} \left( \frac{2}{3} \langle \pi^- p \rangle - \langle \pi^0 n \rangle \right),
\end{align*}$$

(1, 2)

and

where $\xi$ is real. For comparison, we also notice that the charge wave function for $I=3/2$ and $I_3=-1/2$ is

$$\begin{align*}
\frac{(\pi^- p) + \sqrt{2} (\pi^0 n)}{3}, \\
\frac{(\pi^- p) + \sqrt{2} (\pi^0 n)}{3}.
\end{align*}$$

(3)

Let $\delta_1$, $\delta'$ and $\delta_3$ be the eigen phase-shifts for these three eigen states, (1), (2) and (3). Then the $R$-matrix elements for meson-nucleon collisions are easily found and listed in Table I.

<table>
<thead>
<tr>
<th>Process</th>
<th>$R$-matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ + p \rightarrow p + \pi^+$</td>
<td>$R_3$</td>
</tr>
<tr>
<td>$\pi^- + p \rightarrow p + \pi^-$</td>
<td>$\frac{1}{3} (R_3 + 2'' R_1''')$</td>
</tr>
<tr>
<td>$\pi^+ + p \rightarrow n + \pi^+$</td>
<td>$\sqrt{3} (R_3 - &quot; R_1'&quot; )$</td>
</tr>
<tr>
<td>$\pi^- + p \rightarrow n + \pi^-$</td>
<td>$\frac{2}{3} \sin \xi \cos \xi (R_1 - R')$</td>
</tr>
<tr>
<td>$\pi^+ + p \rightarrow n + \pi'$</td>
<td>$(\sin^2 \xi) R_1 + (\cos^2 \xi) R'$</td>
</tr>
<tr>
<td>$\pi^- + p \rightarrow n + \pi'$</td>
<td></td>
</tr>
<tr>
<td>$\pi^+ + n \rightarrow n + \pi'$</td>
<td></td>
</tr>
<tr>
<td>$\pi^- + n \rightarrow n + \pi'$</td>
<td></td>
</tr>
</tbody>
</table>

Table I

In this table we have put

$$R_3 = e^{i \delta_3} - 1, \quad R_1 = e^{i \delta_1}, \quad R' = e^{i \delta'} - 1$$

(4)

and

$$" R_1''' = (\cos^2 \xi) R_1 + (\sin^2 \xi) R'.$$
If $\varepsilon = 0$, the results in Table I will reduce to the usual symmetrical pion case. If $\pi'$ exists, we see that in performing the phase shift analysis, there are more parameters than before. Therefore, the revised phase shift analysis including $\pi'$ is extremely interesting, and we may expect to obtain better agreement between theory and experiment than before.

§ 3. The parity of $\pi'$ and the meson-nucleon scattering

We have not yet specified the parity of $\pi'$. There are two possibilities, the scalar and the pseudo-scalar cases. Let us discuss these two cases in more detail.

$\S$ 3 · 1 $\pi'$: scalar

First we shall discuss the scalar case. The incoming $s$ (or $p$)-wave pion must be converted into an outgoing $p$ (or $s+d$)-wave $\pi'$ in the $\pi N$ collision with $I=1/2$. Hence, $\pi'$ will hardly come out after the collision of the slow $\pi^-$ with the proton and the $s$-wave pion-nucleon scattering near zero energy is essentially same as before. The $\pi^-$ capture by the proton from the $s$-state is similarly unaffected by $\pi'$. The $\pi^-$ capture from the $p$-states may give rise to substantial $\pi'$-emission. If it proves necessary to predict more neutral meson emission in the Panofsky effect, one can appeal to this effect. To do this, we have to assume close equality of masses of the two neutral mesons: $\mu' = \mu \pm 0.2$ MeV. If we do not want the $\pi^-$-emission in the Panofsky effect, one can assume, for instance that $\pi'$ is heavier than the charged pion.

If $\mu' \approx \mu$, one can expect that the incoming $p$-waves of $\pi^-$ on the proton will lead mostly to an outgoing $s$-wave of $\pi'$ at low energy region. In other words, the $p$-wave part of the total cross-section of $\pi^-$-$p$ collision at low energies is due almost entirely to the charge exchange scattering $\pi^- + p \rightarrow n + \pi'$. This is, however, not the case, so that we are inclined to choose $\pi'$ to be considerably heavier than the pion mass.

An annoying thing is the neutral meson production by proton-proton collision. If $\pi'$ is scalar, we would expect sizable $\pi'$-production even near the threshold in contradiction with experiment, unless there is some accidental cancellation of matrix elements near the threshold.

On the other hand, one can check the existence of the scalar $\pi'$ by means of the reactions

$$d + d \rightarrow \alpha + \pi',$$

and

$$\alpha + \alpha \rightarrow 2\alpha \text{ (or } Be^8) + \pi'. \quad (5)$$

The $\pi'$-production in these reactions is forbidden by the isotopic spin conservation.

From these considerations the existence of a scalar $\pi'$ with $\mu' = \mu$ seems to be unlikely.

$\S$ 3 · 2 $\pi'$: pseudoscalar

Next, we proceed to discuss the pseudoscalar case. As is easily seen, there are no difficulties in near zero energy events like the Panofsky effect, the mesic-atom, and the
s-wave $\pi-N$ scattering, if one assumes that the mass $\mu'$ of $\pi'$ is heavier than the charged pion mass $\mu$ by, say, about 30 Mev. Therefore it is only interesting to investigate the case $\mu' \approx \mu^0$. Before going into the detail, let us define:

$$a_3 = \delta_3/\eta, \quad a_1 = \delta_1/\eta, \quad \text{and} \quad a' = \delta'/\eta,$$

where $\delta_3$, $\delta_1$, and $\delta'$ are the eigen phase shifts for s-wave mesons (see Table I) and $\eta = (\text{meson momentum})/(\text{meson mass})$ as usual. For small $\eta$ the $a$'s are practically constant. From the low energy data of

$$\pi^+ + p \rightarrow p + \pi^+$$

and

$$\pi^- + p \rightarrow p + \pi^-$$

one finds $^3$)

$$a_3 = -0.11,$$

$$\"a_1" = (\cos^2 \xi) a_1 + (\sin^2 \xi) a'$$

$$= 0.16.$$ 

While the charge exchange scattering, i.e., the sum of $\pi^- + p \rightarrow n + \pi^0$ and $\pi^- + p \rightarrow n + \pi'$, gives information about the value of

$$\frac{3}{2} |a_3 - \"a_1\"|^2 + \frac{3}{2} (\sin \xi \cos \xi) |a_1 - a'|^2.$$ 

We see that (7) and the experimental value for (8) are consistent with

$$a_3 = -0.11,$$

or

$$a_1 = a' = \"a_1\" = 0.16 \quad \text{and} \quad a_3 = -0.11.$$ 

These results show that the cross section of

$$\pi^- + p \rightarrow n + \pi'$$

vanishes at a very low energy. Thus the Panofsky effect is entirely due to $\pi^0$. We know that (9) seems to be compatible not only with the Panofsky effect but also with information about the $a$'s from the $\pi^-$-mesic atom. The $\pi'$ can of course come out in s-waves at high energies and also in higher waves in $\pi-N$ collisions.

We want to investigate the effect of $\pi'$ on the p-wave $\pi-N$ collisions. For the time being, it would be unnecessary to make fancy calculation. Hence, we shall use the simple $ps(p\pi)$ static models $^4$ for both the symmetrical pions and the $\pi'$, and the Born approximation to the phase shifts. We also assume for simplicity $\mu' = \mu$. Let us denote the (renormalized) coupling constants of $\pi-N$ and $\pi'-N$ interactions by $f$ and $f_0$. Then we can describe the pion-nucleon scattering in terms of the usual phase shifts $\alpha$'s (which are the same as in ref. 4):
\[ \alpha_{11} = -\frac{\theta}{3} \]
\[ \alpha_{21} = \alpha_{12} = -\frac{2}{3} \]
\[ \alpha_{33} = \frac{\pi}{3} \]
\[ \times \frac{f^2}{4\pi} \frac{p^3}{\omega \mu^2} \tag{10} \]

where \( p \) is the meson momentum and \( \omega = \sqrt{p^2 + \mu^2} \) \((b = c = 1)\). The process \( \pi^+ N \rightarrow N + \pi^+ \) with \( I = 1/2 \) can be described by the two phase shifts,

\[ \alpha_{13}' = +\frac{\theta}{3} \]
\[ \alpha_{11}' = -\frac{\theta}{3} \]
\[ \times \frac{f^2}{4\pi} \frac{p^3}{\omega \mu^2} \tag{11} \]

where \( \alpha_{13}' \) is the phase shift for the \( p_N \)-state \((J = 1/2 \text{ or } 3/2)\). Finally, for the process \( \pi^+ N \rightarrow N + \pi^+ \), one finds the phase shifts:

\[ \alpha_{13}'' = +\frac{\theta}{3} \]
\[ \alpha_{11}'' = -\frac{\theta}{3} \]
\[ \times \frac{f^2}{4\pi} \frac{p^3}{\omega \mu^2} \tag{12} \]

where \( \alpha_{13}'' \) is the \( p_N \)-phase shift.

We shall assume throughout in this paper (also see § 4-3)

\[ f^2 \lesssim f^2 \tag{13} \]

The scatterings involving \( \pi^+ \) have relatively small phase shifts, \( \alpha_{13}' \) and \( \alpha_{11}' \), which, however, can be comparable with small phase shifts like \( \alpha_{11} \) or \( \alpha_{13} \). Accordingly, the discrepancy between the old theory\(^1\) and the experiments can now be overcome by the introduction of the \( \pi^+ \). It should be noted that the total cross section \( \sigma (\pi^+ \rightarrow \pi^+) \) for \( \pi^- + p \rightarrow n + \pi^+ \) is \( \lesssim 10\% \) of the \( \pi^- + p \) total cross section\(^5\).

### Table II

<table>
<thead>
<tr>
<th>Total isotopic spin ( I )</th>
<th>Eigen-state (Example only)</th>
<th>Eigen phase shift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((\pi^+ p)) ((\pi^- p) + \sqrt{2}(n^0 n)) (\sqrt{3})</td>
<td>(\delta_3) (\delta_3) (\delta_{33})</td>
</tr>
<tr>
<td>3/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td>((\cos \varepsilon)\frac{\sqrt{2}}{\sqrt{3}}(\pi^- p) - (n^0 n) + (\sin \varepsilon) (\pi^+ n))</td>
<td>(\delta_1) (\delta_{11}) (\delta_{13})</td>
</tr>
<tr>
<td>1/2</td>
<td>((\sin \varepsilon)\frac{\sqrt{2}}{\sqrt{3}}(\pi^- p) - (n^0 n) + (\cos \varepsilon) (\pi^+ n))</td>
<td>(\delta') (\delta_{11}') (\delta_{13}')</td>
</tr>
</tbody>
</table>

Note: The \( \varepsilon \) depends on the state; \( \varepsilon \) for the \( s_{1/2} \) state and \( \varepsilon_{13} \) for the \( p_{J}\)-state \((J = 1/2 \text{ or } 3/2)\).

It is interesting to calculate the eigen phase shifts, and the parameters \( \varepsilon \) (see Table
II). It is easy to find them from eqs. (10) – (12):

\[
\begin{align*}
\varepsilon_{11} &= \varepsilon_{13} = \varepsilon, \quad \tan 2\varepsilon = -2\sqrt{3} \frac{f_{0}/(f^2 + r_{0}^2)}{g_{1} + g_{3}}, \\
\delta_{13} &= \alpha_{33}, \quad \delta_{31} = \alpha_{31}, \\
\delta_{13} &= \left( f_{0}^2 - f^2 + \frac{2\sqrt{3} f_{0}}{\sin 2\varepsilon} \right), \\
\delta_{11} &= \left( 2f^2 + f_{0}^2 - \frac{2\sqrt{3} f_{0}}{\sin 2\varepsilon} \right), \\
\delta_{13}' &= \left( f_{0}^2 - f^2 - \frac{2\sqrt{3} f_{0}}{\sin 2\varepsilon} \right), \\
\delta_{11}' &= \left( 2f^2 + f_{0}^2 - \frac{2\sqrt{3} f_{0}}{\sin 2\varepsilon} \right).
\end{align*}
\]

(14)

In the special case

\[ f_{0} = \pm f, \]

one finds

\[ \sin 2\varepsilon = \mp 1/2, \]

and

\[ \delta_{11} = \frac{1}{2} \delta_{13} = -\delta_{11}' = \frac{1}{2} \delta_{13}' = \frac{1}{7} \delta_{11}' = -\frac{2}{3} \frac{f^2}{4\pi \mu \varepsilon \omega}. \]

(15)

In contrast to the scalar case, there is no difficulty of the neutral meson production by the proton-proton collision near the threshold, since the same reasoning as for the \( \pi^0 \) case can be applied to the pseudoscalar \( \pi' \).

A direct test of the existence of a pseudoscalar \( \pi' \) is rather difficult, because the reactions (5) are now forbidden. So that we have to investigate meson production processes, where both pions and \( \pi' \) are involved, very carefully. For example, we can use the famous reactions

\[
\begin{align*}
p + d &\rightarrow \left\{ \begin{array}{c}
H^3 + \pi^+ \\
He^3 + \pi^0, He^3 + \pi'
\end{array} \right. \quad (16) \\
p + p &\rightarrow d + \pi^+ \\
p + n &\rightarrow d + \pi^0, d + \pi'
\end{align*}
\]

(17)

for this purpose. An experimental accuracy better than \( \sim 5\% \) will be required in order to determine whether the reactions \( p + n \rightarrow d + \pi' \), and \( p + d \rightarrow He^3 + \pi' \) occur or not (see ref. 5) and 6)). We have so far rather poor knowledge about the positive pion to neutral meson ratios for (16) and (17). It is wise to avoid the 33 resonance region to test...
the existence of the \( \pi' \). Thus it is interesting to investigate experimentally the single meson production by the \( \pi N \) or \( N N \) collision at off-resonance energies.

§ 4. Other comments on the \( \pi' \).

§ 4.1 Decay processes of hyperons and \( K \)-mesons.

If the \( \pi' \) exists, it is natural to suppose that one of the decay products of \( \Sigma \), \( \Lambda \) and \( K \)-mesons could be the \( \pi' \). Then, one can expect

\[
\Sigma^+ \rightarrow \begin{cases} p + \pi^0 \\ p + \pi' \end{cases}
\]

These two processes should be easily discriminated by measuring the energy of the proton, if \( \mu^0 \neq \mu' \). Similarly, there are the following decay modes of \( K \)-mesons

\[
K^+ \rightarrow \pi^0 + \pi', \\
K^+ \rightarrow \pi^+ + 2\pi', \\
K^+ \rightarrow 2\pi^0 + \pi', \\
\theta^+ \rightarrow \pi^+ + \pi^- + \pi', \\
\theta^0 \rightarrow \pi^0 + 2\pi', \\
\theta - \pi'.
\]

(18)

It is interesting to note that (18) is consistent with the so-called \( JI = 1/2 \) law. Evidently these decay processes are useful to check the existence of \( \pi' \).

§ 4.2 Dispersion relation.

A few words should be added concerning the dispersion relation for the pion-nucleon scattering. Even if we introduce the \( \pi' \), the form of the dispersion relations (see ref. 7) for the \( \pi^+ + p \rightarrow p + \pi^0 \) does not change at all. In other words, as far as we use experimentally measured values for the forward scattering amplitudes and the total cross sections for \( \pi^+p \) collisions, all effects due to the \( \pi' \) are fully taken into account. Therefore, origins of the difficulty emphasized by Puppi-Stanghellini(7), if it is real, cannot be attributed to the existence of \( \pi' \). Fortunately, Lomon and Zaidi(8) have shown that the difficulty can be removed by using somewhat different values of total cross sections from those used by Puppi-Stanghellini. We believe that the proposal of Lomon-Zaidi is quite reasonable.

§ 4.3 The meson-nucleon coupling constants.

We have two coupling constants \( f \) and \( f_0 \) in our theory. According to Otsuki et al(9), one can determine the strength of the meson-interaction by means of two nucleon data in the low energy region, among which the quadrupole moment of the deuteron plays the most important role. If the \( \pi' \) is scalar, the nuclear forces due to the \( \pi'N \) interaction do not contain non-central force, so that we can say very little about
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the coupling constant $f_0$. On the other hand, if the $\pi'$ is pseudoscalar, one finds

$$\frac{f^2}{4\pi} = \frac{1}{3} \frac{f_0^2}{4\pi} \approx 0.08.$$ 

The accuracy of this determination is rather hard to estimate, but our feeling is that $0.08 \pm 0.03$. If we accept tentatively this determination we find

$$\frac{f_0^2}{4\pi} \lesssim \frac{f^2}{4\pi},$$

a condition which has already been used in §3.2.

Moreover one can expect the resonance for the $\pi'-N$ scattering. The position of this resonance $\omega_0$ will be $\sim (f^2/f_0^2) \omega_r (\omega_r \approx 2.2\mu$ is the 33 resonance energy). This resonance can affect the $\pi-N$ collision with $I=1/2$. If $f_0^2 \approx f^2$, this will be masked by the strong 33 resonance, while if $4f_0^2 \lesssim f^2$ this can partly be responsible to the second resonance of the $\pi-N$ collision with $I=1/2^*$ around 1 Bev.

§ 4·4

We can in principle test the “symmetrical” pion theory by the multiple meson production at extremely high energy nuclear collisions. There, the ratio of the charged pions to the neutral “mesons” is only known within a substantial uncertainty. Therefore there is still room for additional $\pi'$-production.

§ 4·5

We have so far discussed the possible existence of a neutral meson $\pi'$, which can affect low energy pion-phenomena.

Another possibility will be the case $\mu' \gtrsim 2\mu$, in which any explicit effects of the $\pi'$ can hardly be expected on the low energy pion-phenomena. Some interesting discussions of such a case have recently been published by Nambu[10]. Gell-Mann[11] and Schwinger[12] have discussed the possible existence of a neutral meson with isotopic spin 0 from points of view entirely different from ours.

The discussion presented here may or may not be useless. We shall see in the near future which is the case. The existence of both charge triplet and singlet hyperons ($\Sigma^0$ and $\Lambda$) is so impressive that the existence of a charge singlet meson $\pi'$, whatever it may be, seems to be quite natural.

The author would like to acknowledge the hospitality of CERN, Geneve, where the most part of the work was done.

Note added in proof Riddiford et al. (private communication) have recently measured the cross sections for single meson production in $p-p$ and $p-n$ collisions at 970 Mev. In particular they have found the neutral meson production cross section in $p-n$ collisions which is 2--3 times bigger than that expected from the “symmetrical” pion theory. This is a quite interesting result for our proposal presented here, though there remain many possible effects to be considered within the framework of the usual pion theory.

* I am indebted to Dr. Miyazawa for this remark.
References

2) General considerations of $\pi$-$N$ and $\pi'$-$N$ scattering were published long ago by Y. Nambu and the present author, Prog. Theor. Phys. 6 (1951), 1000.
3) J. Oear, Phys. Rev. 100 (1955), 188.
5) This is seen as follows: the $s$-phase shifts $\delta_1$, $\delta_2$ and $\delta'$ are calculated from eqs. (6) and (9), the $p$-phase shifts $\alpha_{2j+l}$ from the effective range formulae given by Chew and Low, ref 4), and the remaining phases $\alpha_{1'j}$ by the Born approximation, eq. (11). Then we can calculate the differential cross sections for the ordinary scatterings

$$\pi^\pm + p \rightarrow p + \pi^\pm$$

and for the sum of two reactions:

$$\pi^\pm + p \rightarrow \begin{cases} n + \pi^0 \\ n + \pi' \end{cases}.$$

These results are in fair agreement with the observed ones in the sub-resonance region (We have assumed here $f_0^2 = f^2$).
6) For $f_0^2/4\pi = f^2/4\pi = 0.08$ one finds,

$$\sigma(\pi^\pm \rightarrow \pi^\pm) \approx \begin{cases} 0.5 \text{ mb at } 40 \text{ MeV} \\ 10 \text{ mb at resonance} \end{cases}$$

7) G. Puppi and A. Stanghellini, Nuovo Cimento 5 (1957), 1305