Effect of interarray elevation differences on infrasound beamforming

Wayne N. Edwards\textsuperscript{1} and David N. Green\textsuperscript{2}

\textsuperscript{1}Natural Resources Canada, Atlantic and Western Canada Branch, Canadian Hazards Information Service, Earth Sciences Sector, 7 Observatory Crescent, Ottawa, Ontario, K1A 0Y3, Canada. E-mail: wayne.edwards@nrcan-rncan.gc.ca

\textsuperscript{2}AWE Blacknest, Brimpton Common, RG7 4RS, Reading, UK

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SUMMARY
The International Monitoring System infrasound network will, upon completion, contain 60 microbarometer arrays with apertures of between 1 and 4 km. These arrays are located within a variety of terrains, leading to large ratios of interelement elevation differences to array aperture for those arrays situated in areas of significant topography. Systematic errors in beamforming estimates caused by neglecting the vertical extent of the arrays, are quantified for both signal backazimuth and apparent velocity. Of the 43 arrays certified as of 2011 January, I14CL on Juan Fernandez Island has the greatest topography across an array, with a least-squares fitted plane through the array elements having an 8.1° slope from the horizontal (compared to a network mean of 1.6°). Beamforming errors (both backazimuth and apparent velocity) are a function of the arrival azimuth and become increasingly large for steeply inclined arrivals, such that systematic errors will be significantly larger for signals returned from the thermosphere compared to those from the stratosphere. At several arrays, azimuthal errors due to array topography are comparable in magnitude to deviations often associated with atmospheric propagation. These findings are illustrated using signals recorded in Greenland at I18DK, where differences between results processed using both full 3-D array geometry and the 2-D (topography neglected) approximation exhibit good correspondence to theoretical predictions.

Key words: Seismic monitoring and test-ban treaty verification; Site effects; Acoustic properties.

1 INTRODUCTION
Analysis of infrasound data requires accurate determination of acoustic wave orientation parameters (signal backazimuth, horizontal apparent speed), as these values are used to constrain source locations and atmospheric propagation paths (e.g. Evers et al. 2007; Ceranna et al. 2009). These parameters are estimated by identifying the speed and direction with which pressure transients propagate across distributed arrays of microbarographs that are designed to ensure signals are coherent across the array, while attempting to reduce noise coherence between microbarograph sites. These constraints lead to the construction of infrasound arrays with apertures a few kilometres in extent, for acoustic waves in the 0.01–10 Hz passband. In some geographical locations arrays of this aperture size are situated in areas of pronounced topography, as has been the case for the global infrasound station network being developed as part of the verification measures for the Comprehensive Nuclear-Test-Ban Treaty (CTBT). These elevation differences between microbarometer locations influence the determination of acoustic wave orientation, and result in errors in inferred backazimuths and apparent speed if these topographic differences are ignored.

The International Monitoring System (IMS) being constructed as part of the verification measures for a CTBT includes seismic, hydro-acoustic, infrasound and radionuclide stations, providing the ability to monitor globally for nuclear explosions located underground, underwater and in the atmosphere (e.g. Committee on Technical Issues CTBT 2002). The proposed infrasound component of the IMS is a network of 60 microbarometer arrays (Christie et al. 2001), of which 43 are operational as of 2011 January. Each infrasound array consists of between four and 15 microbarometers, with apertures ranging between 1.1 and 3.9 km (median 2.2 km). To ensure global coverage and satisfactory geographical spread (e.g. Le Pichon et al. 2009; Green & Bowers 2010), some arrays in the network have had to be located in challenging environments. In such environments it is often difficult to find ideal terrain to place the instruments upon (e.g. remote oceanic islands), and in some cases arrays have been built in areas of significant topography resulting in the array being inclined rather than ideally horizontally level.

Topographic differences are not an issue for all distributed sensor arrays. In hydro-acoustics, topography is not a significant issue as sensors can be placed as a horizontally level array within the water column (Lawrence & Grenard 1998). In seismology, telesismic arrays often have sufficiently large apertures (ranging between 10 and nearly 100 km; Rost & Thomas 2002) in comparison to the topography over which they extend, such that relatively small vertical differences in instrument position can generally be ignored.
Yet Wang et al. (1999) showed that at the Nevada seismic array (NVAR), a smaller than ∼5.5 km aperture seismic array located in mountainous topography, signals exhibited systematic errors in both backazimuth and apparent speed estimates during array processing. These errors are the result of neglecting the nearly 500 m of elevation between sensors within the array using standard array processing. As all IMS infrasound arrays in operation are smaller than NVAR, similar errors can result with smaller elevation differences if the topography is neglected during infrasound data processing.

Incorporation of topographic differences between sensors in array processing for infrasound has been raised previously by Kennett et al. (2003) and more recently Brown et al. (2008) incorporated 3-D array beamforming techniques into a routine for detecting infrasonic signals using the Hough transform, taking into account the vertical extent of an array. Yet neither Kennett et al. nor Brown et al. elaborate on the effects that occur in array processing when significant, systematic array topography is neglected (2-D).

In this study we explore how systematic elevation differences between microbarometers in an array influences beamforming results (Section 2), and assess the impact this influence has at each of the 43 infrasound stations of the IMS infrasound network (Section 3). Finally we provide examples of how backazimuth and apparent velocity estimates vary when considering 2-D and 3-D array processing for signals recorded at station I18DK and compare these results with theoretical predictions (Section 4).

2 INCLINED VERSUS LEVEL PLANE BEAMFORMING

The construction of a distributed array of sensors to record the passage of a wave is a standard method in seismology, hydro-acoustics and infrasound, to estimate a coherent transient wave’s orientation and speed for the purposes of source location, phase identification or simply signal-to-noise enhancement (Rost & Thomas 2002; Douglas 2002). This is made possible by exploiting the fact that the transient wave propagates in the various media with a finite velocity and so is observed by each sensor at different times, Δt. The relative time of this delay will depend upon a sensor’s location, \( r = (r_x, r_y, r_z) \), in relation to a reference location, as well as the propagation direction and speed of the impinging wave front or equivalently its vector slowness, \( S = (s_x, s_y, s_z) \), which is oriented perpendicular to the wave front with a magnitude equal to the inverse of the wave propagation speed. This can be expressed as

\[
\Delta t = -(r \cdot S),
\]

or expanded as

\[
\Delta t = -(r_x s_x + r_y s_y + r_z s_z),
\]

under the assumption that the curvature of the wave front over the aperture of the array is sufficiently small that it may be approximated by a planar wave. The negative sign in eqs (1) and (2) is introduced to associate the direction of the incoming slowness vector, \( S \), with its direction of approach as seen by the array.

Determination of the slowness vector for any arbitrary incident wave is then achieved by identifying the orientation that best reproduces the observed time delays at the known sensor spacing.

<Figure 1. Geometry of an inclined array plane relative to the horizontal plane. The inclined plane is described by the normal to the plane, \( n \), and the gradient vector, \( g \), oriented with an azimuth of, \( \psi \), and inclination, \( \alpha \).>

<Figure 2. A schematic showing the effect of the 2-D approximation on an inclined array. (a) The 3-D array (black dots) observes a wave impinging with the slowness vector shown by the black arrow, recording signals with the time delays exhibited by the synthetic traces in panel (b). The horizontal projection of the true slowness vector is shown in panel (a) by the grey arrow. Panel (c) shows the effect of estimating the horizontal slowness vector assuming that the array is flat (2-D). The observed time delays shown in (b) now need to be explained by this approximate geometry, which shifts the estimated slowness vector to that shown by the black dashed arrow (shown next to the true slowness projection in grey).>
or when definitive signal onsets are unclear, where the computed delays maximize the correlation between recordings. This process is referred to as beamforming, and in practice often requires a search over a region of slowness space to determine the slowness vector components that best reproduce the observed time delays, using a quantitative measure for the goodness of fit (e.g. Blandford 1974; Cansi 1995; Selby 2008). The computational expense of grid searches are strongly dependent upon the number of free parameters which must be determined, therefore beamforming is often carried out only in the horizontal plane under the assumption that the vertical extent of the array is a negligibly small contributor to the observed time delay (2), that is,

$$|r_z \cdot s_z| \ll |r_x \cdot s_x + r_y \cdot s_y|.$$  

(3)

This reduces the problem from a 3-D search to two dimensions such that only the signal’s backazimuth, \(\theta\), and apparent speed across the array, \(v_{app}\), are determined where

$$\tan \theta = \frac{s_x}{s_y}.$$  

(4)

\(v_{app} = \frac{1}{\sqrt{s_x^2 + s_y^2}} = \frac{v}{\sin i},\)  

(5)

where \(v\) is the signal’s wave propagation speed and \(i\) is the incidence of the wave on the array as measured from the vertical.

In the case where an array is sufficiently inclined due to topography, the assumption of eq. (3) is no longer valid and leads to systematic errors in both backazimuth and apparent velocity, becoming increasingly large as the slope of the incline increases. To illustrate this, suppose that an infrasound array is situated upon an inclined plane with a slope of \(\alpha\), measured from the horizontal and defined by unit normal, \(n\). The gradient of this plane, \(g\), is then oriented in the direction, \(\psi\), measured from the \(y\)-axis (North) as shown in Fig. 1. If a plane wave with slowness, \(S\), impinges upon an array the time delay between the wave arriving at each array element is a function of the relative positions of the instruments (eq. 1). An example is shown in Fig. 2, where a plane wave travelling through an atmosphere with an acoustic velocity of 0.33 km s\(^{-1}\) impinges upon a five-element array orientated such that \(\alpha = 5^\circ\) and \(\psi = 0^\circ\). The wave impinges upon the array with a backazimuth, \(\theta = 270^\circ\), and

Figure 3. Azimuth and apparent speed error resulting from 2-D approximation for a planar array inclined at 5\(^\circ\) due North with a local sound speed of 0.33 km s\(^{-1}\) (as in Fig. 2). Left-hand side: Error in azimuth, \(\Delta \theta\). Right-hand side: Error in apparent speed, \(\Delta v_{app}\), as defined in eq. (6). Dashed lines show slowness incidence at 15\(^\circ\) intervals (centre is vertical or 0\(^\circ\) incidence). Only values for slowness incidence > 30\(^\circ\) are shown with contours of 1\(^\circ\) and 0.01 km s\(^{-1}\), respectively.

Figure 4. Schematic diagrams showing the regions of the 3-D slowness space examined by different detection algorithms. For all scenarios the atmospheric acoustic velocity is assumed to vary between 310 and 360 m s\(^{-1}\) and the angle of wave incidence upon the array varies between 45\(^\circ\) and 90\(^\circ\) from the vertical. Panel (a) displays the 2-D annulus searched over if the array geometry is assumed to be flat. Panel (b) shows the 3-D surface which is searched if the atmospheric velocity is assumed known (at 330 m s\(^{-1}\) in this case), while panel (c) shows the 3-D volume which is searched if the atmospheric velocity is assumed to be unknown.
an incidence angle from the vertical, \( i = 60^\circ \) (Fig. 2a). If the time-delay observations (Fig. 2b) are then used in conjunction with the full 3-D location of the array elements, the resulting beamformed slowness will be that of the actual wave—with backazimuth, \( \theta \), and apparent velocity, \( v_{\text{app}} \). If instead the 2-D assumption is made, such that the vertical component of the array is considered negligible, then the same time-delay observations are assumed to have been recorded by an array geometry which is different from that which were recorded. This new geometry results in a slightly different inferred slowness with backazimuth, \( \theta' \), and apparent velocity, \( v'_{\text{app}} \) (Fig. 2c). The difference between the two estimates of \( \theta \) and \( \theta' \) or \( v_{\text{app}} \) and \( v'_{\text{app}} \) are then

\[
\Delta \theta = \theta' - \theta \quad \text{and} \quad \Delta v_{\text{app}} = v'_{\text{app}} - v_{\text{app}}.
\]

These differences represent the inherent systematic error made for each measurement of the slowness vector by assuming a level geometry for an array that is inclined. In general, for all possible orientations of the incident slowness, the backazimuth and apparent

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**Figure 5.** Variation of time-delay residuals for a grid-search determining the best fitting local sound speed \( (C_S) \), incidence angle from vertical \( (i) \) and backazimuth \( (\theta) \) for a synthetic ideal plane wave incident from a 90° backazimuth at the idealized five-element array of Fig. 2. Two sets of synthetic time delays were calculated, representing thermospheric \( (i = 45^\circ) \) and stratospheric \( (i = 75^\circ) \) arrivals. (a) Steeply oriented thermospheric arrivals display a consistent trade-off between \( C_S \) and \( i \) over a wide range of \( C_S \), while (b) grazing stratospheric arrivals become more restricted as \( v_{\text{app}} \) approaches \( C_S \) at 90° incidence. In contrast, backazimuth is significantly less sensitive to \( C_S \). In all diagrams, ‘X’ denotes the location of the global minimum at the true orientation of the incoming wave and residual contour intervals are \(+0.025 \text{ s}\), while the time-delay residual is defined as the sum of the absolute differences between the model and true times at each array element.

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speed errors form a double-lobed structure with the lobes being of opposite sign (Fig. 3). The negative lobe for the backazimuth error, $\Delta \theta$, lies 90° clockwise to the incline gradient, while the negative lobe in apparent speed, $\Delta v_{app}$, is located upslope. The largest variation is seen for steeply oriented slowness vectors for both $\Delta \theta$ and $\Delta v_{app}$, trending towards zero as the slowness becomes horizontal where differences in array elevation no longer contribute to observed time delays and eq. (3) becomes increasingly valid (i.e. $s_y \rightarrow 0$). Lines of zero error also occur when $S$ is parallel to the gradient direction for $\Delta \theta$, or at orientations perpendicular to the gradient for $\Delta v_{app}$. Thus when $|\Delta \theta|$ is at a minima, $|\Delta v_{app}|$ is at a maxima and vice versa (Fig. 3). Since backazimuth and apparent speed error magnitudes increase as the incident wave inclination becomes more vertical, observationally this means that steeply impinging thermospheric infrasonic phases are significantly more affected than stratospheric phases, which often arrive from more grazing inclinations.

The most comprehensive method to avoid the inherent error of the 2-D approximation of eq. (3) for inclined arrays and arrays with significant topography requires a search of slowness space in three dimensions to determine the orientation of the slowness vector, $S$, (referred to as 3-D in this paper). In such a search, all possible orientations (incidence, backazimuth) and magnitudes of the slowness vector are explored, constrained only by a range of permissible atmospheric acoustic propagation speeds, $C_x$. Thus the approximate 2-D search over a region of the slowness plane ($s_x, s_y$; Fig. 4a) becomes a search over the slowness volume (Fig. 4c) in $s_x, s_y$ and $s_z$ defined by

$$s_x = \frac{\sin \theta \sin i}{C_s},$$  \hfill (7)  

$$s_y = \frac{\cos \theta \sin i}{C_s},$$  \hfill (8)  

$$s_z = \frac{\cos i}{C_s},$$  \hfill (9)  

where $i$ and $\theta$ are the incidence and backazimuth of the slowness vector measured from the vertical and Eastward from North, respectively. In instances where $C_s$ is approximately known (as in the case for arrays where local surface temperatures are monitored), the full 3-D volume can be again constrained to two dimensions in terms of a spherical surface rather than a plane (Fig. 4b) thereby avoiding the added computational expense of a volumetric search for $S$.

For infrasound studies, calculation of the acoustic velocity requires knowledge of the ambient temperature of the atmosphere at the array site, from which sound speed can be inferred via

$$C_s = \sqrt{\frac{\gamma RT}{m_o}},$$  \hfill (10)  

where temperature $T$, is measured in degrees Kelvin, $\gamma = 1.4$, the ratio of specific heats of air, $R = 8.31432$ N·m·(mol·K)$^{-1}$ is the Universal gas constant and $m_o$ is the mean molecular weight of (dry) air (0.0289644 kg mol$^{-1}$; United States Committee on Extension to the Standard Atmosphere 1976). As air temperature can vary significantly over timescales of a few hours or greater, this requires the air temperature to be frequently monitored to track the local acoustic velocity over time. We note this is different to seismic array analysis where the propagation velocity beneath the array can be determined using seismic refraction surveys and then considered effectively time-invariant.

A constantly variable sound speed raises concern when deciding the type of search to employ in slowness space to identify the slowness vector. Does fixing of the sound speed, to achieve greater computational speed, introduce significant errors in the resulting slowness estimate? If so then volumetric searches for the slowness vector, rather than surface searches (Fig. 4), may be necessary. The relationship between sound speed and temperature (eq. 10) infers that relatively large changes in temperature are needed to significantly change the local sound speed. For instance, a difference of $\sim 100\degree$ in temperature from 224 to 305 K (−49 to +50°C) modifies the sound speed by only 20 per cent resulting in a similar variation in the vertical component of the inferred slowness vector (i.e. $\Delta s_z$).

### Table 1. Individual array sizes, inclinations and orientations for the 43 stations operational in 2011 January of the CTBTO/IMS infrasound network. Slope and direction are derived from a least-squares fitted plane through the listed array elements’ geodetic coordinates and elevations. $\Delta s_{max}$ and aperture represent the maximum elevation difference and lateral separation between array elements.

<table>
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<th>Station</th>
<th>$\Delta s_{max}$ (m)</th>
<th>Aperture (km)</th>
<th>Slope (‘ from horizon)</th>
<th>Direction (‘ E from N)</th>
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Network mean 53 2.190 1.59
inclination). In the earlier 5° incline example, an ∼18 per cent variation in apparent speed is observed over this thermal range for representative stratospheric (<5 m s⁻¹) and thermospheric (<10 m s⁻¹) arrivals (75° and 45° inclinations, respectively, e.g. Drob et al. 2003). This is often comparable to the variability in apparent speed seen over the duration of a signal (for an example see Section 5). The practical result is that, in general, the region surrounding the minimum time-delay residual in S-i space, employing the 3-D methodology, tends also to be slowly varying with a trade-off between a large |S| (smaller CS) and small i, or small |S| (larger CS) with larger i (Fig. 5a), with the region narrowing at grazing inclinations as the v_app approaches CS (Fig. 5b). Thus fixing of the local sound speed to a constant value to achieve the computational advantage of a slowness surface search (Fig. 4b) can be considered reasonably accurate for many applications so far as the sound speed estimate is appropriate for the array site at the time in question. If necessary, these values could then be used as seeds for a more robust location of the global minimum. Throughout the remaining sections, this 3-D beamforming methodology using a fixed sound speed has been chosen for comparison with the equivalent results using the 2-D approximation.

3 RESULTS FOR IMS ARRAYS

Microbarograph arrays in the IMS have been located in a sparse 60-station network around the Earth to assist in the detection and location of any atmospheric nuclear explosions in a broad band of the infrasonic spectrum between ∼0.01 and 10 Hz. While efforts are made to avoid extreme vertical differences between elements, sometimes it is unavoidable that an array is constructed upon a topographic feature with scale lengths as large as or larger than its aperture (such as a mountainside, broad valley or step). In such cases, the locations of elements will follow the topography such that the general trend of the array may be inclined and so exhibit similar beamforming errors as was shown in the previous section when employing a 2-D approximation. For comparison, we provide an estimate of the best fitting (in the least-squares sense) ideal plane to each of the 43 operational IMS infrasound arrays (Table 1). We note for clarity, however, that all calculations done to estimate the wave parameters in 3-D use the true array element locations and not these planar estimates.

Infrasound arrays in the IMS network have been constructed using several designs with geometries which aim to enhance the performance of array beamforming within the targeted frequency band by improving the array response so that the central lobe is generally small, well-defined and free of significant side lobes (Fig. 6). However, in the case where an array is inclined, this response shifts away from centre (a vertical slowness of sx = 0, sy = 0) when accounting for elevation differences between array elements (Fig. 7). This shift is proportional to the slope of the inclined array plane from the horizontal and provides a readily available test to determine if an array is significantly inclined and the degree beamforming will be affected using a 2-D approximation.

Figure 6. Representative examples of the most common infrasound array configurations (top panels) and their associated responses at 0.1 Hz (bottom panels), employed by the CTBTO/IMS network. (a) Four element triangular array. (b) Triangular array with central four element ‘high frequency’ subarray (c) Pentagon array with central subarray.
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Figure 7. Examples of CTBTO/IMS infrasound arrays (top panels) with significant inclinations due to large-scale topography and their associated responses at 0.1 Hz (bottom panels). Station elements (triangles) are colour-coded relative to elevation differences to the array mean (white: +250 m, black: –250 m). Note that peak of the array responses (black dot) are noticeably offset from the centre of the slowness plane (black cross) along the projected direction of the normal to the array plane. The larger this offset, the greater the backazimuth and apparent speed error will become if the array is processed as if it were horizontally planar (2-D approximation). Black arrows indicate array’s upslope direction.

For each IMS infrasound array, the systematic beamforming error due to array elevation differences was mapped using synthetically generated time delays between elements using their true 3-D geometry and a known slowness vector with a sound speed of 330 m s$^{-1}$. These delay times were then used to estimate the slowness’ backazimuth and apparent speed using the 2-D array approximation by ignoring element elevations in the inversion. If the maximum beamforming error for each IMS infrasound array is plotted as a function of its equivalent slope (Table 1), and 75° and 45° inclinations are taken as representative of stratospheric and thermospheric arrivals, respectively (Drob et al. 2003), it is seen that overall the behaviour of current IMS arrays is consistent with theoretical estimates for the idealized inclined planar array (Fig. 8). Differences between the actual array beamforming error and that of the theoretical inclined plane error are typically small in comparison to the overall error and represent the deviations of the true array topography from the ideal inclined plane.

While at most IMS infrasound arrays the beamforming errors resulting from topography are generally small ($<\sim1°$) and might be considered to be within typical signal beamforming uncertainties.
Figure 8. Maximum error in backazimuth computed using 3-D beamforming as a function of array inclination for the IMS infrasound array network for typical stratospheric and thermospheric arriving waves. Array inclinations are computed using best-fit planes through station element offsets and elevations. Theoretical predictions for ideal inclined planes are shown as long-dashed grey lines. Vertical short-dashed lines indicate the stations at higher inclinations (Table 1) and hence negligible, several stations exhibit azimuthal errors significantly above this level. Unfortunately, these stations are typically in remote regions of the planet where station coverage in the IMS network is scarce, such as the mid-Pacific and Atlantic oceans. Since source location in infrasound is often heavily (sometimes solely) dependent upon observed backazimuths, even comparatively small errors may lead to systematic uncertainty in a source’s location derived from backazimuths determined from 2-D approximated inclined arrays, depending upon the range to the source (Fig. 9). Thus the potential for increased event location bias will be greater where these stations are used and 2-D beamforming is employed due to the array–topography effect. Such errors will be particularly significant if the event lies along backazimuths nearly perpendicular to the gradient of the array incline, where $|\Delta \theta|$ nears its maxima (Fig. 3), that is, $90^\circ$ from the azimuths provided in Table 1.

4 APPLICATION TO I18DK, GREENLAND

The eight-element I18DK array in Greenland provides an excellent example for displaying the effects of topography as the array is located on an incline of approximately $7^\circ$ from the horizontal, oriented at an azimuth of $\sim 013^\circ$ (Figs 7 and 10 and Table 1). Consequently, the beamforming errors generated by neglecting topography are expected to be significant for the azimuths near the perpendicular to this direction, and for apparent speed in those directions parallel with the slope (Figs 8 and 10). Here an analysis is performed on two signals, impinging on the array from approximately these orthogonal directions, to illustrate the topographic effect upon recorded data using (1) infrasound from the 2008 Kasatochi volcanic eruption and (2) the launch of STS-131 from Cape Canaveral. In both cases we assume a constant acoustic speed of 0.320 km s$^{-1}$ during 3-D processing.

Infrasound from the onset of the Kasatochi, Alaska volcanic eruption on 2008 August 7 (Fee et al. 2010) arrives at I18DK with a backazimuth of approximately $30^\circ$, while an infrasonic signal from the launch of the U.S. space shuttle Discovery during STS-131 from Cape Canaveral, Florida, on 2010 April 5 arrives at approximately a backazimuth of $192^\circ$. Due to the inclination of the array it is expected that array processing results using a 2-D approximation for Kasatochi signals will exhibit large positive azimuthal errors but small negative apparent velocity errors (Fig. 10, eq. 6). In contrast, the STS-131 signal should exhibit extremely small errors in azimuth, but significant positive apparent velocity errors.
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Signal parameter estimates for the Kasatochi signal (Figs 11b and c) show clear systematic differences between the 2-D approximation and 3-D calculations, with the backazimuth estimates always being more southerly (positive error) by 1.5–4° and apparent speed being faster (negative error) by 1–10 m s\(^{-1}\) in the 3-D case. This is to be expected as for I18DK, inclined array theory predicts that a wave front from Kasatochi impinges upon the positive azimuthal error lobe and grazes the negative apparent speed lobe of the error surface (Fig. 10). For the signal detected from the STS-131 launch there are again systematic differences between the two parameter estimates. In this case, the azimuthal estimates are approximately equal (<0.2° difference), whereas the apparent speed estimates show large (5–10 m s\(^{-1}\)) differences with the 2-D approximation being always overestimating the velocity values. Once more this is consistent with theoretical predictions based upon the anticipated direction of the arriving signal and the directionality of backazimuth and apparent speed errors due to the array’s incline (Fig. 10).

We note that in both cases the \(F\)-statistic values (Figs 11d and h) are insensitive to the choice of either 2- or 3-D processing. This is due to the low-frequency content of the two signals. As wavelengths of these signals are greater than the aperture of the array (~1.2 km, Table 1), the array response function exhibits a broad maximum (e.g. Fig. 7) and so the beamset using the 2-D approximation can account for the time delays with a low time residual, yet still estimates the incorrect backazimuth and apparent speed values.

Inspection of the differences between the signal parameter estimates of the 2-D approximation and 3-D processing show that these estimates are highly dependent upon the incidence angle of the incoming acoustic wave as predicted theoretically (Fig. 12). For the Kasatochi signal this is especially pronounced for the backazimuth variations (Fig. 12a) as the signal impinges close to the centre of the backazimuth error lobe (Fig. 10). As predicted, the steeper the acoustic wave impinges on the array, the larger the backazimuthal error due to the increased influence of the array topography. The STS-131 signal, in contrast, displays little backazimuthal error, changing little with signal incidence (Fig. 12b), as the signal arrives at the array almost parallel to the slope where predicted backazimuthal error reaches a minimum (Fig. 10). This arrival direction, however, means the signal impinges upon the array where the apparent speed errors are large, and small changes in incidence have a large effect on the magnitude of the error (Fig. 12d). Once more we can see that the trend in apparent speed error is well predicted by theory.
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Figure 11. The signal and detection parameters of the Kasatochi eruption at the I18DK array (Panels a–d). Panel (a) shows the best-fitting beam accounting for the 3-D nature of the array, with the beam formed along a backazimuth of 300° assuming an apparent horizontal velocity of 0.330 km s\(^{-1}\) and an acoustic speed of 0.320 km s\(^{-1}\). The beam is bandpass filtered between 0.03 and 0.3 Hz. Panels (b) and (c) show the best-estimates of the backazimuth and apparent speed of a plane wave passing across the array at each time interval, and panel (d) shows the associated \(F\)-statistic. In each panel the grey symbols are associated with the 2-D approximation and the black dots are associated with the 3-D array topography. Panels (e)–(h) show the same for the signal recorded by I18DK from the STS-131 launch of the space shuttle Discovery. Here the beam is formed along a backazimuth of 192° and the beam is bandpass filtered between 0.06 and 1 Hz; other parameters remain the same.

5 DISCUSSION AND CONCLUSIONS

Significantly inclined arrays of either infrasound or seismic instruments in the CTBTO/IMS network are not common. However, in the case of infrasound, because events are typically located through the intersection of beamformed backazimuths, the potential for event mislocation is greater. This particularly affects events in the Pacific and southern Atlantic oceans, where several of the most inclined arrays in the IMS network (Fig. 7, Table 1) are located due to the constraints of available locations. Yet, this issue is a well-understood, predictable problem and the consistency of the IMS infrasound arrays with the simple inclined plane theory suggests minor changes in processing procedures that include array topography during beamforming (e.g. Brown et al. 2008) might readily be adopted to correct backazimuth results for such systematic topographic effects. While some knowledge or estimate of the local ambient temperature (and hence sound speed) is required for 3-D beamforming procedures, azimuthal corrections are insensitive and apparent speed only marginally sensitive to this choice if appropriate values for the array in question are taken. As many arrays currently log ambient temperature continuously, this is not a significant issue.

Left uncorrected, 2-D approximation methods at current IMS arrays with significantly inclined topography result in azimuthally dependent backazimuth errors for grazing stratospheric arrivals of up to the order of a degree, becoming increasing larger as the incidence of the wave front steepens. These backazimuth errors are of similar magnitude (1.5°–3.5°) to backazimuth deviations interpreted to be due to the influence of upper air cross-winds during atmospheric propagation (e.g. Garces et al. 1998; Le Pichon et al. 2002), and left unidentified and uncorrected could easily be mis-interpreted as atmospheric effects (e.g. the signal of the Kasatochi eruption at I18DK analysed in Section 4). As concerted efforts are invested in understanding the global propagation of various infrasound generating sources (e.g. Drob et al. 2003; Willis et al. 2004; Arrowsmith et al. 2008; Ottemöller & Evers 2008) and how global atmospheric changes affect the sensitivity of the IMS network (Le Pichon et al. 2009; Green & Bowers 2010), identification and removal of non-propagation-related biases and systematic effects is a necessity as our understanding of global propagation increases and the IMS global infrasound network nears completion.

Although the final determination of the necessity to incorporate array topography into beamforming procedures for a particular array is a matter for the individual operator, the benefits of removing a systematic bias in beam results and hence source location estimates appear to outweigh the modest amount of added computational complexity. If the IMS infrasound network is to be as effective as possible, and to allow the data to be used to improve our understanding of detailed infrasonic propagation, such minor known biases in the inferred results of array data collection should be removed whenever possible.

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Figure 12. The difference in backazimuths (a,b) and apparent speeds (c,d) estimated using the 2-D approximation and the 3-D array topography as a function of the horizontal apparent speed estimated using the 3-D scheme. Data from the Kasatochi eruption and STS-131 shuttle launch data are shown as black dots. If the 3-D processing is assumed to be the ‘true’ value, then the difference is equivalent to the azimuthal error, $\Delta\theta$, and the apparent speed difference is equivalent to the apparent speed error, $\Delta v_{\text{app}}$ (eq. 6). The dashed lines are the theoretical relationship predicted assuming an atmospheric acoustic speed of 0.320 km s$^{-1}$ and the I18DK geometry. Scatter between theory and data is largely due to noise within the data recordings.

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