

## Travel time of curved parallel hillslopes

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### ABSTRACT

Travel time is an important variable that influences the hydrologic response of watersheds. Hydrologic models often do not take the variation of slope along the flow paths, i.e. profile curvature, directly into account. In this study, using a kinematic wave-based model, the effect of profile curvature on the travel time in parallel hillslopes is investigated. Also, the profiles associated with minima travel times are derived. Moreover, the range of profile curvature for which this error becomes negligible is determined. Finally, a kinematic wave-based geomorphic index is derived which considers profile curvature and plan curvature. Given that a rather similar index has been previously developed, the two indices are compared. It is concluded that there is no universal rule about which type of profile (curvature, concave or convex) has the longer travel time. In fact, this strongly depends on the degree of curvature. However, travel times for convex hillslopes are generally more than those of the straight hillslopes. In contrast, there is no consistent trend regarding the variation of travel time with the degree of curvature in concave hillslopes. Furthermore, ignoring slope variation in travel time determination may potentially cause an underestimation by up to 12% in convex hillslopes.

**Key words** | concave profile, convex profile, geomorphic index, kinematic wave, overland flow, parallel hillslopes, travel time

### INTRODUCTION

Most watershed rainfall–runoff models, directly or indirectly, require some sort of parameters describing the response time of the watershed. Time of concentration, lag time and time to equilibrium are the most common response time parameters used in hydrologic models. For example, the design of urban drainage systems using rational formulas requires an estimate of the time of concentration to determine the critical rainfall intensity (Ben-Zvi 1984). Snyder, Clark and SCS (Soil Conservation Service) unit hydrograph methods all require an estimate of the watershed response time. The Muskingum flood routing method also makes use of the travel time in river reaches. Meynink (1978) showed that varying the time of concentration from one half to twice the initial value, respectively, changed the peak discharge by 1.64 to 0.48 times that corresponding to the initial value for a typical 5 km<sup>2</sup> watershed on the Darling Downs in Australia. Since

the value of design flood is often in reverse proportion to the response time, a more efficient design relies on a better estimate of the time of concentration.

There are several relationships in the literature for estimating the response time based on the kinematic wave (KW) theory. Ever since Woolhiser & Liggett (1967) showed that KW equations can be applied to most overland flow situations, there has been a growing interest in using the KW equations to derive travel time formulas. One of the greatest strengths of this approach is the feasibility of obtaining physically based formulas without the need for any experimental data (Wong 2006). Further, physically based formulas are suitable for general use and the assumption involved in the formulation can be obviously stated (Akan 1986). In the above studies, the researchers focused on a specific geometry and formulated response time for that special case.

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A few authors have developed general analytical formulations which are applicable to every surface with spatially variable topographic features. [Saghafian & Julien \(1995\)](#) and [Agnese et al. \(2007\)](#) presented integral equations to calculate the travel time of overland flow, considering the effects of the geometric and non-geometric parameters.

Different functional forms were assumed in order to describe the slope variability along the hillslope. In order to account for the influence of slope curvature on soil erosion process, [Di Stefano et al. \(2000\)](#) assumed a power law to describe convex and concave hillslope profiles. A different power law for the hillslope profile was proposed by [Carson & Kirkby \(1972\)](#) and [Ridolfi et al. \(2003\)](#). [Philip \(1991\)](#) rather described the profile shape of the hillslope with a circle arc. [Evans \(1980\)](#) presented a comprehensive geometry model that allows investigation of the effects of the varying of bed slope and flow width on the hydrologic response of hillslopes by producing the hillslopes of different geometric characteristics (complex hillslopes). Evans' equation has been previously employed in subsurface flow analysis by [Troch et al. \(2002\)](#), [Talebi et al. \(2008\)](#), and [Sabzevari et al. \(2010b\)](#). They applied Evans' equation to produce nine basic geometries with three different profile curvatures (concave, straight, and convex) and three different plan curvatures (converging, parallel, and diverging). [Troch et al. \(2002\)](#) developed analytical solutions to a hillslope-storage KW equation for subsurface flow. They derived characteristics of drainage response functions for nine geometries. [Talebi et al. \(2008\)](#) investigated the stability of complex hillslopes and introduced a width function for converging and diverging hillslopes. Their results proved that the geometry of hillslopes has a direct effect on the stability of hillslopes. More recently, [Sabzevari et al. \(2010b\)](#) studied the saturation of complex hillslopes due to subsurface flow. Based on their results, convex hillslopes have longer travel time in comparison to straight and concave hillslopes. An equation was presented for calculating subsurface travel time for all complex hillslopes. They stated that the role played by the geometry of hillslopes in subsurface flows seems to be of great importance, yet more work is needed in the case of surface flow. This study is conducted based on the Evans' equation for the special case of parallel hillslopes. It should be noted that, for parallel hillslopes,

Evans' equation is reduced to the equation employed by [Di Stefano et al. \(2000\)](#).

Slope is one of the most common variables considered in the travel time equations. However, most past reports have ignored the variation of slope, i.e., profile curvature, along the flow path by relying on the mean slope. The first objective of this paper is to address the question of 'which type of profile (curvature, convex, or concave) has the longest/shortest travel time?' In order to investigate the effect of the profile curvature, [Troch et al. \(2002\)](#), [Talebi et al. \(2008\)](#), and [Sabzevari et al. \(2010b\)](#) considered one slope as a representative of convex slopes and another slope as a representative of concave slopes. In this study, however, instead of only two profile curvatures, a wide range of profile curvatures is tested. The results will show that the relative time response of convex versus concave slopes may not be easily generalized.

The second objective of this study is the derivation of a dimensionless parameter, called 'geomorphic index', to which travel time is linearly related. This parameter considers the effect of plan curvature and profile curvature. [Agnese et al. \(2007\)](#) have already presented an equation to calculate the travel time over complex hillslopes using two basic assumptions. They introduced a similar dimensionless parameter called 'shape factor'. In this paper, a discussion on the assumptions of [Agnese et al. \(2007\)](#) travel time equation is presented and the geomorphic index and shape factor are compared.

## GENERAL TRAVEL TIME RELATIONSHIP

[Saghafian & Julien \(1995\)](#) derived a general relationship to determine the KW time to equilibrium for spatially variable watersheds. They investigated the time to equilibrium in two distinct phases, overland and channel flows. Their formulation resulted in the following relationship:

$$t_e = \int_0^L (1 - \gamma) \left( \frac{a_1}{Q_e} \right)^\gamma \left( \frac{n}{a_2^{2/3} S^{1/2}} \right)^{1-\gamma} dx \quad (1)$$

where  $S$  is the bed slope,  $Q_e$  is the equilibrium discharge,  $n$  is the Manning roughness coefficient,  $L$  is the flow length, and

$a_1, a_2,$  and  $\gamma$  are derived from the relations of discharge per unit width ( $q$ ), the flow cross-section area ( $A$ ) and the hydraulic radius ( $R$ ), as expressed in Equations (2) to (5):

$$q = ah^m \tag{2}$$

$$A = a_1h^{b_1} \tag{3}$$

$$R = a_2h^{b_2} \tag{4}$$

$$\gamma = \frac{m - 1}{m + b_1 - 1} \tag{5}$$

where  $m$  is the flow regime constant ( $m$  is usually taken to be equal to 5/3 for turbulent flow, 2 for transitional flow, and 3 for laminar flow),  $a = \sqrt{S}/n$  is the friction parameter and  $a_1, a_2, b_1,$  and  $b_2$  are constants for a given cross-section. In fact, there is no simplifying assumption in Equation (1) on the geometry of cross-section or any other aspect of flow, except validity of KW. Substituting  $a_1 = w(x), Q_e = rA(x)$ , ( $r$  is the excess rainfall intensity and  $A(x)$  and  $w(x)$ , respectively, are the drainage area and the hillslope width at the distance  $x$  (Figure 1) and  $b_1 = b_2 = a_2 = 1$  for overland equilibrium time, Equation (1), yields:

$$t_e = \frac{1}{m} r^{\frac{1-m}{m}} n^{\frac{1}{m}} \int_0^L \left(\frac{A(x)}{w(x)}\right)^{1/m-1} \left(\frac{1}{\sqrt{S(x)}}\right)^{1/m} dx \tag{6}$$

Saghafian (1992) tested Equation (6) for converging and diverging planes. The calculated values of the time to equilibrium matched those reported by Agiralioğlu (1984, 1988). Saghafian (1992) also developed a raster-based algorithm for surface runoff calculations that was tested against runoff measurements from small overland flow planes, and experimental data of Schaake (1965), Dickinson et al. (1967), and Woolhiser (1969).

Assuming that  $k_* = \sqrt{S}/(nL)$  ( $L$  length of overland plane and  $\bar{S} = H/L$  mean slope) accounts for the ‘gross’ hillslope geometry and by normalizing  $x$ , Equation (6) may be expressed as a function of geomorphic functions such that:

$$t_e = \frac{1}{m} r^{\frac{1-m}{m}} k_*^{\frac{-1}{m}} \int_0^1 \frac{[\varphi_{pl}(\xi)]^{1/m-1}}{[\sqrt{\varphi_{pr}(\xi)}]^{1/m}} d\xi \tag{7}$$

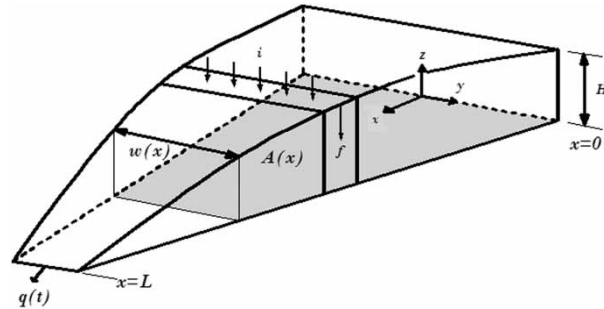


Figure 1 | Three-dimensional view of a converging hillslope with a convex surface profile (after Sabzevari 2010a).

where  $\xi = x/L$  is the normalized distance,  $\varphi_{pl}$  is the ‘planform geometry function’ and  $\varphi_{pl}(\xi) = L^{-1} \varphi_{pl}(x)$ ,  $\varphi_{pl}(x) = A(x)/w(x)$ .  $\varphi_{pl}$  represents the drainage area per unit contour line width.  $\varphi_{pr}(\xi) = |S(x)|/\bar{S}$  is the so-called ‘profile shape function’. It should be noted that ‘planform geometry function’,  $\varphi_{pl}$ , and ‘profile shape function’,  $\varphi_{pr}$ , are exactly those defined by Agnese et al. (2007). From Equation (7) it can be inferred that the time to equilibrium grows linearly with a shape factor that is given by:

$$\varphi_g = \int_0^1 \frac{[\varphi_{pl}(\xi)]^{1/m-1}}{[\sqrt{\varphi_{pr}(\xi)}]^{1/m}} d\xi \tag{8}$$

where  $\varphi_g$ , that we call *geomorphic index* hereafter, is a dimensionless parameter that accounts for both the planform geometry and profile shape of the hillslope.

Profile curvature is the topographic curvature along the steepest descent flow path. The variability of profile curvature within hillslopes and watersheds is directly related to the governing geomorphic processes. Landscapes where diffusional erosion dominates will have domed-shaped hills with flat tops and steeper slope bases, but will be convex in overall shape (Bogaart & Troch 2006).

One can usually approximate a certain proportion of the topographic surface of a watershed by a continuous function. In this study, the special form of Evans (1980) comprehensive model is applied to produce the parallel surfaces of different profile curvatures:

$$z = H\left(1 - \frac{x}{L}\right)^N \tag{9}$$

where  $z$  is the elevation (m),  $x$  is the horizontal distance measured in the downstream direction of the surface (m),  $y$  is the horizontal distance (m) from the slope center in the direction perpendicular to the downstream direction (the width function),  $H$  is the maximum elevation (m) difference defined by the surface,  $L$  is the total horizontal length of the hillslope (m),  $N$  is a profile curvature parameter, and  $\omega$  is a plan curvature parameter. We allow the profile curvature (defined by  $N$ ) to assume values less than, equal to, or greater than unity corresponding to convex, straight, and concave hillslopes, respectively.

### DERIVATION OF TRAVEL TIME RELATIONSHIP FOR DIFFERENT CURVATURES

In order to obtain the travel time relationship for the geometry of interest, all constituting elements should be obtained first. Equation (9) can be written as:

$$\eta = (1 - \xi)^N \quad (10)$$

where  $\eta = z/H$  is the normalized elevation.  $\varphi_{pl}$  and  $\varphi_{pr}$  are obtained by:

$$\varphi_{pl}(\xi) = \frac{A(x)}{w(x)L} = \frac{x}{L} = \xi \quad (11)$$

$$\varphi_{pr}(\xi) = \frac{|S(x)|}{S} = N(1 - \xi)^{N-1} \quad (12)$$

Therefore,  $\varphi_g$  may be calculated by:

$$\begin{aligned} \varphi_g(\xi) &= \int_0^\xi \frac{[\varphi_{pl}(u)]^{\frac{1}{m}-1}}{[\sqrt{\varphi_{pr}(u)}]^{1/m}} du = \int_0^\xi \frac{u^{\frac{1}{m}-1}}{[\sqrt{N(1-u)^{N-1}}]^{1/m}} du \\ &= N^{\frac{-1}{2m}} B_\xi \left( \frac{1}{m}, 1 + \frac{1-N}{2m} \right) \end{aligned} \quad (13)$$

where  $B_\xi(\cdot)$  is the incomplete beta function of arguments ( $\cdot$ ). It should be noted that Equation (13) generalizes Equation

(8) to any distance from the top of the hillslope. Finally, the travel time can be obtained by:

$$t_e(x = \xi L) = \frac{1}{m} r^{\frac{1-m}{m}} k_*^{\frac{-1}{m}} N^{\frac{-1}{2m}} B_\xi \left( \frac{1}{m}, 1 + \frac{1-N}{2m} \right) \quad (14)$$

### FASTEST RESPONSE CURVE

In the previous section, we defined a function to describe the bed profile a priori. If the profile function is not predefined, unlimited curves can be drawn through two given points. Among these curves, there is one curve which is of the shortest travel time. This curve is called *fastest response curve (FRC)* in this paper. To derive the FRC,  $\varphi_g$  has to be minimized. Towards this aim, the differential equation of Euler-Lagrange can be used (Elsgolts 1981):

$$\frac{d}{d\xi} \frac{\partial f}{\partial \eta'} = \frac{\partial f}{\partial \eta} \quad (15)$$

where generally  $f = f(\xi, \eta, \eta')$  and in this case:

$$f = \frac{\xi^{1/m-1}}{\eta'(\xi)^{1/(2m)}} \quad (16)$$

After substituting and simplifying, the function of FRC can be easily obtained:

$$\eta(\xi) = 1 - \xi^\beta \quad (17)$$

where

$$\beta = 3/(1 + 2m) \quad (18)$$

### RESULTS

In order to evaluate the effect of both the type and the degree of profile curvature on the travel time, the travel time of the planes of different curvatures, Equation (14),

are divided by that of the equivalent parallel straight planes ( $t_e/t_{e\text{Straight}}$ ). This ratio can be expressed as a function of normalized distance. The travel time for straight planes can be obtained by substituting  $N = 1$  in Equation (14) as follows:

$$t_{e\text{Straight}}(x = \xi L) = r \frac{1-m}{m} k_*^{\frac{-1}{m}} \xi^{\frac{1}{m}} \tag{19}$$

Therefore:

$$\frac{t_e(x = \xi L)}{t_{e\text{Straight}}(x = \xi L)} = \frac{\varphi_g(\xi)}{m \xi^{\frac{1}{m}}} \tag{20}$$

Figure 2 illustrates the variation of the  $t_e/t_{e\text{Straight}}$  ratio with respect to the profile curvature parameter along the plane for different flow regimes, while Figure 3 shows the same ratio at the bottom of the plane ( $\xi = 1$ ) for  $m = 5/3$ .

From Figures 2 and 3, it is concluded that for  $m = 5/3$ , which holds in most situations, the travel times of concave and convex slopes are completely dependent on the degree of profile curvature. Thus, there is no universal rule stating the ratio of overland flow travel times over concave versus convex slopes. However, travel time for the convex plane is always more than that of the straight hillslopes. Moreover,  $t_e/t_{e\text{Straight}}$  at the bottom of the slope decreases as the profile curvature parameter increases from 1 to 1.3, although the trend is reversed for the profile curvatures more than 1.3 so that at  $N = 1.6$ , the travel time of the curved plane becomes more than that of the straight plane. For profile curvatures more than 1.6, the

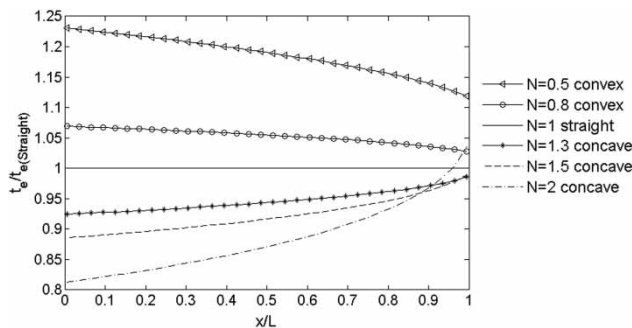


Figure 2 | Variation of  $t_e/t_{e\text{Straight}}$  with profile curvature parameter (shown in legend) along the plane.

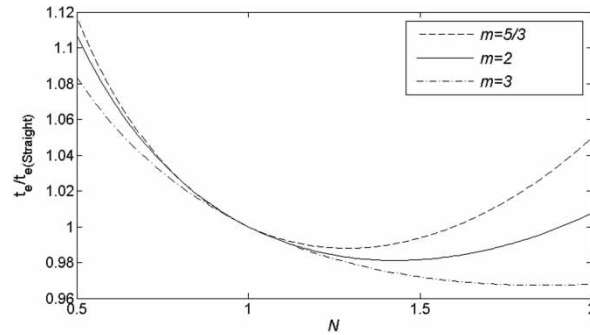


Figure 3 | Variation of  $t_e/t_{e\text{Straight}}$  with profile curvature parameter at the bottom of the plane for different flow regimes.

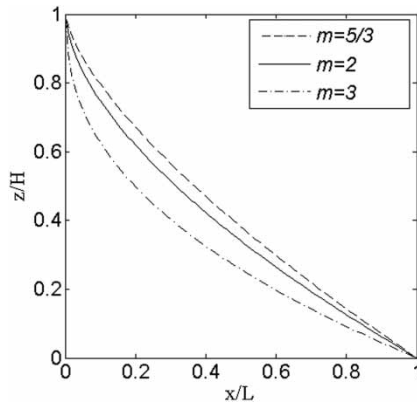
ratio continues to increase. This irregular trend can be justified in that the general travel time relation (Equation (7)) is based on two factors: the slope ( $\varphi_{pr}$ ) and the drainage area ( $\varphi_{pl}$ ). In fact, these two factors cause the mass of water to accelerate on the plane. In contrast to convex planes, these two factors act in opposite directions along the plane in concave planes. Indeed, while the drainage area increases down the concave plane, the slope decreases. The ultimate trend is influenced by the conjunctive effect of the two factors. In concave hillslopes with profile curvatures more than 1.6, there is a point of inflection at the middle of the plane where the effect of slope overcomes that of the drainage area. At such point, the travel time for concave plane begins to exceed that of the straight plane. It should be noted that  $\varphi_{pl}(\xi)$  is independent of the profile curvature parameter ( $N$ ) (Equation (11)). Therefore, the effect of drainage area remains constant as the degree of curvature increases in concave planes. While the effect of drainage area is constant, the effect of slope,  $\varphi_{pr}(\xi)$ , varies (Equation (12)).

It is worth noting that in Figure 3, in contrast to the curves corresponding to  $m = 5/3$  and  $m = 2$ , the curve related to  $m = 3$  monotonically decreases with the profile curvature parameter.

The powers ( $\beta$ ) of FRCs, described in the previous section, for various flow regimes are illustrated in Table 1. Figure 4 shows the profiles associated with the values listed in Table 1. According to Figure 4, FRCs are concave for all flow regimes. The values of  $t_e/t_{e\text{Straight}}$  for FRCs in different flow regimes are listed in Table 2.

**Table 1** | Powers of FRCs for parallel hillslopes, for different flow regimes

$m$	$\beta = 3/(1 + 2m)$
5/3	0.69
2	0.60
3	0.43

**Figure 4** | FRCs for parallel concave/convex hillslopes, for different flow regimes.**Table 2** | The ratio of  $t_e$  to  $t_{e(\text{straight})}$  for FRCs in different flow regimes

	$m = 5/3$	$m = 2$	$m = 3$
$t_e/t_{e(\text{straight})}$	0.968	0.947	0.896

## IMPROVEMENTS OVER PREVIOUS WORKS

Agnese *et al.* (2001) presented a simple model to simulate overland flow. The applicability of their model is restricted to simple parallel planes and is based on two basic assumptions:

- KW equations govern the overland flow.
- At any time the rate of change of the discharge per unit width with distance down the plane is constant and equal to the outflow rate at the bottom of the plane (an approximation known as Rose's hypothesis).

Agnese *et al.* (2001) employed the second assumption to separate temporal and spatial variation of discharge along the plane. They stated that although this simplified approach does not capture the complete hydraulics of the motion, it does lead to an analytical solution.

Agnese *et al.* (2007) extended their previously developed model to complex hillslopes, preserving the governing assumptions, to obtain the following relationship:

$$t_{\text{eag}} = \left( \sum_{j=0}^{\infty} \frac{0.95^{\frac{1+mj}{m}}}{1+mj} \right) r^{\frac{1-m}{m}} k_*^{\frac{-1}{m}} \varphi_s \quad (21)$$

where  $t_{\text{eag}}$  is the travel time based on the Agnese *et al.* (2007) model and  $\varphi_s$  is a dimensionless parameter, the so-called *shape factor*.  $\varphi_s$  may be derived as follows:

$$\varphi_s = \int_0^1 \left[ \frac{\varphi_{pl}(\xi)}{\sqrt{\varphi_{pr}(\xi)}} \right]^{1/m} d\xi \quad (22)$$

In this paper, Rose's hypothesis as the second approximation made by Agnese *et al.* (2007), was relaxed and a new geomorphic index was introduced. Comparing the definition of shape factor and geomorphic index results in some major remarks. According to  $\varphi_g$  (Equation (8)), equilibrium time is inversely proportional to the planform geometry function,  $\varphi_{pl}$ , regardless of the flow regime type. On the other hand, in this equation, the effect of flow accumulation is properly taken into account. As the water deepens, the effective resistance of the bed on the flow diminishes because the hydraulic radius increases and the velocity increases. However, according to  $\varphi_s$  in Equation (22), travel time is directly proportional to planform geometry function. This fact contradicts the effect of flow accumulation.

Since Saghafian & Julien (1995) formulation considers the advance of the wave from upstream to downstream, the discharge at every point of flow path at any time is a function of the length of the portion of flow path which has reached equilibrium by that time. However, in the formulation of Agnese *et al.* (2001, 2007), the discharge at every point of flow path at any time is a function of the total length of the flow path (this can be inferred from the second assumption of their model). This difference is illustrated in Table 1. The equations in this table hold until the steady state is reached ( $t \leq t_e$ ). On the other hand, Table 1 equations may be used to derive the rising limb of the hydrograph at the bottom of a plane subject to a rainfall of very long duration. For ease of understanding, the equations related to the parallel plane are shown in Table 1, where

**Table 3** | Comparison between Saghafian & Julien (1995) and Agnese et al. (2001) models

Saghafian & Julien (1995)	Agnese et al. (2001)
$q_x(t) = \begin{cases} \frac{x}{L_e(t)} q(t) & x < L_e(t) \\ q(t) & x \geq L_e(t) \end{cases}$	$q_x(t) = \frac{x}{L} q(t)$

$q_x(t)$  is the discharge per unit width at time  $t$  at distance  $x$ , and  $L_e(t)$  is the length of the portion of the plane which reaches equilibrium until the time  $t$ . It can be easily inferred that Saghafian & Julien (1995) formulation considers the advance of steady zone and the movement of the KW originating from the hydraulically most remote point traveling to the outlet. According to the equation in the second column of Table 3, the second assumption of Agnese et al. (2001) model is valid, only if  $t = t_e$ , when  $L_e(t) = L$ . In fact, this assumption forces all points on the flow path to reach steady state at the same time.

At this stage, in order to gain insight into the difference between the two models, the effect of the profile curvature using the equilibrium time model presented by Agnese et al. (2007) is also investigated. Assuming that the plane is defined by Equations (10) and (21) yields:

$$t_{eag}(x = \xi L) = r^{\frac{1-m}{m}} k_*^{\frac{1}{m}}(\xi) \times 0.95^{\frac{1}{m}} \text{Hypergeometric2F1} \times \left[ \frac{1}{m}, 1, 1 + \frac{1}{m}, 0.95 \right] \tag{23}$$

where Hypergeometric2F1(.) is the Gauss hypergeometric function of arguments (.), and:

$$\varphi_s(\xi) = N^{\frac{-1}{2m}} B_{\xi} \left( 1 + \frac{1}{m}, 1 + \frac{1-N}{2m} \right) \tag{24}$$

The ratio of  $t_{eag}$  to  $t_e$  may be simplified by:

$$\frac{t_{eag}}{t_e} = \frac{0.95^{\frac{1}{m}} B_{\xi} \left( 1 + \frac{1}{m}, 1 + \frac{1-N}{2m} \right) \text{Hypergeometric2F1} \times \left[ \frac{1}{m}, 1, 1 + \frac{1}{m}, 0.95 \right]}{B_{\xi} \left( \frac{1}{m}, 1 + \frac{1-N}{2m} \right)} \tag{25}$$

**Table 4** |  $t_{eag}/t_e$  for different curvatures when  $m = 5/3$

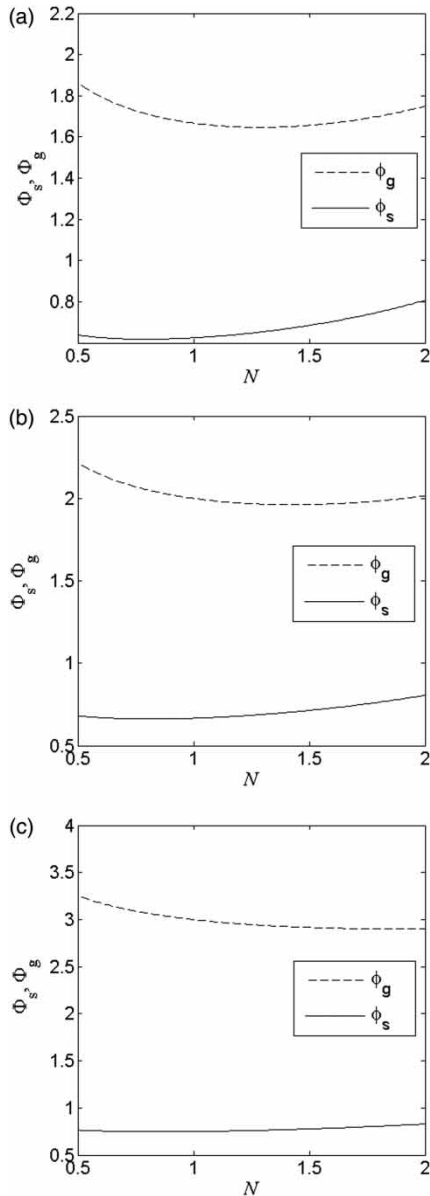
$N_{\text{concave}}$	$t_{eag}/t_e$	$N_{\text{convex}}$	$t_{eag}/t_e$
1.48	1	1.48	1
1.45	0.9	1.51	1.1
1.42	0.8	1.53	1.2
1.40	0.7	1.57	1.3
1.37	0.6	1.60	1.4
1.35	0.5	1.63	1.5

Assuming that  $m = 5/3$ , the value of  $t_{eag}/t_e$  for various profile curvatures are listed in Table 4. It should be noted that for all hillslopes, the mean slopes ( $\bar{S}$ ) are the same.

It is concluded that, in general, the travel time obtained by Equation (23) is more than that of Equation (17). Moreover, the difference between the two equations becomes greater as the degree of curvature increases in convex hillslopes or as the degree of curvature decreases in concave hillslopes.

The relationships between the geomorphic index, the shape factor, and the profile curvature of parallel planes for different flow regimes are illustrated in Figure 5, where  $\varphi_g$  is different from  $\varphi_s$  not only in terms of absolute value but also in terms of the trend of variation with profile curvature. In fact, the profile curvatures at which the minima of  $\varphi_g$  and  $\varphi_s$  occur do not coincide. While the minimum of  $\varphi_s$  occurs in a convex profile, the minimum of  $\varphi_g$  occurs in concave domain, independently of the  $m$  value.

Agnese et al. (2007) obtained minima shape factors associated with the profiles of minima travel time or the so-called ‘brachistochrones’. The brachistochrones are similar to the profiles derived in this paper, in terms of governing general relationship (Equation (17)). However, the power of relationship is  $(3 + 2m)/(1 + 2m)$  representing convex slopes. It should be noted that, although the assumptions of Saghafian & Julien (1995) and Agnese et al. (2007) models are quite different and their derived profiles of minima travel time are completely different in terms of the power, it is very interesting that FRCs of Equation (10) have the same form as the brachistochrones found by Agnese et al. (2007).



**Figure 5** | Relationship between geomorphic index and shape factor and the profile curvatures in parallel hillslopes when (a)  $m = 5/3$ , (b)  $m = 2$ , and (c)  $m = 3$ .

## COMPARISON WITH TIME OF CONCENTRATION FORMULA FOR A SERIES OF PLANES

By means of equilibrium detention storage and lag time, Overton (1971) and Overton & Meadows (1976) developed a time of concentration formula for a series of planes.

Overton's formula which is basically the same as Akan (1993) formula can be expressed as follows:

$$t_c = \sum_{j=1}^N \frac{7}{r_j^{0.4}} \left( \frac{n_j}{S_j^{0.5}} \right)^{0.6} \left[ \frac{\left( \sum_{k=1}^j L_k \right)^{1.6} - \left( \sum_{k=1}^{j-1} L_k \right)^{1.6}}{\sum_{k=1}^j L_k - \sum_{k=1}^{j-1} L_k} \right] \quad (26)$$

where  $t_c$  is the time of concentration,  $N$  is the number of planes,  $k$  represents the  $k$ th plane under consideration in the direction of flow for calculating the equilibrium outflow of the  $j$ th plane, and  $L_k =$  length of  $k$ th plane. We tried to compare the results from Equation (26) with those from Equation (14). However, it was observed that Equation (26) is strongly sensitive to the selected number of planes when applied to the curved parallel hillslopes defined by Equation (9). Therefore, it is not possible to obtain a general result using Equation (26) in this case.

## SUMMARY AND CONCLUSIONS

Travel time of surface flow is a critical parameter in most rainfall-runoff models. It is important to study the effect of the geometry and the shape of complex hillslope on the travel time. Although a number of equations have been proposed to estimate the travel time, there are only a few physically based equations which directly consider the geometry of hillslopes (profile curvature and plan curvature). In this paper, the pure effect of profile curvature ( $N$ ) on the travel time was evaluated by comparing the travel time of the plane of a given profile curvature ( $t_e$ ) with that of an equivalent straight plane under the same condition (excess rainfall intensity, mean slope, roughness, and flow regime),  $t_{e\text{Straight}}$ , using a KW-based formula (i.e., Saghafian & Julien (1995) equilibrium time model). It was concluded that the response to the question as to 'which of convex or concave slopes has a longer response time' is conditional on their profile curvatures and no general conclusion may be drawn in this regard. Results prove that the profile curvature affects the travel time in an irregular, but quite identifiable in quantitative terms, manner. In other



words, if the value of profile curvature parameters of convex and concave slopes is known, then the plane of the longer/shorter travel time can be identified.

Moreover,  $t_e/t_{e\text{Straight}}$  for convex hillslopes is always greater than unity and decreases as the degree of profile curvature increases so that for the convex hillslope with  $N = 1.5$ , the travel time is more than that of the straight hillslope by 12%. In contrast, the variation of travel time with the degree of curvature is not consistent for concave hillslopes, in that at small degrees of curvature the  $t_e/t_{e\text{Straight}}$  decreases as  $N$  increases from 1 to 1.3 (at  $N = 1.3$  the ratio reaches 98%). However, for  $N$  greater than 1.3 the trend is reversed and  $t_e/t_{e\text{Straight}}$  increases as the degree of curvature increases. At  $N = 2$ , the travel time of concave hillslopes becomes longer than that of the straight hillslopes and  $t_e/t_{e\text{Straight}}$  reaches 1.05. In general, ignoring the variation of bed slope in parallel hillslopes can result in an underestimation by 12%. The error is negligible and less than 2% for the planes with  $N$  from 0.84 to 1.8. The profiles associated with minima travel times, FRCs, for different flow regimes are derived. It is concluded that FRCs are concave, independently of the flow regime.

A KW-based geomorphic index accounting for both profile and plan curvature was derived from Saghafian & Julien (1995) equilibrium time model. The geomorphic index was compared to the shape factor already proposed by Agnese et al. (2007) in terms of the governing assumptions and the physical justification. It is concluded the two indices can result in contradictory judgments in some cases. Since, according to the evidence documented in this study, the derivation of geomorphic index is physically based without any particular assumption, it is expected that its results are reliable. It is also concluded that, due to the strong sensitivity of the time of concentration formula for a series of planes to the number of planes, use of this formula does not lead to a general result for curved parallel hillslopes.

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