The difference engines

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This is a survey paper which deals with the development of the difference engines from their conception by Müller in 1786, through the attempts of Charles Babbage, their realisation by the Scheutz team, and their final practical use by Comrie in the period before the Second World War. The circumstances behind the development of the most famous engines are described in moderate detail, while the machines of secondary importance are mentioned in an attempt to cover the overall development of these special purpose devices.

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The late eighteenth century saw the start of the large scale publication of mathematical tables. These tables spanned the whole spectrum from simple addition and subtraction tables, to tables of logarithms of over 20 digits. Whatever the content of the tables, two factors were always present:
1. The tables were intended to reduce the labour of calculation
2. The tables were full of errors.

Because of the inherent difficulty of some of the calculations he might be called upon to perform, it was not uncommon to find a scientist of the time having a private library of over 125 volumes of tables of different kinds. A survey done in 1835 of one scientist's library showed it to contain 140 volumes of arithmetical and trigonometric tables; of these a sample of 40 volumes was taken and found to contain over 3,700 known errors (Lardner, 1834).

The mathematicians of the late eighteenth century were well aware of the fallibility of their tables and several studies were started to find a method to remedy the situation. The technique most often used was that of publishing a table of errata; however this did not always solve the problem as some of the errata were known to have a higher percentage of errors than the original tables. The only foolproof method of cutting out the inaccuracies arising during the computations was for them to be done by automatic machinery rather than by hand.

The method of differences, although once the main tool of all table makers, has, of late, fell into disfavour. Thus a few words about the method itself might well be in order for the majority of readers. If a function, such as $F(x) = 2x + 3$, is evaluated for successive values of $x$, and then the difference noted between each adjacent value of $F(x)$, one finds:

\[
x = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad \ldots
\]

\[
F(x) = 5 \quad 7 \quad 9 \quad 11 \quad 13 \quad 15 \quad 17 \quad 19 \quad 21 \quad \ldots
\]

\[
\text{differences} = 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad \ldots
\]

If the function was $F(x) = x^2 + 2x + 3$ then it would have been necessary to obtain the differences of the differences (or second differences) before a constant difference was obtained. For example:

\[
x = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad \ldots
\]

\[
F(x) = 6 \quad 11 \quad 18 \quad 27 \quad 38 \quad 51 \quad 66 \quad 83 \quad 102 \quad \ldots
\]

\[
\text{first difference} = 5 \quad 7 \quad 9 \quad 11 \quad 13 \quad 15 \quad 17 \quad 19 \quad \ldots
\]

\[
\text{second difference} = 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad \ldots
\]

In general, if the polynomial to be evaluated has a term of $x^n$ in it, then it will require the $n$th difference to be taken before a constant is obtained. If one has to evaluate a polynomial for many values of $x$ (such as when computing tables) it is easier to do it by adding the constant difference to the difference above it, then add that difference to the one above it, etc. until the value of the function is reached. This results in a procedure in which only additions are performed, rather than the many multiplications which would have to be done if the function itself was evaluated for each value of $x$.

Although all polynomials have a constant difference, functions such as logarithms and the trigonometric functions do not, in general, have this property. Thus to produce tables for these functions, by difference methods, it is necessary to approximate them by some polynomial function and then evaluate this polynomial.

A difference engine is simply a machine which is capable of both storing a series of numbers and performing additions with these numbers. The numbers will represent the function value, its first difference, second difference, third difference, etc. By performing a series of additions on these numbers, the engine is capable of generating successive values of the function.

At Frankfurt in 1786 a Mr. E. Klipstein published a small book whose title translates as: 'Description of a Newly Invented Calculation Machine' (Klipstein, 1786). This book describes the operation of a mechanical calculator invented by J. H. Müller who was a Captain of Engineers in the Hessian Army.

The book contains an appendix in which Müller describes a much more ambitious calculating machine which he could construct if only someone would provide the finances. This calculator was to be a difference engine operating from a constant third difference. The device was designed to print out its results on a piece of paper. Müller figured that his device would be capable of one addition per second and that a table of the cubes of the integers from 1 to 100,000 could be produced by a 'common labourer' in about 10-5 days. Although it appears that his plea for financial aid was never answered, it is certainly the case that he deserves recognition as the one who first published the basic idea of a difference engine.

The English machine

Charles Babbage had a passion for accuracy. He was continually being distressed by the errors he found in the mathematical tables available during his lifetime. He was responsible for one of the earliest sets of accurate logarithm tables, which he published in 1827 (Babbage, 1827). His passion for accuracy passed into most other aspects of his life and he was able to combine it with his great inventive genius to produce a number of worthwhile studies and inventions. One of these was, of course, a difference engine.

The origin of Babbage's ideas is obscure. In his *Passages from the Life of a Philosopher* he states that the idea of a difference engine came to him, while he was still a student at Cambridge, in 1812 or 1813 when:

'One evening I was sitting in the rooms of the Analytical Society, at Cambridge, my head leaning forward on the table in a kind of dreamy mood, with a table of logarithms lying open before me. Another member, coming into the room, and seeing me half asleep, called out, "Well Babbage, what are...
you dreaming about?" to which I replied, "I am thinking that all these tables (pointing to the logarithms) might be calculated by machinery" (1864, Chapter V; reprinted in Morrison and Morrison, 1961, page 33-34).

Babbage admits that he does not remember this incident, which was related to him much later by a friend. Thus the idea of a difference engine probably came about a few years later during discussions with the astronomers Wollaston (Babbage, 1864) and Herschel (Collier, 1970), both of whom were close friends. It is almost certain that Babbage had no knowledge of Müller's proposal for a difference engine made more than a quarter of a century previously.

In the early 1820's, spurred by his experience at publishing his table of logarithms, he took up the challenge of attempting to design a working machine to both compute and print a set of mathematical tables. The mechanism had to be able to set the type for the printing of the tables, for, as Babbage was well aware, it was possible to introduce many errors in going from the manuscript copy to the final galleys of print. Being of moderate independent means, he was able to devote full time to this exercise and, by 1822, was in a position to show a working model of a difference engine to his friends. He had constructed a machine which was capable of working with 6 figure numbers and capable of evaluating any function having a constant second difference. Babbage wrote to Sir Humphry Davy, the president of the Royal Society, on July 3, 1822, and described his machine as:

'producing figures at the rate of 44 per minute, and performing with rapidity and precision all these calculations for which it was designed' (paraphrase by Weld, 1848), letter printed in full in (Morrison and Morrison, 1961) and (Babbage, 1889).

The main purpose of this letter was to seek the aid of the Royal Society in a petition to the government for financial aid in the construction of a full scale difference engine.

The Royal Society was asked by the government to look into the project and so appointed 12 of its members to prepare a report. By May of 1823 they were able to send a letter to the Lords of the Treasury stating that:

'they consider Mr. Babbage as highly deserving of public encouragement in the prosecution of his arduous undertakings' (Weld, 1848).

This report was not entirely unanimous; a certain Dr. Young was of the opinion that any money given to Babbage would be better used by investing it and using the dividends to pay human calculators.

By July 1823 (shortly after Babbage had been awarded the Astronomical Society of London Gold Medal for his idea of the difference engine (Babbage, 1889)) the Chancellor of the Exchequer agreed to advance £1,500 for the project, and Babbage agreed to put up between £3,000 and £5,000 out of his own private fortune. This was not a stroke of generosity on his part, rather it was to cover what he thought would be the total cost of the detailed design and construction of the machine and the government would, when the machine was complete, reimburse Babbage for the money he had spent. It was thought that the machine would be ready for practical use in two to three years.

Babbage quickly found out that it was one thing to make a few model parts in a basement workshop to demonstrate his ideas, and quite another to make machinery sufficiently advanced to execute the many highly complex movements required of his design. He set about to examine all the major manufacturing workshops, and came to the conclusion that, before he could attempt to construct a difference engine, he had to spend some of his resources in attempting to advance the art of construction itself. During the next few years he would carefully design each part, and then design and construct the tools for making this part. The two fold operation would, as often as not, suggest alternate and often simpler mechanisms for either the object or the tools needed to make it, and the whole process would be repeated for this new version. By this process Babbage and his staff of engineers advanced the tool making trade faster than it had ever been known to move before. The training he gave to the men working for him was, in time, disseminated throughout other British workshops and, even though he never managed to complete his full design, the money advanced by the British government was well repaid by the by-products of the difference engine development. These by-products included Babbage's system of mechanical notation, devices which were later used in the cotton spinning industry, and a large number of new tools used in machine shops (Babbage 1851; Lardner, 1834).

The constant effort of design, tool making, and redesign soon took its toll. In October 1827 (four years after starting the construction) Babbage's health broke down. His doctors advised him to go to a warmer climate to recover, so he moved to Italy for a time. While there, he had a chance to go over his accounts and discovered that he had spent £3,475. He therefore set about petitioning the government to recover the £1,975 that he had put out of his own pocket. Unfortunately the Chancellor of the Exchequer had forgotten about his verbal agreement to refund the money that Babbage had spent, and it required the intervention of several of his influential friends to get the Government to act. After requesting the Royal Society to make another report on the progress of the difference engine, and receiving one which highly praised all Babbage's efforts, the Chancellor of the Exchequer agreed to advance a further £1,500. A further appeal by some of Babbage's friends to the Duke of Wellington (then Prime Minister) resulted in a further £3,000 and a suggestion that Babbage 'get on with it' (Babbage, 1864).

These financial troubles were a constant source of concern to Babbage. He was always very scrupulous in all his dealings, getting all his accounts audited by the committee of the Royal Society before presenting them to the Government. Unfortunately this procedure introduced long delays between the expenditure for materials and the money being forthcoming from the Treasury. He managed to keep the project moving by spending his own money and then attempting to claim it back. There were, however, several occasions when, through lack of his own funds, and government ministers holding up their funds while debating the future of the project, he had to stop work altogether. During these enforced periods of idleness, some of which lasted up to four years, he had to let most of his engineering staff go and then, when more money was forthcoming, hire and train new ones.

It was during one of these breaks in production that Mr. Clement's men worked the first model for Babbage (and had been kept on through thick and thin), made some demands concerning the uncomfortable working conditions in a new fire-proof building the Government had erected to house the partially completed engine. Babbage refused to agree to Mr. Clement's demands and the two men broke off their association. This was particularly unfortunate because, under British law, mechanics possess the right of property to all tools they have constructed, even if the cost of their construction has been paid by their employers. Mr. Clement exercised his right and removed all the tools Babbage had designed to help in the construction of his dream (Babbage, 1889). He also removed, but later returned, all the mechanical engineering drawings Babbage had made. In fairness to Mr. Clement, he did attempt to come to some settlement, but Babbage refused to have any further dealings with him, even if it meant losing all his tools (Weld, 1848).

It was during the absence of all his engineering drawings that Babbage, while attempting yet another modification of the design, conceived the idea of his famous analytical engine. He realised at once that, if he was ever to construct such a machine, it would require a much more sophisticated arithmetic
mechanism than that present in the design for the difference engine. Accordingly he set about further experimentation with mechanical adding devices. After designing and testing over twenty different mechanisms he produced one which he thought could not be improved. He informed the Government that, in his opinion, it would take less time and money to construct a difference engine to his improved design than it would to finish the old one (Babbage, 1864, page 90).

The news of yet another redesign was not received warmly. Rather than come to an immediate decision on the future of the difference engine, the ministers asked Babbage to wait for their further deliberations. During these deliberations a general election was called, the government lost its majority, and Babbage had to start all over again in dealing with a new group of ministers. After further delays, this time lasting a total of nine years, an appeal was made to the Prime Minister to decide one way or the other. As a result of this appeal Babbage received a letter on 3 November 1842 informing him that Sir Robert Peel had decided to abandon all government support and offered to let Babbage keep all the drawings, tools and parts of the machine which then existed. However, mindful of his original agreement of over twenty years ago, he reminded the government that the machine was their property and that they should be responsible for its future. The partially completed machine (Fig. 1) and its drawings were then given to the Museum of King’s College, London, where they remained for a further 20 years, after which they were removed to the Science Museum at South Kensington where they are presently on display. Most of the rest of Babbage’s sample gears, wheels, shafts, etc. were sold for scrap or melted down when Mr. Clement’s nephew took over the engineering firm (Babbage, 1889, page 341).

After a final accounting, it was determined that the project had cost the British taxpayer a total of £17,000, of this amount only £11,000 to £12,000 were actually spent on the machinery while the rest went for the purchase of land and erection of the fireproof buildings (Babbage, 1889, page 340). Although it is not known exactly, it is estimated that Babbage had contributed a further £20,000 from his own resources. When Sir Robert Peel was asked, in the House of Commons, to explain the action of his government, he turned the question aside by a joke about how ‘the machine should be set to calculate the time at which it would be of use’ (DeMorgan, 1848).

Although Babbage’s descriptions of the machine are difficult to follow, Lardner (1834) has described, in general terms, how the completed engine would have looked and its mode of operation. The entire mechanism would have been about ten feet high, ten feet wide and five feet deep (Buxton, 1933). It was to be composed of several vertical steel axles, each of which would carry eighteen brass wheels about five inches in diameter. Each vertical axle would represent one of the six orders of difference while the seventh axle would represent the value of the function being computed. These values were represented on the axles by the positions of the eighteen brass wheels. Each wheel was engraved with the digits from 0 to 9 around its circumference and thus, by simply turning the wheels so that they displayed their various digits, any 18 digit number could be represented on each vertical axle (the units on the lowest wheel, the tens on the next etc.). This arrangement was decided upon because the friction generated by the gears and wheels was less than it would have been for a horizontal arrangement of the numbers.

Behind the first set of vertical axles, which contained, besides the number wheels, the mechanism for ten’s carries, was a second set of vertical axles supporting the mechanism which could perform the addition of numbers from one column to the next. Behind these were yet another set of seven axles which served to engage and disengage the adding mechanism, as and when required.

The engine was to operate in four distinct cycles, each one corresponding to a $\frac{1}{4}$turn of the drive wheel. The first cycle caused the numbers represented by the first, third and fifth difference axles to be added to the numbers stored on the result, second and fourth difference axles respectively. The second cycle took care of any ‘carrys’ which might have resulted from the first cycle additions. The third cycle caused the addition of the second, fourth and sixth differences to the first, third and fifth while the last cycle again took care of any carries generated.

The system was designed with several fail-safe devices to protect the machine and to stop it generating errors. If one of the wheels got slightly out of place, it was forced back into its exact position by a system of springs and pins. If the malfunction was severe enough to prevent this automatic readjustment, then the machine would simply lock up and prevent any further computations taking place until after the difficulty had been found and corrected.

There were a number of ingenious additions to the basic system which would allow the computation of tables which did not have a constant difference. For example, to calculate a table of logarithms, it was necessary to approximate the logarithm function by a polynomial, evaluate this polynomial for about 100 values, then determine a new polynomial for the next portion of the table. The mechanism could be set so that, after a predetermined number of calculations, a bell would ring to inform the operator that it was time to reset the difference wheels for the new polynomial. There were also several adjustments which would enable the device to add any difference (any number of times) to a difference of any other order. This enabled the computation of tables which did not have a constant difference but whose fourth difference (say) was some form of geometric series, or tables of astronomical data which fitted observed phenomena but whose analytical solution was not known. A series of comparison units were attached to each axle.
which would be able to detect when any column contained a specific value. These would then either cause an automatic change (positive or negative) in the constant (sixth) difference wheels, or ring a bell informing the operator that it was time to make such a change.

Although eighteen digit numbers were sufficient for most computations, Babbage foresaw that there could be troubles with accumulating round off errors. Consequently he ensured that the addition mechanism would correctly round off the eighteenth digit whenever necessary. There was also an adjustment which would allow two columns to act in unison to enable numbers of up to thirty digits to be used when required. This latter adjustment would, of course, reduce the machine to operating on a constant third difference.

The printing mechanism was to consist of a series of cam-like devices attached to the wheels representing the final result. These acted against levers, each of which was raised to one of ten different positions corresponding to the digits 0 to 9. The other end of each lever moved an arm containing ten steel punches (one for each digit); these in turn were driven onto a copper or lead plate and left an impression which could then be used to cast a stereotyped plate for the printing press. The copper or lead plate was automatically shifted up one place with each turn of the drive wheel.

In 1849, while working on the plans for his analytical engine, Babbage took time off to draw up a complete set of plans for his new improved difference engine. These plans, which are now in the Science Museum, were the most complex mechanical drawings produced up to that time, covering over one thousand square feet of paper. He presented these plans to the government in the hope that they might one day decide to have it constructed. The Chancellor of the Exchequer, however, described the plan as:

'indefinitely expensive, the success problematical, and the expenditure incapable of being calculated' (Babbage, 1864).

Thus the difference engine was put to rest. It is an irony of history that only a few years were to elapse before the government was to finance the construction of a difference engine designed in Sweden.

The Swedish machine
In 1834 a Dr. Dionysius Lardner wrote an article for the Edinburgh Review describing Charles Babbage’s attempt to construct a difference engine (Lardner, 1834). A Swede, George Scheutz, who, as editor of a Stockholm technical journal, made it his business to keep informed on such matters, saw at once that the machine Dr. Lardner described would be a great help to virtually every branch of science. Dr. Lardner’s article was designed to be read by the general public and was therefore only a general description of what a difference engine should do, and not how it was designed. Scheutz was fascinated by the concept and, rather than write to Babbage asking for details of its construction, set out to design one for himself.

He started out by designing the basic components of a machine and then constructed several models in wood, pasteboard and wire. After verifying that his design was practical, he was forced, through pressure of work, to set aside the construction of a proper machine. In the summer of 1837 his son, Edvard, returned from the Royal Technological Institute where he was an engineering student. Having been inspired by his father, Edvard set about the construction of a working machine from metal. Towards the end of October they were in a position to see that the construction of a fully working model was beyond their financial means. Accordingly they made application to the Swedish Government for a grant to aid them in their work, but, perhaps because of the British experience in these matters, they were refused (Scheutz and Scheutz, 1857). The Scheutzes continued to spend what time and money they could afford and, by 1840, had completed a small machine which operated from only the first difference with each of the two registers capable of holding a five figure number. By 1842 they had extended it to a machine capable of dealing with three orders of differences. Finally in 1843 the printing device was attached and the whole mechanism was submitted to the Royal Swedish Academy of Sciences for their approval.

Although the machine was well received by the Academy, and a certificate issued by them recommending the device, nobody appeared interested in ordering a complete machine, despite the fact that orders were solicited from several different countries (Anon, 1866).

In 1851, George Scheutz, encouraged by friends, again asked his Government for a grant to enable the full machine to be constructed. The Swedish Ministers, again following the British example, referred the request to the Swedish Royal Academy which produced a very favourable report and the suggestion that money be made available. However, the Government again decided that no public funds were available for the project. Later that same year a member of the Diet (the Swedish Parliament) put forward a motion which would supply the Scheutzes with a grant, but only after the Swedish King had, on due examination, given it as his opinion that the machine was complete and answered its purpose. The inventors now tried to obtain the money in advance, and, towards the end of 1851, the Government relented and advanced the money on the condition that fifteen of the Scheutzes’ friends (all members of the Swedish Academy of Sciences) guarantee it if the machine was not finished to the King’s satisfaction before the end of 1853.

The difference engine (actually called the Tabulating Machine by the Scheutzes) was completed in October 1853 (Fig. 2). It differed slightly from the initial model. Various improvements were incorporated to enable it to do computations, not only in the decimal scale, but also in the sexagesimal scale (Anon, 1855). These changes, which were to prove valuable in calculating hour and angle related tables, were set up in working drawings by Edvard Scheutz and the final machine constructed by the engineering firm of Bergström in Stockholm. In a spirit of generosity not often found in these matters, the Royal Academy suggested to the King that, as expenses had been heavy, a second grant, equal to the first, should be awarded. His Majesty agreed and, by 1854, the Scheutzes had produced the world’s first, fully working, nontrivial difference engine and had been partially compensated for their expenditure by the Government parting with the princely sum of £560 (Archibald, 1947).

Back in London, an engineer named Donkin came to hear of the progress being made in Sweden. Mr. Donkin had a long
association with Babbage and his ordeal. In fact, he had been on both the committees of the Royal Society which judged Babbage's progress and had also been one of the three members of the Royal Society who audited all of Babbage's accounts.

In 1854 Mr. Donkin was instrumental in having the Swedish machine shipped to London and bringing it to the notice of the Royal Society. Babbage, after inspecting it, gave his unreserved praise to both the machine and its inventors. The Scheutzes, in return, never concealed their admiration of Babbage for his early work on difference engines.

After residing in London for almost a year, showing the machine to the scientific community and hoping to find a buyer for it, the Scheutzes shipped it to Paris to be exhibited in the Great Exhibition then being held. The jury, composed of men from all over Europe, unanimously awarded it the Exhibition's Gold Medal. This award brought it to the notice of Professor B. A. Gould of the Dudley Observatory in Albany, New York. He, in turn, managed to convince a local philanthropist to purchase the device for £5,000 (about £1,000 at the time) and present it to the Observatory (Babbage, 1864, page 48; Anon, 1946). After remaining idle in the Observatory for 68 years (Archibald, 1947a), it was sold to a Dr. Dorr E. Felt, the inventor of the Comptometer, for his private collection of calculating devices. The explanation for the fact that the engine was ignored during its life in the Dudley Observatory may be found in a remark made by a member of the Observatory staff:

“Our past experience has led us to believe that such a machine would be greatly enhanced in value by the addition of one or two more orders of differences...” (Anon, 1866).

The Swedish Difference Engine No. 1, as it is officially called, now finds a home in the museum of the Smithsonian Institute, Washington, D.C.

The general organisation of the components was similar to that envisaged by Babbage. It was constructed so that each register could contain a fifteen digit number. These were represented by a series of fifteen circular silvered brass rings aligned in a horizontal row. There were five rows of rings, four for the four orders of differences and one for the final result. The entire mechanism was mounted on fifteen vertical steel axles. The most interesting aspect of the construction was that the figure rings were not actually attached to the steel axles, rather they simply sat on small platforms with the axles running through them. This method of construction did away with the need for special devices to stop the figure rings from rotating too far, and friction between the rings and its supporting platform stopped any excess rotation.

Fig. 3 shows a model made by the Scheutzes to demonstrate the workings of their machine. The steel axle (marked A) and the platforms (marked B) upon which the numbered rings (marked C) rest, are all easily seen. A bit of imagination will help to see that the rings (C) were turned by the trigger pieces (D) which came into gear with the cogs (E) whenever the revolving shaft brought the tail piece (F) of the trigger (D) in contact with the rising piece (G) on the ring. The circular ring (C) had a projection which, when a carry was generated by the ring turning from 9 to 0, contacted the lever (H) which set up the mechanism so that the next ring would be turned one place (the actual carry being done by the bar, which was driven back and forth in front of the figure wheels by a chain, which is easily seen in Fig. 2).

The printing mechanism, although different in detail to Babbage's was similar in its mode of operation, both having modelled on the mechanism used to control the striking of the hours in chiming clocks (Babbage, 1889, page 250, Anon, 1866). It was positioned at the top and slightly behind the main calculating part of the engine. Although the machine was capable of computing with fifteen figure numbers, the typesetting mechanism would round off each number to eight figures before creating the mould from which the type would be cast. The principle employed by Babbage of first adding the even orders of differences to the odd, then vice versa, was repeated in the Scheutz design (Anon, 1855).

When being worked by an experienced operator, it could generate 120 lines of a table in each hour (Anon, 1866). In one experiment it generated logarithms of the numbers 1 to 10,000 in just 80 hours, including the time taken to reset the differences for the 20 different approximating polynomials that were used (Scheutz and Scheutz, 1857).

In the middle 1850's the British Registrar General, the man responsible for the collection and publication of vital statistics, decided that it was time to produce a new set of tables for the insurance industry. It was found that the required tables could, for the most part, be easily approximated by polynomials. This naturally led to the conclusion that a difference engine should be used in the calculations (Farr, 1864: preface). The Registrar General arranged for the British Government to put up £1,200 towards the cost of constructing a calculating machine. He had the support of the Astronomer Royal in his application for this money—it is tempting to assume that this support was given so that the Astronomer Royal would have access to the same facilities as the Dudley Observatory. Mr. Donkin's engineering firm agreed to the price of £1,200 (a sum which did not come close to the actual cost (Anon, 1870)) for constructing a second difference engine to the Scheutz design.

This Swedish Difference Engine No. 2 was, in most respects, identical to the previous one. It consisted of just over 4,000 separate pieces (only about 1,000 pieces were actually used in the mechanism, the rest being nuts, bolts, screws, and links in the chain drive), weighed about 1,000 pounds, and was the size of a small upright piano. Of all the early difference engines,
this one was given the most use. During its working life it computed and stereotyped over 600 different types of tables, 238 pages of which were for the original insurance industry project (Anon, 1870). Although it undoubtedly eased the production of these tables, it was not without its faults. As Mr. W. Farr (1864), the editor of the English Life Tables, noted:

'The Machine required incessant attention. The differences had to be inserted at the proper terms of the various series, checking was required, and when the mechanism got out of order it had to be set right. . . . its work had to be watched with anxiety, and its mathematical music had to be elicited by frequent tuning and skillful handling, in the quietmost congenial to such productions.' (Farr, 1864).

This second Swedish engine was eventually given to the Science Museum where it joins Babbage's early models in a display of computers and calculating equipment.

**The other machines**

There were several other attempts, both professional and private, to construct difference engines. For example, a Mr. Alfred Deacon of London was inspired by the same article in the Edinburgh Review that started Scheutz on his efforts. Mr. Deacon attempted to construct a small model, now lost, capable of storing three orders of differences with the numbers being kept to twenty digits (Scheutz and Scheutz, 1857). Because the machine was intended simply as a model, he did not incorporate any form of printing mechanism. He built the device entirely for his own satisfaction and, as far as can be deduced, it was only shown to a few friends (Babbage, 1864). It is possible that Babbage acquired Mr. Deacon's machine, for he offered to loan

'a small difference engine, made in London, in consequence of its author having read Dr. Lardner's article in the Edinburgh Review' (Babbage, 1889, page 197)

for the great Exhibition of 1862.

Another Swede, Martin Wiber, redesigned the Scheutz machine and, in so doing, managed to reduce its size and weight (Archibald, 1947b). His device was first used to publish, in 1860, a set of interest tables. He was not happy with the style of these tables and spent the next ten years redesigning the print mechanism. In 1875 he published a set of tables of seven-place logarithms of numbers from 1 to 100,000, logarithms of sines, cosines, tangents and cotangents, all produced on his engine. Wiber's machine eventually ended its days in the possession of the academy of sciences in Paris (Delaunay, 1863; Jacob, 1911, d'Ocagne, 1905; Anon, 1946).

Mr. G. B. Grant (1871), who eventually became the founder of the American gear cutting industry, was engaged in the late 1860's in computing a series of tables for 'cut and fill' problems when he first thought of the possibilities of a machine to do the same job. He had, at that time, not heard of any of the previous attempts and, after experimenting for a short time, gave up the idea. Grant, who was a student at the time, was encouraged by his professors and with the help of Mr. J. N. Bachelber, who was in charge of the Scheutz machine at the Dudley Observatory, was able to design and build a small model. Grant's professors managed to arrange for a sum of $10,000 (about £2,000 at the time) to be used in building a full scale machine which, when completed, was to be given to the University of Pennsylvania. This machine, which weighed over a ton, stood 5 feet high by 8 feet long and contained 15,000 parts (Hawkes, 1971), was exhibited at the Philadelphia Centennial Exhibition of 1876 (Chase, 1952). It drew praises from the Exhibition's judges as 'being the finest machine of its kind'. A machine to Grant's design was sold to The Provident Mutual Life Insurance Company who used it to produce tables similar to those computed by the Office of the Registrar General in Britain.

Mr. Percy E. Ludgate (1915), an Irish accountant, is known to have designed several calculating machines during the early part of this century (Randell, 1971). Although his interest was mainly directed towards 'analytical engines', it is known that he designed at least one machine whose main purpose was to act as a difference engine operating to sixteen orders of differences. The lack of funds and his early death prevented him from ever attempting the construction of these machines and it would appear that all his personal papers and designs have been lost.

The only other serious design was proposed by the French mechanical genius, Leon Bollee. Bollee's youth was highlighted when, at the age of 18 (in 1887), he designed and built the first machine which could directly multiply two numbers together. Bollee later turned his attention to the developing automobile industry and, after his early death, there were discovered among his papers the complete plans for a difference engine capable of operating to 27 orders of differences—by far the most extensive machine of its type ever envisaged (Jacob, 1911; d'Ocagne, 1915).

Although Grant's was the last 'large' difference engine actually constructed, the technique did not die out. In 1890 the Royal Prussian Academy of Sciences and the Imperial Academy of Sciences of Vienna sponsored two German scientists, J. Bauschinger (1910) and J. Peters, in their efforts to produce a new set of eight figure logarithmic and trigonometrical tables. Bauschinger and Peters contacted a Mr. Hamann in Berlin who had become famous for his desk calculators, and asked his company to design and build a small machine which would act as a difference engine capable of operating from a constant second difference and printing the results of its computations (Comrie, 1928). The device was complete in 1909. It consisted of three separate components, one for holding the value of the second difference, one for the first difference and one for the function value and the printing mechanism. It was operated by turning one crank (marked (a) in Fig. 4) to add the second difference to the first, then turning another crank (marked (b)) to add the first difference to the function value and print this result out on a strip of paper. This machine, now unfortunately lost, was capable of producing one function value every ten seconds and was used to prepare tables which were published in 1910 (Bauschinger, 1910). These tables and their derivatives were, because of their accuracy and convenient layout, one of the standard reference works in the earlier part of this century.

The period between the two World Wars saw the manufacture and distribution of many different kinds of desk top calculating and accounting machines. It did not take long for scientists to realise that these machines, with slight modifications, could become the modern version of Babbage's engine (Comrie, 1946).
method of differences (Comrie, 1946). In 1931 he recognised that the National Accounting Machine (produced by the National Cash Register Company) would provide an ideal basis on which to construct a new difference engine (Fig. 7). The basic ‘National’ consisted of an adding machine with twelve column keyboard and six separate registers designed for accumulating subtotals. The features which made the machine so valuable were that any number set on the keyboard could be entered into any of the six registers, numbers already stored in the registers could be transferred to any of the others, and the machine would print the result of each computation. The ‘National’ had a movable carriage which, as it moved from left to right, contacted various ‘tab-stops’, which would activate or deactivate the various registers. Thus, as the carriage moved from left to right, the stops would engage the correct registers to cause the first difference to be added to the function value and the result printed, the next stop would cause the second difference to be added to the first and the result printed, etc. Comrie’s description of this mechanism before the Royal Statistical Society (Comrie, 1936) made it clear that, almost 100 years after Babbage’s failure, an efficient, inexpensive difference engine had, at last, been produced.

The introduction of ledger-posting machines gave a great advance to the calculation and printing of mathematical tables. Although some tables had been produced by specially made difference engines, it was Comrie’s discovery that inexpensive commercial accounting machines could be made into difference engines which entirely revolutionised the art of table making. These machines were not a lot faster than the specially made difference engines (the Burroughs Class II ledger machine used by H.M. Almanac Office in the early 1930’s was capable of producing a list of argument, function—thirteen significant figures, first and second differences, at the rate of 500 to 600 lines per hour (Comrie, 1932a)), but the reliability and low cost of these devices enabled a lot of computation which, otherwise, would never have taken place. These machines were the work horses of the scientific table makers until the advent of the modern computer.

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